

I. Consider the following LP:

$$\begin{array}{cccccccccc}
 x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & -z & & \text{rhs} \\
 0 & 1 & 0 & \alpha & 1 & 0 & 3 & 0 & = & \beta \\
 0 & 0 & 1 & -2 & 2 & \Delta & -1 & 0 & = & 2 \\
 1 & 0 & 0 & 0 & -1 & 2 & 1 & 0 & = & 3 \\
 0 & 0 & 0 & \delta & 3 & \gamma & \xi & 1 & = & 0
 \end{array}$$

What are the necessary and sufficient conditions on  $\alpha, \beta, \gamma, \delta, \xi,$  and  $\Delta$  for:

- (i) the problem to be feasible.
  - (ii) for the problem to be unbounded.
  - (iii) for the problem to have one or more optimal solutions.
- II. Show that the LP:  $[\min cx : x \geq 0; Ax = b]$  is unbounded iff it is feasible and  $\exists y \geq 0$  satisfying  $Ay = 0; cy < 0$ .
- III. Solve the LP:  $[\min \sum_{j=1}^n jx_j : \sum_{j=1}^i x_j \geq i; 1 \leq i \leq n; x \geq 0]$ .
- IV. State whether the following statements are true or false giving reasons or counterexamples in each case:
- (i) If the problem  $[\min cx : x \geq 0; Ax = b]$  has an optimal solution then so does  $[\min cx : x \geq 0; Ax = b' = b + \sum_j A_{.j}]$ .
  - (iii) If a variable leaves the basis then it can not reenter at the next iteration.
  - (iv) The value of the objective function changes iff the step is nondegenerate.
  - (v) The regular simplex algorithm (not using any modification for degeneracy) will terminate in finitely many steps if there are no ties at any step for the variable leaving the basis.

V. The following LP:

$$\begin{array}{cccccccccc}
 x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & -z & & \text{rhs} \\
 3 & -3 & 2 & 8 & 1 & 0 & 0 & 0 & = & 2 \\
 4 & 6 & -4 & -4 & 0 & 1 & 0 & 0 & = & 3 \\
 5 & -2 & 1 & 3 & 0 & 0 & 1 & 0 & = & 5 \\
 -3 & -1 & -5 & -4 & 0 & 0 & 0 & 1 & = & 0
 \end{array}$$

was solved using the simplex method and after several iterations the following incomplete information is available at some iteration.

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$-z$		rhs
?	?	?	?	1	0	-2	?	=	?
?	?	?	?	0	1	4	?	=	?
?	?	?	?	0	0	1	?	=	?
?	?	?	?	0	0	?	?	=	?

Can you complete the table?