# Introduction-continued

R. Chandrasekaran*

Rasmussen (Chapter1)
Key ingredients in a Game: Players, Actions, Payoffs, Information (PAPI)

**Definition 1** *A **strategy** for a player in a game consists of a plan that selects actions depending on information that is available at each instant and each possible situation (whether or not the situation actually occurs).*

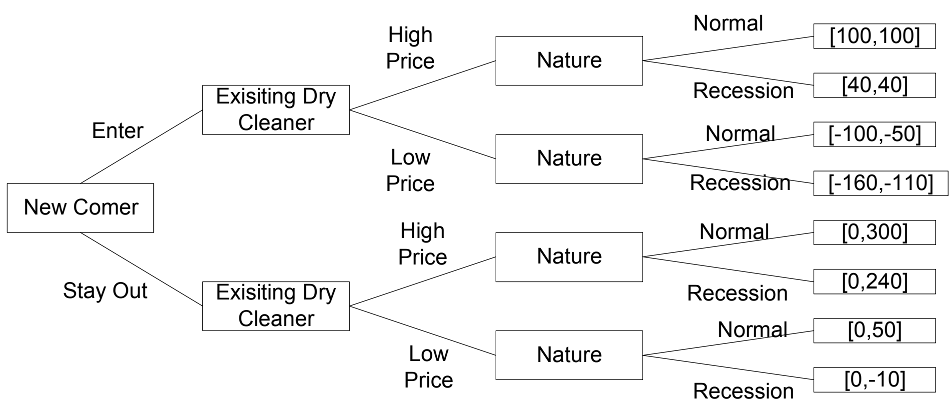
**Definition 2** *A **player** in a game is an individual or entity consisting of a set of individuals who act in unison. Each player's goal is (assumed to be) to maximize his utility by the choice of his actions.*

In some games, nature is also considered as one of the players; nature only makes random choices among the set of actions possible at specified points in a game. It is assumed that the corresponding probability distributions are known to all players when nature acts.

**Example 1** *"Dry Cleaner Game": (Rasmussen): In a town there is an existing dry cleaner and one possible new entrant. The new guy wants to know if he should enter this business in this town. So he has two possible choices; enter or stay out. The existing guy has also two choices: continue his (high) price or enter into a price war with the new guy (low price). Moreover, the economy has its own uncertainties: there my be a recession. This choice is controlled by nature. The probability of recession is 0.3 and no recession is 0.7 and this is know to all (and that everyone knows this is known to all and so on.) More rules: 1) First the new comer decides whether to enter or not; 2:Then the existing cleaner decides whether or not to lower price; and 3)nature decides whether there is recession or not. All this is known to all.*

*School of Management, University of Texas at Dallas, Richardson, Texas, U.S.A..

*This information together with the corresponding payoffs is shown below in the game tree:*




$Tree.wmf$ Game Tree

*Please note that this tree is a directed tree with a root and all paths are directed away from the root. The numbers at the leaf nodes are payoffs. This form of representing a game is called an **extensive form**. In this game, each node is "assigned to or controlled by" some player. Each of the players knows which nodes belong to him. Since the rules prescribed that the new guy makes his move first and the existing guy knows the move made by the new guy when it is his turn to move, in this game existing player has full knowledge during his move. This may not be the case in general. This brings us to the concept of an "information set". An information set belongs to one player and consists of all nodes that correspond to a move of this player. The player does not know which of these nodes he is in at that time. So the set of alternatives available to this player are the same in each of these nodes. If each informations set of a player has only one node, this player is said to have **perfect information**. If every player has perfect information, then the game is said to have perfect information.*

**Definition 3** *A strategy $s_i$ for player $i$ is called a **best response** or **best reply** to a combination of strategies $s_{-i}$ of all other players if*

$$\pi_i(s_i, s_{-i}) \geq \pi_i(s_i', s_{-i}) \qquad \forall s_i' \neq s_i$$

*It is **strongly best** if no others are equally good; **weakly best** otherwise.*

**Definition 4** *A strategy profile in which each player's strategy is the best response to the combination of other players' strategies is called a* ***Nash (Cournot) equilibrium****.*

