Identifying Exchange Rate Common Factors*

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Abstract

We determine that exchange rate returns are driven by a two-factor model and identify these factors to consist of dollar and euro components. This implies exchange rates are driven by global, US and Euro-zone stochastic discount factors. Identification also motivates multilateral models for bilateral exchange rates. Out-of-sample forecast accuracy of empirically identified multilateral models dominate the random walk and a bilateral purchasing power parity fundamentals prediction model. Forecast accuracy of the multilateral model is roughly equivalent to a principal components forecasting model.

Keywords: Exchange rates, common factors, factor identification multilateral exchange rate model.

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Introduction

Exchange rate returns (first differences of log exchange rates) show substantial cross-sectional correlation. In a sample of 27 monthly exchange rate returns from 1999.01 to 2012.12, the average correlation is 0.44 when the U.S. dollar (USD) is the numeraire currency. Similarly, the average correlation is 0.31 when the euro is numeraire and 0.33 when the Canadian dollar is the numeraire. The cross-sectional correlation of exchange rate returns has been recognized in research at least since O'Connell (1988) but has primarily been treated as a nuisance parameter in panel data models (Mark and Sul (2001), Engel et al. (2007)). More recent research, however, has turned attention towards understanding the source of co-movements across exchange rates. Engel et al. (2015) estimate a common factor structure by principal components and find the evolution of exchange rates to be well described by a small number of common factors. Moreover, these factors remain significant even after controlling for macroeconomic fundamental determinants. Statistical factor analysis of this sort, however, is primarily descriptive and very reduced form. Because the estimated factors are not identified with economic variables, the methodology cannot give a direct economic interpretation of the underlying economic mechanisms that drive exchange rate dynamics.

In this paper, we address the identification issue using econometric methods developed by Bai and Ng (2002, 2006) and Parker and Sul (2015). Because significant institutional changes occurred upon adoption of the common currency, our sample starts in 1999 to coincide with the euro epoch. Our analysis seeks to uncover the relationship between the vector of true but unobserved factors and a vector of economic variables put forth as candidates for empirical factors. The first step in the two-step identification methodology uses an information criterion proposed by Bai and Ng (2002) to determine the number k, of common factors in a panel of exchange rate returns. The second step involves determination of the number of common factors in the panel of residuals from regressions of the exchange rate return on unique combinations of $k-$element groupings of the candidate economic variables. Identification is based on the idea that if this particular group of k variables are empirical factors, then there are no common factors in the residuals. If one or more common factors are found in the residual panel, this particular set of variables is rejected as the empirical factors. If more than one $k-$element group is so identified, a face-off procedure, which we describe below, is implemented.

The candidate list of economic variables is potentially large and searching over all possibilities is not feasible. We therefore limit candidates for empirical factors to exchange rate returns. This is not unreasonable because exchange rate returns, being the difference between country's (possibly unobservable) log stochastic discount factors (SDF), may contain information that is difficult to observe in other macroeconomic fundamentals. Consideration of exchange rate returns as candidate empirical factors identifies a dollar factor and a euro factor as the pair of common factors driving exchange rates in our sample. Our identification is robust to the choice of the numeraire currency.

What is the value-added of empirical factor identification? One value is that it admits an economic interpretation to the source of exchange rate co-movements across currencies. Drawing on the stochastic discount factor (SDF) approach to the exchange rate, as do Lustig et al. (2011) and Verdelhan (2015), implies that the exchange rate return is driven by the global, the US and the Euro-zone log SDFs. The dollar and euro factors imply that co-movements of log SDFs across countries are heavily influenced, if not dominated by the dynamics of the log SDF of the US and the Euro zone. Driven by the same underlying set of unobserved factors, heterogenous behavior in cross-country log SDFs arise because they respond differently to the factors. A second value to the identification is that it can be exploited to improve the performance of empirical exchange rate models. Our dollar and euro factor identification suggests a multilateral model of bilateral exchange rates. This contrasts with typical bilateral formulations. That is, bilateral exchange rates in conventional models are determined by variables from the pair of countries associated with bilateral the exchange rate.[1] Instead fixating on details of every bilateral country pair, knowing the determinants of the dollar and the euro allows one to understand a substantial proportion of the variation in any bilateral exchange rate. To assess empirical model performance of the multilateral model, we employ an out-of-sample forecasting methodology. Ever since Meese and Rogoff (1983) out-of-sample forecasting has become standard procedure for exchange-rate model validation.

In the forecasting analysis, we compare our multilateral 'dollar-euro' model with alternative models considered in the literature. The first, is the bilateral purchasing-power parity (PPP) based fundamentals model. We use this as a comparison model because Engel et al. (2007) find that it gives the best forecast accuracy among several bilateral fundamentals-based formulations considered in the literature. Reserving the period from 2005.01 to 2012.12 for out-of-sample forecast evaluation, we find that prediction accuracy from our dollar-euro model dominate those from the PPP-based model as well as those from the driftless random walk.[2]

The empirical exchange rate literature finds that sample size matters for forecast accuracy. Rapach and Wohar (2001) and Lothian and Taylor (1996) report significant predictive power when working with long historical time-series data. To obtain more observations within the post Bretton Woods floating regime, a first-generation of papers (Mark and Sul, 2001, Rapach and Wohar, 2004, and Groen, 2005) expanded observations cross-sectionally with the use of panel-data methods.[3] The

[1] Berg and Mark (2015) is an exception. They argue that bilateral exchange rates are driven in part by third-country (rest of world) shocks.

[2] Drawing motivation from the present value model of exchange rates, Chen et al. (2010) and Sarno and Schmeling (2013) find evidence that today's exchange rate predicts future fundamentals.

[3] The importance of cross-sectional information has been recognized since Bilson (1981) who used seemingly unrelated regression to estimate his exchange rate equation. Frankel and Rose (1996) initiated a literature on the panel data analysis of PPP, which is surveyed by Caporale and Cerrato (2006). Cerra and Saxena (2010) employed a panel

panel aspect of our data expands observations by exploiting the cross section. Our identification also suggests a model suitable for forecasting daily exchange rates. Using daily observations also increases observations in the time-series dimension. The daily version of our multilateral dollar-euro factor model is also able to forecast as well.

