Assignment #6:
Due November 19

1. 23.1-2; 23.1-3; 23.1-4
2. 23.2-8
3. 23-4
4. Consider the minimum spanning tree problem on a connected undirected graph. Show that Boruvka algorithm produces a spanning tree if all weights are distinct (equally if they are totally ordered). Give a counter-example to show that if edges have equal weight, we may not get a spanning tree.
5. Let $G = [V, E]$ be a directed graph and $w[e]$ ($= w[u, v]$ if $e = (u, v)$) (not necessarily nonnegative) be weight on edge $e \in E$. Let K be a constant satisfying the condition that

$$r[e] = w[e] + K > 0 \qquad \forall e \in E$$

   (a) Give an example to show that the shortest path in G from s to all other nodes depends on whether we use the weights $w[e]$ or $r[e]$.

   (b) We know that lengths of the shortest path from s satisfy the relations:

$$\delta(s, v) \leq \delta(s, u) + w(u, v) \qquad \forall (u, v) \in E$$

   Suppose $\{\pi_v\}; v \in V$ satisfy the relations:

$$\pi_v \leq \pi_u + w(u, v) \qquad \forall (u, v) \in E$$

   Does this imply $\pi_v = \delta(s, v)$ for all $v \in V$?

   (c) Let $r[u, v] = w[u, v] + \pi_u - \pi_v$ for $(u, v) \in E$ with the above $\{\pi_v\}$. Now $r[u, v] \geq 0$ for all $(u, v) \in E$. So we can apply Dijkstra algorithm to the problem with r. Are these paths also shortest paths with w?

   (d) In case (c), do we get to do less work in determining the shortest paths from s to all other nodes?

   (e) In case (c), there was no mention of negative cycles in the problem with w – how come?

6. 26.2-9
7. 26-1
8. 26-4