Assignment #6:
Due November 26

[Please Note the change of dates. This date is firm!!! ]

[Please keep a copy with you since this may not be returned in time for Exam III]

1. 23.1-2; 23.1-3; 23.1-4
2. 23.2-8
3. 23-4
4. Consider the minimum spanning tree problem on a connected undirected graph. Show that Boruvka algorithm produces a spanning tree if all weights are distinct (equally if they are totally ordered). Give a counterexample to show that if edges have equal weight, we may not get a spanning tree.
5. Let $G = [V, E]$ be a directed graph and $w[e]$ ($= w[u,v]$ if $e = (u,v)$) (not necessarily nonnegative) be weight on edge $e \in E$. Let $K$ be a constant satisfying the condition that

\[ r[e] = w[e] + K > 0 \quad \forall e \in E \]

(a) Give an example to show that the shortest path in $G$ from $s$ to all other nodes depends on whether we use the weights $w[e]$ or $r[e]$.

(b) We know that lengths of the shortest path from $s$ satisfy the relations:

\[ \delta(s, v) \leq \delta(s, u) + w(u, v) \quad \forall (u, v) \in E \]

Suppose \( \{x_v\}; v \in V \) satisfy the relations:

\[ x_v \leq x_u + w(u, v) \quad \forall (u, v) \in E \]

Does this imply $x_v = \delta(s, v)$ for all $v \in V$?

(c) Let $r[u, v] = w[u, v] + x_u - x_v$ for $(u, v) \in E$ with the above $\{x_v\}$. Now $r[u, v] \geq 0$ for all $(u, v) \in E$. So we can apply Dijkstra algorithm to the problem with $r$. Are these paths also shortest paths with $w$?

(d) In case (c), do we get to do less work in determining the shortest paths from $s$ to all other nodes?

(e) In case (c), there was no mention of negative cycles in the problem with $w$ – how come?

6. 26.2-9
7. 26-1
8. 26-4