Improved forecast performance over the random walk and the bilateral PPP-based model does not fully answer the question of whether identification has predictive value in empirical modeling since the factor structure can also be estimated by principal components and used to forecast. Engel et al. (2015) found that quarterly forecasts from a two-principal components model were significantly more accurate than random walk predictions over the 1999 to 2007 period. Hence, we also compare forecasts from our multilateral model to the two-principal components forecasting model. We find, on balance, that the dollar-euro model has lower mean-square forecast error (MSFE) than the principal components model at the longer (12 and 24 month) horizons.

Our paper is also related to Verdelhan (2015). While he is not concerned with forecasting, he engages in empirical factor identification for exchange rates and proposes an alternative multilateral identification. Using a regression-based method, Verdelhan also concludes that exchange rates are driven by two-factor model and his first empirical factor, the dollar factor, coincides with our identified US factor. Using the dollar as the numeraire currency, it is the cross-sectional average of all bilateral exchange rate returns. His second empirical factor, the carry factor, differs from ours. It is the cross-exchange rate return between a portfolio of high and low interest rate currencies and is also multilateral in nature. Our analysis includes consideration Verdelhan's carry as a candidate economic variable for empirical factor identification. For our data set, in the identification phase, the evidence for the carry is weaker than the evidence in support for the euro factor. However, forecast accuracy of a multilateral dollar-carry model is on par with the dollar-euro model.

The paper is organized as follows. The next section presents the common factor structure that we assume and the identification methodology that we use. Our data set is described in Section 2. Empirical factor identification results are presented in Section 3. Forecasting results are presented in Section 4 and Section 5 concludes.

1 Common Factors in Exchange Rate Variation

This section develops the factor structure for exchange rate returns that guides our empirical work. To fix notation, let f_t be the $k-$dimensional vector of the true but unobserved common factors and f_t^p be an $m-$dimensional vector of economic variables considered to be potential empirical factors. m is potentially very large. The goal is to identify a unique set of k elements from f_t^p that describe

data set with a large number (98) of countries in a study of the monetary model of exchange rates.

the evolution of f_t.

Ideas are developed for the nominal exchange rate. Developing the parallel development for real exchange rates is straightforward. Let there be $N+1$ currencies with currency '0' as the numeraire. If the USD (U.S. dollar) is the numeraire, then s_{it} is the logarithm of the nominal exchange rate between the US and country i and an increase in s_{it} means a USD increase. If within a country, markets are complete or if markets are incomplete but the law-of-one price holds and there is no arbitrage, the country will have a unique SDF. SDFs differ across countries if there are cross-country risks that cannot be insured.[4] Let n_{it} be the log nominal stochastic discount factor for country $i = 0, ..., N$. In the SDF approach to exchange rates, the exchange rate return is

$$\Delta s_{it} = n_{it} - n_{0t}. \tag{1}$$

Because Δs_{it} varies (quite a bit) over time, we know that SDFs evolve differently across countries. A representation of the log SDF, consistent with such cross-country heterogeneity is the factor structure,

$$n_{it} = \delta_i' f_t + n_{it}^o, \tag{2}$$

where $f_t = (f_{1t}, ..., f_{kt})'$ is the $k-$element vector of true but unobserved common factors, δ_i is a $k-$element vector of factor loadings and n_{it}^o is the idiosyncratic component of the country i log SDF. The latent factors may be correlated with each other $\text{Cov}(f_{it}, f_{jt}) \neq 0$, for $i \neq j$, while the idiosyncratic components are uncorrelated across countries, $\text{Cov}\left(n_{it}^o, n_{jt}^o\right) = 0$. Heterogeneous responses to factor movements across countries are necessary for exchange rate returns Δs_{it} to vary over time. If there were no cross-country differences in factor loadings δ_i, the exchange return would be driven only by idiosyncratic components of the log SDF and would then be cross-sectionally uncorrelated. Because the factors f_t drive common movements in every country's log SDF, the common factors f_t are global in nature. Lustig et al. (2011) and Verdelhan (2015) also decompose the log SDF into a common or global component and a country-specific idiosyncratic component. We take eqs. (1) and (2) to represent the *truth*.

Substituting (2) into (1) gives the factor representation for exchange rate returns,

$$\Delta s_{it} = \left(\delta_i' - \delta_0'\right) f_t + n_{it}^o - n_{0t}^o. \tag{3}$$

Notice from (3) that the idiosyncratic part of the numeraire country's log SDF n_{0t}^o, appears for all i and is also a common source of exchange rate co-movement. Our interest is in the identification of f_t, not n_{0t}^o. The influence of n_{0t}^o is attenuated by transforming the observations into deviations

[4] Markets are complete in Backus and Smith but home and foreign SDFs differ because risks associated with non-traded goods consumption shocks cannot be internationally traded.

from the cross-sectional mean. In doing so, let the cross-sectional average of factor loadings be

$$\bar{\delta}' = \left(\frac{1}{N}\sum_{i=1}^{N}\delta_{i,1}, ..., \frac{1}{N}\sum_{i=1}^{N}\delta_{i,k}\right)$$

and $\tilde{\delta}_i' = \left(\delta_i' - \bar{\delta}'\right)$ be the deviation from the mean loadings. Then the cross-sectional mean of the exchange rate return is

$$\frac{1}{N}\sum_{i=1}^{N}\Delta s_{it} = \Delta\bar{s}_t = \left(\bar{\delta}' - \delta_0'\right) f_t - n_{0t}^o. \tag{4}$$

In deviations from the mean form $\Delta\tilde{s}_{it} = \Delta s_{it} - \Delta\bar{s}_t$, n_{0t}^o is removed and f_t becomes the only common factor component of the exchange rate return,

$$\Delta\tilde{s}_{it} = \tilde{\delta}_i' f_t + n_{it}^o. \tag{5}$$

We also note that the underlying factor structure for observations $\Delta\tilde{s}_{it}$ in deviation from the mean form, is numeraire invariant when N is large, but in any finite sample, changing the numeraire currency will result in some variation in the $\tilde{\delta}_i$ factor loadings.[5]

A parallel development for real exchange rate returns follows from the relation between the log real SDF m_{it} and the log nominal SDF,

$$m_{it} = n_{it} + \pi_{it}$$

where π_{it} is country i inflation rate in period t. It follows that

$$\Delta\tilde{q}_{it} = \tilde{m}_{it} = \tilde{n}_{it} + \tilde{\pi}_{it}. \tag{6}$$

The set of factors that drive nominal exchange rate returns are likely to drive real exchange rate returns as well.

