CS 7301 Assignment#2 FALL 2008 R.Chandrasekaran

1. Let x^0 be an optimal solution to the linear program:

$$\min cx$$
$$Ax = b; x \geq 0$$

and let y^0 be an optimal solution to its dual. Now consider the LP which is the same as above except the i^{th} constraint in the new problem is the old constraint multiplied by $k \neq 0$. Can you find an optimal solution to the new problem and its dual in terms of the solutions to the old primal and dual and the value of k?

2. Solve M.Beale's example shown below by lexicographic method:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	$-z$		rhs
$\frac{1}{4}$	-60	$-\frac{1}{25}$	9	1	0	0	0	=	0
$\frac{1}{2}$	-90	$-\frac{1}{50}$	3	0	1	0	0	=	0
0	0	1	0	0	0	1	0	=	1
$-\frac{3}{4}$	150	$-\frac{1}{50}$	6	0	0	0	1	=	0

This example cycles if regular simplex method is used.

3. Consider the following problem known in the literature as the two-person-zero-sum game:

$$\min \ M$$
$$\sum_{j=1}^{n} a_{i,j} y_j \leq M; 1 \leq i \leq m$$
$$\sum_{j=1}^{n} y_j = 1; y_j \geq 0; 1 \leq j \leq n$$

Show that its dual is:

$$\max \ N$$
$$\sum_{i=1}^{m} a_{i,j} x_i \geq N; 1 \leq j \leq n$$
$$\sum_{i=1}^{m} x_i = 1; x_i \geq 0; 1 \leq i \leq m$$

Prove that :

(i) $N \leq M$ for any pair of feasible solutions. .

(ii) both problems are feasible and hence have optimal solutions and $N^* = M^*$.

(iii) $N^* = M^* > 0$ if $A > 0$.

4. Prove that the optimal dual solution is never unique if the optimal primal basic solution is degenerate and the optimal dual is not.

5. Let the LP: $[\min cx : Ax \geq b; x \geq 0]$ have an optimal solution. Let the LP with the same constraint matrix and objective function but with b' as the right hand side be feasible. Show that it has an optimal solution if it is feasible.