**Definition 5** *A strategy* $s_i^d$ *is* ***dominated*** *if it is strictly inferior to some other strategy (this must be a single strategy not a combination of strategies) no matter what strategies other players choose; i.e if* $\exists s_i'$ *such that*

$$\pi_i(s_i^d, s_{-i}) < \pi_i(s_i', s_{-i}) \qquad \forall s_{-i}$$

**Definition 6** *A strategy is* ***dominant*** *if all other strategies are strictly dominated by it.*

**Definition 7** *A profile of strategies in which each component is a dominant strategy for that player is called an* ***dominant equilibrium****. If a dominant equilibrium exists, it is unique. In this case it is the predicted solution.*

**Definition 8** *A strategy* $s_i'$ *is* ***weakly dominated*** *if there exists a strategy* $s_i''$ *such that*

$$\begin{aligned} \pi_i(s_i'', s_{-i}) &\geq \pi_i(s_i', s_{-i}) \qquad \forall s_{-i} \\ \pi_i(s_i'', s_{-i}) &> \pi_i(s_i', s_{-i}) \qquad \textit{for some } s_{-i} \end{aligned}$$

*The notion of a weakly dominant strategy, and that of a weakly dominant equilibrium follow along similar lines.*

**Example 2** *Consider a 2 player game in which each player has 2 strategies given in normal form as below;*

| | $L$ | $R$ |
|---|---|---|
| $U$ | $[1, 1]$ | $[-1, 2]$ |
| $D$ | $[2, -1]$ | $[0, 0]$ |

*This table is to be read as two matrices one for each player:*

| | $L$ | $R$ |
|---|---|---|
| $U$ | 1 | $-1$ |
| $D$ | 2 | 0 |

*is player 1's matrix and*

| | $L$ | $R$ |
|---|---|---|
| $U$ | 1 | 2 |
| $D$ | $-1$ | 0 |

*for player 2. It should be clear that D dominates U for player 1 and R dominates L for player 2. So there is a dominant equilibrium in this example and it is the predicted result. it is easy to verify that this is also a Nash equilibrium.*

**Example 3**

| | $L$ | $R$ |
|---|---|---|
| $U$ | $[1, 3]$ | $[4, 1]$ |
| $D$ | $[0, 2]$ | $[3, 4]$ |

*U dominates D for player 1. But between L and R there is no domination. However, if we remove D for player 1 since it is dominated by U (and player 2 knows that player 1 does this), then we are left with the following table:*

| | $L$ | $R$ |
|---|---|---|
| $U$ | $[1, 3]$ | $[4, 1]$ |

*and now we see that L dominates R for player 2. The pair [U,L] is the result of (repeated) elimination of dominated strategies. It is also a Nash equilibrium; there was no dominant equilibrium in the original table.*

A dominant equilibrium is arrived at even when each player does not know the thought process of others. Here, however, player 2 needs to know that player 1 will not play a dominated strategy. This process is known as *iterated elimination of dominated strategies.*
Drawbacks: (i) requires a larger amount of *common knowledge.* (ii) imprecise prediction.

For the second of the drawbacks consider:

**Example 4**

| | $L$ | $C$ | $R$ |
|---|---|---|---|
| $U$ | $[0, 4]$ | $[4, 0]$ | $[5, 3]$ |
| $M$ | $[4, 0]$ | $[0, 4]$ | $[5, 3]$ |
| $B$ | $[3, 5]$ | $[3, 5]$ | $[6, 6]$ |

*There is no domination and so no iterated domination either. Hence no dominant equilibrium. But [B,R] is a pareto-optimal Nash equilibrium and so it is the predicted outcome in this example!*

**Example 5**

| | *Mum* | *Fink* |
|---|---|---|
| *Mum* | $[-1,-1]$ | $[-9,0]$ |
| *Fink* | $[0,-9]$ | $[-6,-6]$ |

*Fink dominates Mum for each player and so [Fink,Fink] is a dominant equilibrium. But both players prefer the outcome for [Mum,Mum] combination!*