1.1 Identification Method

The common factor representation has successfully been used as the statistical foundation for modeling co-movements across exchange rates but because the factors are not identified, it does not give an economic interpretation for the underlying mechanism. To address this issue, Bai and Ng (2006) and Parker and Sul (2015) develop methods to identify the unobserved common factors with observed economic variables. Drawing on these methods, this section undertakes a two-step procedure to identify empirical counterparts to the unobserved common factors. The first step

[5]If the US is the numeraire country, $\bar{\delta}$ is the average of all other (not US) country factor loadings. If instead, Canada is used as the numeraire, Canada's factor loadings are replaced by the US's δ in computing the average, $\bar{\delta}$. The effect of swapping numeraires on $\tilde{\delta}_i$ vanishes when N is large.

identifies the number of common global factors k present in the data. The second step identifies economic variables that closely mimic the true latent factors by evaluating restrictions imposed on candidate empirical factors by the factor representation.

In deviations from the mean form, the panel data $\Delta\tilde{s}_{it}$ consists of N exchange rates observed over T time periods. Identification of the number of common factors, k, in the data employs the Bai and Ng (2002) IC_2 information criterion on standardized observations. Bai and Ng (2002), Hallini and Liska (2007), Onatski (2009, 2010), Ahn and Horenstein (2013) propose alternative methods to determine the number of common factors. We employ Bai and Ng's (2002) IC_2 because Parker and Sul (2015) showed that it has good robustness properties.

Let $C_{NT} = \min(N, T)$, and λ_i be the ith largest eigen value of the sample covariance matrix. In step 1, the number of common factors in the panel is the value of k that minimizes

$$\mathrm{IC}_2 = \ln\left(\sum_{i=k+1}^{C_{NT}} \lambda_i\right) + k\left(\frac{N+T}{NT}\right)\ln C_{NT}. \tag{7}$$

For concreteness in discussing step 2, assume that we determined in step 1 that exchange rates $\Delta\tilde{s}_{it}$ are driven by $k = 2$ common factors (the analysis for other values of k follow directly). In step two, let $f_t^p = \{f_{1t}^p, ..., f_{mt}^p\}$ be the set of candidate variables for the empirical factors. Viewing eq.(5) as the true factor representation for exchange rates, we test the null hypothesis that a unique pair of economic variables (f_{jt}^p, f_{st}^p) span the same space as the two true common factors (f_{1t}, f_{2t}),

$$f_{1t} = a_{11}f_{jt}^p + a_{12}f_{st}^p + \epsilon_{1t}, \tag{8}$$

$$f_{2t} = a_{21}f_{jt}^p + a_{22}f_{st}^p + \epsilon_{2t}, \tag{9}$$

where for $j = 1, 2$, $\mathrm{Var}(\epsilon_{jt}) \to 0$ as $T \to \infty$. Asymptotically, the economic variables give an exact identification of the factors in the sense that the error terms are $O_p\left(\sqrt{T}^{-1}\right)$. It is also possible that some of the a_{js} coefficients are zero. If, for example, $a_{12} = a_{21} = 0$, the latent factors are uniquely identified.

Empirical identification employs regressions of $\Delta\tilde{s}_{it}$ on (f_{jt}^p, f_{st}^p),

$$\Delta\tilde{s}_{it} = a_i + b_{1i}f_{jt}^p + b_{2i}f_{st}^p + \Delta\tilde{s}_{it}^o \tag{10}$$

where $\Delta\tilde{s}_{it}^o$ is the regression residual. Parker and Sul (2015) establish the following two results:

1. If there are no (zero) common factors in the panel of residuals $\Delta\tilde{s}_{it}^o$, then (f_{jt}^p, f_{st}^p) are the true common factors.

2. If there are one or more common factors in the panel of residuals $\Delta\tilde{s}_{it}^o$, then either (f_{jt}^p or f_{st}^p), or both (f_{jt}^p, f_{st}^p) are not the true common factors.

Hence we examine whether or not that a pair of economic variables are approximately the true unobserved factors by regressing $\Delta\tilde{s}_{it}$ on all combinations of two candidates f^p_{st} and f^p_{jt} and use the IC_2 information criteria (7) to determine the number of common factors in the panel of regression residuals. If there are no common factors in the residual panel, then f^p_{st} and f^p_{jt} are identified as empirical factors. If more than one pair of economic variables are determined to be common factors, the candidate variables are put through a face-off procedure which we describe below.

2 Data

Because of the important role played by the euro in international finance, we use observations only from the euro epoch. The sample begins 1999.01 and ends 2012.12 for 27 exchange rates expressed as USD prices of foreign currency. We use the currencies of Australia (AUS), Brazil (BRA), Canada (CAN), Chile (CHI), Columbia (COL), the Czech Republic (CZE), the Euro (EUR), Hungary (HUN), Iceland (ICE), India (IND), Israel (ISR), Japan (JPN), Korea (KOR), Norway (NOR), New Zealand (NZL), the Philippines (PHI), Poland (POL), Romania (ROM), Singapore (SIN), South Africa (RSA), Sweden (SWE), Switzerland (SUI), Taiwan (TAI), Thailand (THL), Turkey (TUR), the U.K. (GBR) and the U.S. (USA). As seen in Table 1, the euro has consistently been the second most important currency (behind the U.S. dollar) in terms of foreign exchange market turnover. Although the time-span of our sample is relatively short, it does not extend across different regimes or institutional structures.

Currency selection was based on data availability and whether or not countries allowed their exchange rate to float. Currencies included in the sample were consistently classified as either "floating" or "managed floating without a predetermined path" in the IMF *Annual Report on Exchange Arrangements and Exchange Restrictions*.[6] Exchange rates are expressed as monthly averages and were obtained from IHS Global insight. To construct real exchange rates, we use consumer price indices of the associated regions from the OECD economic indicators and IHS Global Insight. For New Zealand and Australia, monthly consumer price indexes are not available so they are estimated by taking a linear interpolation using quarterly consumer price indexes.

We also make use of interest rates to construct the carry factor exchange rate return. From IHS Global Insight, we have one-month eurocurrency deposit rates for the euro, Australia, Canada, Czech Republic, Japan, New Zealand, Norway, Poland, Singapore, Sweden, Switzerland, the UK and the US. and interbank rates for Iceland, India and Indonesia. Interest rates for the remaining

[6]The IMF report does not cover Taiwan since it is not part of the IMF. We include it in the sample however since the central bank of Taiwan states it uses a managed floating regime. In any case, the standard deviation of monthly returns of the USD/New Taiwan dollar is 1.20%, which is of similar order of magnitude as that of the Thai Bhat 1.47%, which has consistently been classified as a "managed float with no pre-determined path" by the IMF.

Table 1: Top Ten Currencies Ranked By Global Foreign Exchange Market Volume

Percentage Shares Of Average Daily Volume							
	1998	2001	2004	2007	2010	2013	Average
US dollar	86.8	89.9	88	85.6	84.9	87	87.0
Euro	...	37.9	37.4	37	39.1	33.4	37.0
Yen	21.7	23.5	20.8	17.2	19	23	20.9
Pound	11	13	16.5	14.9	12.9	11.8	13.4
Swiss Franc	7.1	6	6	6.8	6.3	5.2	6.2
Australian dollar	3	4.3	6	6.6	7.6	8.6	6.0
Canadian dollar	3.5	4.5	4.2	4.3	5.3	4.6	4.4
Swedish Krona	0.3	2.5	2.2	2.7	2.2	1.8	2.0
Norwegian Krone	0.2	1.5	1.4	2.1	1.3	1.4	1.3
Other	65.4	14.7	15.7	20.1	19	21.8	20.0
Total	200	200	200	200	200	200	200

countries are obtained from *Datastream.* They are, the one-month interbank middle rate for Brazil, Hungary, Israel, Romania, Korea, Taiwan, Thailand and Turkey, the overnight interbank middle rate for Chile and Colombia, the 61-90 day time-deposit rate for the Philippines, one-month deposit middle rate for South Africa.

3 Empirical Factor Identification

A large number of macro and financial variables potentially have influence on bilateral exchange rates. What economic variables should we include in the vector f_t^p? To narrow the search, we restrict attention to exchange rate returns. One of the exchange rate returns we consider is the carry suggested by Verdelhan (2015). Studying nominal exchange rate returns with the USD as the numeraire currency, he concludes that exchange rates have a two-factor representation consisting of a dollar factor, which is the average of the cross-section of exchange rate returns $\Delta \bar{s}_t$, and a 'carry factor,' which is the cross-rate currency return on a portfolio of high interest rate countries relative to a portfolio of low interest rate countries. He calls this exchange rate return the carry, because a (portfolio) carry trade is formed by taking a short position in the low interest rate portfolio and using the proceeds to take a long postion in the high interest rate portfolio.

For each time period, we construct exchange rate carry returns by ranking the 28 countries in the sample by their interest rate. Sorting from low to high and divided into 4 portfolios, let P_4

be the portfolio formed by the 7 countries with the highest interest rates and P_1 is the portfolio formed by the 7 countries with the lowest interest rates.[7] The nominal carry exchange rate return Δs_t^c is the cross exchange rate return between P_{4t} and P_{1t} currencies,[8]

$$\Delta s_t^c \equiv \frac{1}{7}\sum_{j\in P_{4t}} \Delta s_{jt} - \frac{1}{7}\sum_{i\in P_{1t}} \Delta s_{it}$$

The other variables in our candidate list f_t^p, are the 27 individual bilateral exchange rate returns and the cross-sectional average of the exchange rate returns. Altogether, for a given numeraire currency, f_t^p contains 29 candidates.

Empirical identification with the USD as numeraire currency. We run the identification procedure for both nominal and real exchange rate returns using the USD as the numeraire currency. We do not show the specifics of the k identification (the number of factors) but simply report here in the text that both real and nominal exchange rate returns are determined to be driven by $k = 2$ factors. This is consistent with Verdelhan (2015) and also Engel et al. (2015) who find little difference between their 2 factor and 3 factor models. Taking into account the robustness of the IC_2 criteria and findings across studies, determination that exchange rate returns are driven by $k = 2$ appears to be reasonably robust.

Moving to step 2, empirical factor identification determines that $(\Delta\bar{s}_t, \Delta\tilde{s}_{\mathrm{EUR}t})$ are empirical factors for nominal exchange rates and $(\Delta\bar{q}_t, \Delta\tilde{q}_{\mathrm{EUR}t})$ for real exchange rates, where $s_{\mathrm{EUR}t}$ ($q_{\mathrm{EUR}t}$) is the log nominal (real) USD-Euro exchange rate. Following Verdelhan (2015), we refer to the cross-sectional average of exchange rate returns as a 'dollar' factor. By analogy, we call the dollar-euro identification a 'euro' factor.

This identification turns out not to be unique, however. For the nominal exchange rate panel, alternative exchange rate pairs consisting of the dollar and Swiss franc, the euro and Singapore dollar, and the euro and Taiwan dollar were not rejected as potential empirical factors. The carry exchange rate return was not identified as an empirical factor for nominal exchanges. For the real exchange rate panel, alternative unrejected empirical pairs are dollar-Swiss franc, euro-Singapore dollar and euro-carry. Because multiple pairs of variables were identified as empirical factors, we pit the competing pairs against each other in a face-off.

[7] The carry trade takes a USD short position in the P_1 portfolio and use the proceeds to take a corresponding USD long position in the P_4 portfolio. This return is accessible to investors in any country.

[8] The real carry exchange rate return subtracts portfolio inflation rates from the exchange rate returns,

$$\Delta q_t^c \equiv \frac{1}{7}\sum_{j\in P_{4t}} (\Delta s_{jt} - \pi_{jt}) - \frac{1}{7}\sum_{i\in P_{1t}} (\Delta s_{it} - \pi_{it})$$

where π_{jt} is country j's inflation rate.

Face-off using alternative numeraire currencies. Although identification should be robust to numeraire choice, this is true only asymptotically as $N \to \infty$. In any finite sample, there will be some sampling variability. We exploit this variability for the face-off by running the identification procedure through the other 27 possible numeraire currencies.

The face-off conditions on the dollar as one of the factors and is therefore not exhaustive. We do this based on accumulation of three pieces of evidence supporting the dollar factor as a dominant empirical factor. First, the cross-sectional average is not exactly, but is close to the first principal component. Second, both our analysis and Verdelhan's finds the dollar to be an empirical factor. Third, the dominance of the USD in foreign exchange volume and the importance of the US in the world economy lend further but indirect weight to support the dollar as one of the two factors. Hence, the face-off asks if the dollar combined with any of the other exchange rates form the pair of empirical common factors.

When the dollar was the numeraire, the dollar factor was the cross-sectional average of exchange rates. Under an alternative currency numeraire, the dollar factor is the bilateral USD exchange rate against the numeraire currency. For example, when the Australian dollar (AUD) is the numeraire, the dollar factor is the AUD/USD exchange rate, as a deviation from the cross-section mean. Conditional on the dollar factor, we ask if the dollar and euro are the factors, if the dollar and Swiss franc are factors, and if the dollar and the carry exchange rate return are the two empirical factors. As an additional check for robustness, for each numeraire, the identification procedure is performed by recursively expanding the sample, beginning in 2009.01 through 2012.12 for a total of 48 identifications.

The results of the face-off are shown in Table 2. The table reports the frequency (out of 48 trials) that a common factor is detected in the panel of residuals. Regardless of the numeraire, we do not find a common factor in the residuals from the exchange rate return regressed on dollar and euro factors. That is, conditional on the dollar factor, we always identify the euro as the second empirical factor. Similarly, conditional on the dollar factor, the face-off does not identify the carry as the second factor. For many numeraires, however, the Swiss franc is identified as the second factor. This occurs in about a third of the (48) cases. The identification results in the face-off are similar for both nominal and real exchange rates.

In our data, the face-off provides little evidence that the carry is a global exchange rate factor. We give further consideration to the carry, by consulting Verdelhan's (2015) identification methodology. Conditioning on the (foreign, denoted by *) country i and US interest differential $i_t^* - i_t$, he identifies the dollar and carry as empirical factors because the t-ratios for β_3 and β_4 in the regression

$$\Delta s_{it} = \alpha + \beta_1 \left(i_t^* - i_t\right) + \beta_2 \left(i_t^* - i_t\right) \Delta s_t^c + \beta_3 \Delta s_t^c + \beta_4 \Delta \bar{s}_t + \epsilon_{it}, \tag{11}$$

Table 2: Frequency of Common Factors Detection in Residual Panel Conditional on Dollar Factor

	Nominal Exchange Rates			Real Exchange Rates		
Numeraire	EUR	SUI	Δs_t^c	EUR	SUI	Δq_t^c
AUS	0	0.31	1	0	0	1
BRA	0	0.33	1	0	0.15	1
CAN	0	0.27	1	0	0	1
CHI	0	0.33	1	0	0.31	1
COL	0	0	1	0	0	1
CZE	0	0.31	1	0	0	1
EUR	0	0	1	0	0	1
GBR	0	0.33	1	0	0.31	1
HUN	0	0	1	0	0	1
ICE	0	0	1	0	0	1
IND	0	0.33	1	0	0.1	1
ISR	0	0.31	1	0	0	1
JPN	0	0.33	1	0	0.25	1
KOR	0	0.33	1	0	0.31	1
MEX	0	0.31	1	0	0	1
NOR	0	0.33	1	0	0.31	1
NZL	0	0.33	1	0	0.1	1
PHI	0	0	1	0	0	1
POL	0	0.31	1	0	0	1
ROM	0	0.21	1	0	0	1
RSA	0	0.31	1	0	0	1
SIN	0	0.06	1	0	0	1
SUI	0	0	1	0	0	1
SWE	0	0.33	1	0	0.15	1
TAI	0	0	1	0	0	1
THL	0	0.33	1	0	0	1
TUR	0	0.31	1	0	0	1

Note: For each numeraire currency, the identification procedure is applied to 48 recursively updated samples. Sample begins 1999.1. The first sample ends 2009.1 and the last ends 2012.12. Table shows frequency with which a common factor is detected in the residual panel out of 48 trials.

are significant. Using our data, we run (11) bilaterally for each exchange rate with the US dollar as numeraire. The t-ratios on β_3 are shown in Table 3, and is statistically significant at the 5 percent level in 15 of the 27 regressions. Next, we add $\Delta s_{\mathrm{EUR}t}$, the USD-euro exchange rate to the regression,

$$\Delta s_{it} = \alpha + \beta_1 \left(i_t^* - i_t\right) + \beta_2 \left(i_t^* - i_t\right) \Delta s_t^c + \beta_3 \Delta s_t^c + \beta_4 \Delta \bar{s}_t + \beta_5 \Delta s_{\mathrm{EURt}} + \epsilon_{it} \tag{12}$$

The table also shows t-ratios on β_3 and β_5 from the regression in (12) that includes both the carry and euro factors, Now, the carry is significant in only 11 of 27 cases while the euro is significant in 18 of 27. The last two columns of the table show the frequency with which a country is represented in the carry factor, an element of P_4 or P_1. Instances in which the carry is significant are typically those where that particular currency is heavily represented in the carry exchange rate return. For example, the carry is highly significant for Japan and the yen is always represented in the funding portfolio P_1. In a sense, the carry reflects the yen being regressed (at least partially) on the yen. Using Verdelhan's regression strategy on our data, the evidence that the euro is the second factor is more compelling than the evidence for the carry.[9]

Another comparison between the euro and the carry as empirical factors looks at how well they explain statistically estimated factors. Here, we regress principal components estimates of the factors, $\hat{f}_{1t}^{pc}$ and $\hat{f}_{2t}^{pc}$ on the dollar and euro and on dollar and carry. The regressions take the form of eqs.(8) and (9) with the principal components substituted in for the unobserved true factors as dependent variables. Figure 1 shows the principal components and fitted values of the regressions. Panel A shows the results for the first principal component $\hat{f}_{1t}^{pc}$, and panel B shows the results for the second $\hat{f}_{2t}^{pc}$. As can be seen, the dollar-euro pair and the dollar-carry pair both explain the first principal component well. In panel B, the dollar-euro pair are seen to do a much better job of explaining the second principal component than the dollar-carry pair.

