1. Given a directed graph \(G = (V, E)\) with edge capacities \(u\), and an origin-destination pair \((s, t)\), let \((S, \bar{S}), (T, \bar{T})\) be two minimum cuts separating \(s\) and \(t\). Show that \((S \cup T, S \cup \bar{T})\) and \((S \cap T, S \cap \bar{T})\) are also minimum cuts separating \(s\) and \(t\). [This renders the set of minimum cuts a lattice]

2. Prove that the maximum number of nonzero entries, no two of which are in the same row or same column of a square matrix is equal to the minimum number of lines that include all nonzero entries. (Here a line is either a row or a column). [Hint: Set up as a maximum flow problem.]

3. Let \(Q\) be a finite set and let \((S_1, S_2, ..., S_k)\) be a family of subsets of \(Q\). A system of distinct representatives (SDR) is a set \((q_1, q_2, ..., q_k)\) of distinct elements of \(Q\), such that \(q_i \in S_i; i = 1, 2, ..., k\). Show how to test if an SDR exists and how to find one if it does. [Hint: same as in (2)]

4. Using Ford-Fulkerson algorithm compute maximum flow in the following network and show minimum cut (obtained by breadth-first search type labeling) separating the origin \(S\) and the destination \(T\): Decompose this flow into chain flows.

![Network Diagram](image)

5. Solve the maximum flow problem with \(s\) as origin and \(t\) as destination in the (undirected) network below using (a) Layered network Dinic’s algorithm;
(b) Pre-flow-PUSH-RELABEL algorithm: