NETWORK FLOWS AND COMBINATORIAL OPTIMIZATION

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Chapter 1

TOPICS

1. Graph Theory
2. Matrices Associated with Graphs
3. Maximum Flow Problem – Single Commodity
   (a) Formulation as an L.P.
   (b) Max-Flow Min-Cut Theorem
   (c) Labeling Algorithm
   (d) Finite Termination of Maximum Flow Algorithms
      i. Ford-Fulkerson Example
      ii. Queyranne's example
   (e) Strongly polynomial Algorithms:
      i. Edmonds-Karp
      ii. Dinic
      iii. Karzanov
      iv. Maheshwari et al.
   (f) Special cases:
      i. Undirected Networks
      ii. Parallel Arcs
      iii. Multiple Origins/Destinations
      iv. Node Capacities
   (g) Arc Chain Formualtion and Chain Decomposition of Flows
   (h) Lower Bounds and Exogeneous Flows
   (i) Picard-Queyranne work and the structure of cuts

(j) MIT work

4. Spanning Tree Problem
5. Multi-terminal Flows
6. Shortest Path Problems
7. Minimum Cost Flows
   (a) Transportation and Primal-Dual Algorithm
   (b) Out-of-kilter Algorithm
   (c) Klein's Algorithm
   (d) Edmonds-Karp Scaling
   (e) Zadeh's Examples
   (f) Strongly Polynomial Algorithms:
      i. Eva Tardos
      ii. Fujishige
      iii. Simplex
   (g) PERT-CPM
8. Unimodularity
9. Matching
   (a) Bipartite
   (b) General
   (c) Applications:
      i. Chinese postman
      ii. Scheduling
      iii. Odd and even shortest paths
      iv. Shortest paths in undirected graphs without negative cycles
      v. Geometric applications
   (d) Ellipsoid Algorithm and the Odd-Cut Problem
   (e) T-joins and T-cuts
10. Blocking Systems
   (a) Bottleneck Problems
   (b) Blocking Systems
   (c) Blocking and Antiblocking Polyhedra

(d) Perfect Graph Theorem

11. Multicommodity flows

(a) General Problem

(b) Two Commodity:

i. T.C. Hu

ii. M. Sakarovitch

(c) Work of Karzanov, Lomonosov, Papernov, Seymour, Lovasz, Schrijver, Sebo et al

12. Matroids

(a) Introduction

(b) Axioms and their equivalence:

i. Independence

ii. Circuit

iii. Bases

iv. Rank function

v. Greedy algorithm

vi. Hyperplanes, flats etc.

(c) Optimization Problems:

i. Covering

ii. Partition

iii. Intersection

iv. Parity

(d) Polyhedral Characterizations

(e) Operations:

i. Deletion and Contraction

ii. Minors

iii. Duals

iv. Decomposition and Composition

v. Splitters

(f) Applications of Decomposition to Testing:

i. Graphicness

ii. Regularity

iii. Max-Flow Min-Cut Matroids

(g) Generalizations to Pseudomatroids

13. Total Dual Integrality

    (a) General Theory
    (b) Integer Rounding
    (c) General Algorithm for Testing and Solving Optimization Problems
    (d) Special Cases
    (e) Nature of Hilbert bases.

14. Geometry of Numbers

    (a) Introduction
    (b) Lattice Basis Reduction Techniques
    (c) Lenstra's Algorithm and its Relationship to Scarf's Results
    (d) Other Applications