In regard to Swiss identification, a Swiss factor is occasionally identified in the face-off using real exchange rates, but not in the majority of cases. It seems unlikely that exchange rates are governed by a Swiss factor instead of a euro factor. Switzerland is too small a country for its currency or its log SDF to dominate global exchange rate pricing. Looking at Table 1, the Swiss franc also accounts for a relatively low share of foreign exchange market volume. The identification scheme appears sometimes to confound the Swiss factor with the euro factor due to the high correlation between the euro and Swiss franc return rates of 0.74. We conclude that the dollar and the euro are the empirical factors in our data.

To give some context for our identification, the implied relationship between the latent factors

[9] Verdelhan's data is monthly from 1983.11 to 2010.12.

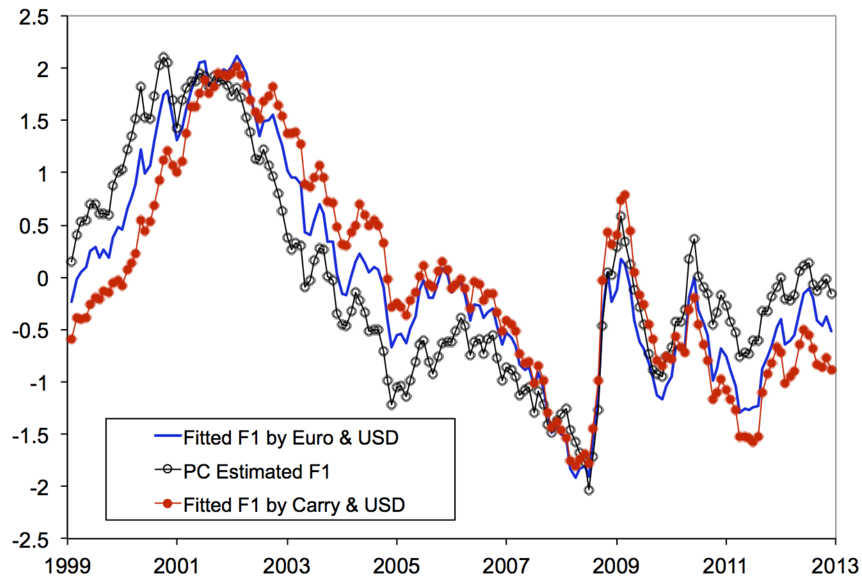
Table 3: Newey-West t-ratios on the Carry and Euro

	Eq.(11)	Eq.(12)		Freq.	Freq.
	t_{β_3}	t_{β_3}	t_{β_5}	P_4	P_1
AUS	1.276	1.080	1.326	0.07	0.00
BRA	**-2.371**	-1.067	**3.988**	0.81	0.00
CAN	**-2.534**	-1.032	**2.129**	0.00	0.24
CHI	-0.423	1.546	**4.451**	0.07	0.27
COL	**-4.510**	**-2.509**	**3.903**	0.37	0.00
CZE	**5.441**	1.288	**-9.083**	0.00	0.43
EUR	**5.225**	**4.656**	∞	0.00	0.37
GBR	0.234	-1.282	**-3.107**	0.00	0.19
HUN	-0.983	**-3.487**	**-5.513**	0.85	0.00
ICE	**-7.005**	**-8.334**	**-5.329**	0.68	0.00
IND	-1.584	-0.852	**3.289**	0.29	0.00
ISR	**2.140**	**1.983**	-0.171	0.11	0.02
JPN	**9.880**	**9.086**	1.468	0.00	1.00
KOR	0.734	1.603	**4.163**	0.00	0.00
MEX	**-4.540**	**-3.069**	**5.337**	0.47	0.00
NOR	**2.287**	1.326	**-5.510**	0.00	0.13
NZL	-0.812	-0.645	0.292	0.22	0.00
PHI	-1.158	0.088	**2.225**	0.00	0.10
POL	-0.051	-0.543	-0.946	0.30	0.00
ROM	0.646	-1.382	**-4.099**	0.81	0.00
RSA	**-2.725**	**-2.252**	0.605	0.88	0.00
SIN	**4.315**	**4.840**	1.954	0.00	0.92
SUI	**7.501**	**4.505**	**-9.270**	0.00	0.42
SWE	**4.957**	1.460	**-6.338**	0.00	1.00
TAI	0.141	1.384	**3.412**	0.00	0.62
THL	0.665	1.515	**2.233**	0.00	0.56
TUR	**-2.732**	**-2.392**	-0.085	0.46	0.00

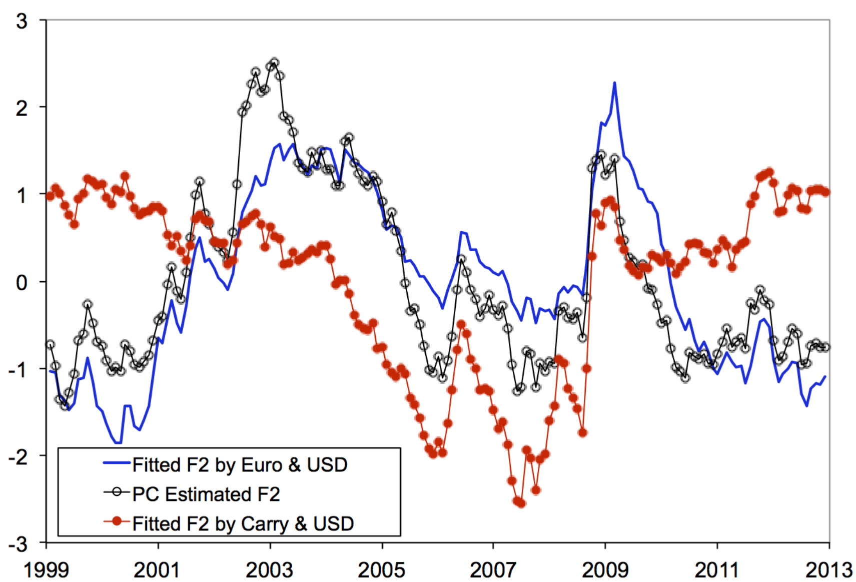
Note: Freq. P_4 is the frequency with which the currency is in the P_4 portfolio. Similarly for Freq. P_1.

