LINEAR PROGRAMMING AND EXTENSIONS

R. Chandrasekaran
The University of Texas at Dallas

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Chapter 1

Revised Simplex Method

This is also known as *the simplex method using multipliers*. Consider a linear program:

$$\min\ z : [Bx^B + A^{NB}x^{NB}] = b$$
$$c^B x^B + c^{NB}x^{NB} - z = 0$$

where B is current basis. Assume that the equations are linearly independent. Hence B^{-1} exists. Hence the inverse of $\begin{bmatrix} B & 0 \\ c^B & 1 \end{bmatrix}$ exists and is given by:

$$\begin{bmatrix} B^{-1} & 0 \\ -c^B B^{-1} & 1 \end{bmatrix} = \begin{bmatrix} B^{-1} & 0 \\ \pi & 1 \end{bmatrix}$$

The canonical form corresponding to this basis is obtained by multiplying the system (enlarged by including the z row) by $\begin{bmatrix} B^{-1} & 0 \\ -c^B B^{-1} & 1 \end{bmatrix}$. The last row of the result is $[\bar{c}, 1]$ and is obtained by multiplying the vector $[\pi, 1]$ times the original (enlarged) matrix. Using this we can check if $\bar{c} \geq 0$ or not. If it is not, we can also find s that satisfies $\bar{c}_s = \min \bar{c}_j < 0$. This yields the *entering variable*. To find the updated column (that in the canonical form) of this variable we need to multiply $\begin{bmatrix} B^{-1} & 0 \\ -c^B B^{-1} & 1 \end{bmatrix} \begin{bmatrix} A_{.s} \\ c_s \end{bmatrix}$. Similar process gives the updated RHS by the relation $\begin{bmatrix} B^{-1} & 0 \\ -c^B B^{-1} & 1 \end{bmatrix} \begin{bmatrix} b \\ -z^0 \end{bmatrix}$. Now doing a pivot operation on the subsystem that includes these three yields the new inverse and the process is repeated until the usual termination conditions are observed. That the new inverse is obtained in this manner follows by observing that if an identity matrix is appended to the system we always have the inverse in its position.

One advantage of this procedure is that many entries in the updated form need not be calculated; this results in saving computations if the number of variables exceeds the number of equations by a significant amount. *While this* *advantage is stressed in many texts, what is more important is that we can formulate problems with enormous number of variables that arise from combinatorial explosion in some applications; these are, in my opinion, the main advantage of this method.* This often goes under the name of *(delayed) column generation technique.*

Instead of storing the entire inverse, we can store the inverse between successive steps. These are elementary matrices and hence easy to store. This is discussed in your book under *product form of the inverse.*