Assignment #1:
Due January 18

1. Which pairs of statements of the type \( f(n) = A(g(n)) \) are incompatible? Here \( A \in \{ \Theta, \Omega, O, \omega, o \} \). Give reasons.

2. Is the statement \([f(n) = O(g(n))]\) imply the statement \([2^{f(n)} = O(2^{g(n)})]\)? Is the converse true? Give proofs.

3. Let \( f(n) = n^{\log_b a}(\lg n)^k; b > 1; a \geq 1 \). Which of the following statements are true:
   
   (a) \( f(n) = O(n^{\log_b a - \epsilon}) \) for some \( \epsilon > 0 \)
   
   (b) \( f(n) = \Theta(n^{\log_b a}) \)
   
   (c) \( f(n) = \Omega(n^{\log_b a + \epsilon}) \) for some \( \epsilon > 0 \)

4. Exercises 3.1-4 (page 50): Is \( 2^{n+1} = O(2^n) \)? Is \( 2^{2n} = O(2^n) \)?

Exercise 3.2-4 (page 57): Is the function \([\lg n]!\) polynomially bounded? Is the function \([\lg \lg n]!\) polynomially bounded?

5. Problems: 3-1: Let \( p(n) = \sum_{i=0}^{d} a_i n^i \) where \( a_d > 0 \), be a degree-\( d \) polynomial in \( n \) and let \( k \) be a constant. Show that:
   
   (a) If \( k \geq d \), then \( p(n) = O(n^k) \)
   
   (b) If \( k \leq d \), then \( p(n) = \Omega(n^k) \)
   
   (c) If \( k = d \), then \( p(n) = \Theta(n^k) \)
   
   (d) If \( k > d \), then \( p(n) = o(n^k) \)
   
   (e) If \( k < d \), then \( p(n) = \omega(n^k) \)

Problem 3-2 Fill in the following table with yes/no in each slot: Assume \( k \geq 1; \epsilon > 0; c > 1 \)

<table>
<thead>
<tr>
<th>( f(n) = A )</th>
<th>( g(n) = B )</th>
<th>( O )</th>
<th>( o )</th>
<th>( \Omega )</th>
<th>( \omega )</th>
<th>( \Theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) ( n^k )</td>
<td>( n^\epsilon )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>( b ) ( n^\epsilon )</td>
<td>( e^n )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>( c ) ( \sqrt{n} )</td>
<td>( n^{\ln n} )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>( d ) ( 2^n )</td>
<td>( 2^{\frac{n}{2}} )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>( e ) ( n^{\ln m} )</td>
<td>( m^{\ln n} )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>( f ) ( \lg(n!) )</td>
<td>( \lg(n^c) )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
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</tbody>
</table>