Figure 1: Fitted from Regression of Principal Components on Potential Empirical Factors

A. Regression of $\hat{f}_{1t}$ on Potential Empirical Factors



B. Regression of $\hat{f}_{2t}$ on Potential Empirical Factors



and the dollar and euro empirical factors is

$$f_{1t} = a_{11}\Delta\bar{s}_t + a_{12}\Delta\tilde{s}_{\mathrm{EUR}t} + \epsilon_{1t}, \tag{13}$$

$$f_{2t} = a_{21}\Delta\bar{s}_t + a_{22}\Delta\tilde{s}_{\mathrm{EUR}t} + \epsilon_{2t}. \tag{14}$$

Recall from (2), country i's log SDF has a two-factor structure. From eqs.(13),(14) and (3) it is also related to the US, euro-zone and the global log SDF,

$$n_{it} = b_{i1}\bar{n}_t + b_{i2}n_{\mathrm{EUR}t} + b_{i3}n_{\mathrm{USA}t} + \tilde{\delta}_{i,1}\epsilon_{1t} + \tilde{\delta}_{i,2}\epsilon_{2t} + n_{it}^o \tag{15}$$

where

$$\begin{aligned}
b_{i1} &= \left(1 + \left(\tilde{\delta}_{i,1}a_{11} + \tilde{\delta}_{i,2}a_{21}\right) - \left(\tilde{\delta}_{i,1}a_{12} + \tilde{\delta}_{i,2}a_{22}\right)\right) \\
b_{i2} &= \left(\tilde{\delta}_{i,1}a_{12} + \tilde{\delta}_{i,2}a_{22}\right) \\
b_{i3} &= -\left(\tilde{\delta}_{i,1}a_{11} + \tilde{\delta}_{i,2}a_{21}\right).
\end{aligned}$$

Every country's log SDF is seen to be connected to the global log SDF $\bar{n}_t$, the US log SDF $n_{\mathrm{USA}t}$ and the Euro-zone log SDF $n_{\mathrm{EUR}t}$. By substituting (15) into (1), it can be seen that exchange rate returns are governed by the US, euro and a global ($\bar{n}_t$) log SDF.

A parallel structure for the log SDFs is obtained when the USD is not the numeraire currency. For example, if we let country 1 (Australia, say) be the numeraire country and write the exchange rates from this alternative normalization using the upper case S to denote log exchange rates with an alternative (non USD) numeraire,

$$S_{it}^1 = s_{it} - s_{1t} \tag{16}$$

for $i = 0, ..., N$, then an increase in S_{it}^1 is an increase in the AUD. In the identification procedure, we found unambiguously that the dollar and euro exchange rates $\Delta\tilde{S}_{\mathrm{USA}t}^1$ and $\Delta\tilde{S}_{\mathrm{EUR}t}^1$, form the empirical factor pair. This is the expected result since the invariance of the true latent factor representation on deviations from the mean form gives

$$\Delta\tilde{S}_{it}^1 = n_{it} - \bar{n}_t = \tilde{\delta}_i' f_t + n_{it}^o.$$

This means the implied relation between the economic variables and the latent factors is

$$f_{1t} = b_{11}\Delta\tilde{S}_{\mathrm{USA}t}^1 + b_{12}\Delta\tilde{S}_{\mathrm{EUR}t}^1 + v_{1t} \tag{17}$$

$$f_{2t} = b_{21}\Delta\tilde{S}_{\mathrm{USA}t}^1 + b_{22}\Delta\tilde{S}_{\mathrm{EUR}t}^1 + v_{2t} \tag{18}$$

where

$$\Delta\tilde{S}_{\mathrm{USA}t}^1 = n_{\mathrm{USA}t} - \bar{n}_t \tag{19}$$

$$\Delta\tilde{S}_{\mathrm{EUR}t}^1 = n_{\mathrm{EUR}t} - \bar{n}_t \tag{20}$$

Notice that $n_{\mathrm{USA}t}$ enters (17) and (18) with the opposite sign from (13) and (14), so the b_{ij} coefficients are not the same as the a_{ij} coefficients. Together, (17)-(20) give country log SDFs

$$\begin{aligned} n_{it} = & \left(\tilde{\delta}_{i,1} b_{11} + \tilde{\delta}_{i,2} b_{21}\right) n_{\mathrm{USA}t} - \left(\tilde{\delta}_{i,1} b_{12} + \tilde{\delta}_{i,2} b_{22}\right) n_{\mathrm{EUR}t} \\ & + \left(1 - \left(\tilde{\delta}_{i,1} b_{12} + \tilde{\delta}_{i,2} b_{22} + \tilde{\delta}_{i,1} b_{11} + \tilde{\delta}_{i,2} b_{21}\right)\right) \bar{n}_t + \tilde{\delta}_{i,1} v_{1t} + \tilde{\delta}_{i,2} v_{2t} \end{aligned} \tag{21}$$

We have seen that the empirical factor identification is useful in that it helps to give an economic interpretation for cross-currency comovements of exchange rates. In the next section, we show that the identification can also work to improve empirical exchange rate models in terms of their ability to forecast.

4 Multilateral Empirical Exchange Rate Modeling

In empirical exchange rate modeling, research aims to understand the relationship between exchange rate movements and macroeconomic fundamentals. One criterion for this aim is to examine the predictability of models. Although Inoue and Kilian (2004) points out that In-sample test is more powerful than out-of-sample tests in testing the predictability of exchange rates, it has been customary to evaluate empirical models by out-of-sample forecast accuracy since Meese and Rogoff (1983). We therefore adopt the out-of-sample test to examine the predictability of exchange rates. Forecasting is for nominal exchange rate returns over horizons from 1 to 24 months ahead, with the USD as numeraire. Forecast ability for any pair of exchange rates implies forecast ability for the associated cross rate. Our dollar-euro factor identification motivates a particular *multilateral* forecasting model for bilateral exchange rates.

Exchange rates are an asset price. As in other asset-pricing research, exchange rate forecasting aims to exploit information contained in the deviation of the exchange rate from a fundamental value which is thought to be a measure of central tendency. The strategy shares much with studies of stock prices where variables such as the dividend-price ratio or book value relative to market value of firms predict future equity returns. For stock prices, a certain multiple of dividends (or book value) plays the role of the central tendency for price. To develop the forecasting model, rewrite the factor representation (3) in levels,

$$\begin{aligned} s_{it+1} &= \alpha_i + \left(\delta_i' - \delta_0'\right) F_{t+1} + s_{it+1}^o \\ &= \alpha_i + \left(\delta_i' - \delta_0'\right) F_t + s_{it}^o + \left(\delta_i' - \delta_0'\right) f_{t+1} + \omega_{it+1} \end{aligned} \tag{22}$$

where α_i is a constant of integration, $F_{t+1} = (F_{1t+1}, F_{2t+1})' = F_t + f_{t+1}$ is the cumulated value of the true latent factors from the first-differenced specification and the idiosyncratic part is similarly cumulated $s_{it+1}^o = \sum_{j=1}^{t+1} \Delta s_{ij}^o = s_{it}^o + \omega_{it+1}$ and $\Delta s_{it}^o = n_{it}^o - n_{0t}^o$. Estimation of the true factors must be done in first-difference form because the idiosyncratic component s_{it}^o, in levels is unit-root nonstationary.

Denote currency identification of f_t (F_t) by the dollar and euro factors as $\hat{f}_t^c$ ($\hat{F}_t^c$). To implement empirically, we set the dollar factor as before and the euro factor as the cross-sectional average of exchange rates with the euro as numeraire,

$$\begin{aligned}\hat{F}_{1t}^c =& \bar{s}_t = \frac{1}{N}\sum_i s_{it},\\ \hat{F}_{2t}^c =& \bar{S}_t^{\text{EUR}} = \left(\frac{1}{N}\sum_{i\neq \text{EUR}}^{N+1} s_{it}\right) - s_{\text{EUR}t}.\end{aligned}$$

Since $\hat{F}_t^c$ is built from the differenced specification f_t^c, it is consistent for F_t.[10] Eq.(22) can be represented as

$$s_{it+1} = \alpha_i + \left(\delta_i' - \delta_0'\right)\hat{F}_t^c + \left(\omega_{it+1} + \left(\delta_i' - \delta_0'\right)\left(f_{t+1} + F_t - \hat{F}_t^c\right) + s_{it}^o\right) \tag{23}$$

We still have the problem that s_{it}^o in the composite error term is non stationary. By analogy to using the dollar and euro factors to estimate F_t, we use the cross-sectional average of exchange rates with currency i as numeraire

$$\bar{S}_t^i = \left((N-1)^{-1}\sum_{j\neq i} s_{jt}\right) - s_{it},$$

to estimate the idiosyncratic piece s_{it}^o where $s_{it}^o - \phi_i \bar{S}_t^i = \tilde{s}_{it}^o$ is stationary. Adding and subtracting $\phi_i \bar{S}_t^i$ from (23) gives

$$s_{it+1} = \alpha_i + \left(\bar{s}_t, \bar{S}_t^{\text{EUR}}\right)\left(\delta_i - \delta_0\right) + \phi_i \bar{S}_t^i + \epsilon_{it+1} \tag{24}$$

where $\epsilon_{it+1} = \omega_{it+1} + (\delta_i' - \delta_0')\left(f_{t+1} + F_t - \hat{F}_t^c\right) + \tilde{s}_{it}^o$ is stationary. Subtracting s_{it} from both sides of (24) gives the predictive regression

$$s_{it+1} - s_{it} = \left(\alpha_i + \left(\bar{s}_t, \bar{S}_t^{\text{EUR}}\right)\left(\delta_i - \delta_0\right) + \phi_i \bar{S}_t^i - s_{it}\right) + \epsilon_{it+1} \tag{25}$$

We view the systematic part of (25) as the deviation from the fundamental value (playing the role of an error-correction term) which contains the information content for forecasting. Implementation of a 'restricted' forecasting model recursively estimates

$$s_{it} = \alpha_i + \left(\bar{s}_t, \bar{S}_t^{\text{EUR}}\right)\left(\delta_i - \delta_0\right) + \phi_i \bar{S}_t^i + \tilde{s}_{it}^o$$

which is inputted into the forecasting model. Heavily parameterized models forecast poorly due to uncertainty from estimation error and the restricted model involves two-stages of estimation,

[10] We place the initial value $N^{-1}\sum_i s_{i0}$ in the constant α_i.

however. Instead, our forecast analysis employs the more parsimonious unrestricted model,

$$s_{it+1} - s_{it} = \alpha_i + \beta_{i1}\bar{s}_t + \beta_{i2}\bar{S}_t^{\mathrm{EUR}} + \beta_{i3}\bar{S}_t^i + \beta_{i4}s_{it} + \epsilon_{it+1}. \tag{26}$$

The unrestricted version of the model involves only one-stage of estimation and reduces the estimation uncertainty.

We compare forecast accuracy of the multilateral exchange rate model to predictions from the driftless random walk. Theil's U statistics, the ratio of root-mean-square forecast error (MSFE) from the model to those from the random walk, are used to assess accuracy of point forecasts. Assessing statistical significance of forecasts in comparison to the random walk involves comparing nested models. This leads to greater bias in the mean-square forecast error (MSFE) of larger models than smaller models due to the fact that the larger model has more parameters to be estimated with the same amount of data. Hence, we use the Clark and West (2006) test of forecast accuracy, which adjusts the MSFE to account for this bias.

To investigate value-added from identification, we compare forecast accuracy of the multilateral exchange rate model with alternative models shown to be useful in the literature. The first is the bilateral purchasing-power parity (PPP) fundamentals model. It posits that the fundamental value of s_{it} is the PPP $p_{it} - p_{\mathrm{USA}t}$, where p_{it} is the log price level of country i. Although s_{it} may deviate from its PPP over the short and medium term, if over the long term they both have a common trend, the real exchange rate will be stationary and mean-reverting. The PPP-based fundamentals model is thus an error correction without the short-run dynamics,

$$s_{it+k} - s_{it} = \alpha_i + \beta_i q_{it} + v_{it+k}. \tag{27}$$

If the nominal exchange rate is not weakly exogenous, we expect $\beta_i > 0$ so that the exchange rate s_{it} moves towards the PPP value $p_{it} - p_{\mathrm{USA}t}$ over time.

This is an example of a bilateral model in the sense that the fundamentals $p_{it} - p_{\mathrm{USA}t}$ depends only on variables from the associated bilateral pair of countries. Exchange rate models are typically formulated in bilateral terms. Examples of alternative bilateral fundamentals-based models include monetary-based models (Mark, 1995) and Taylor Rule models augmented with the real exchange rate (Molodtsova and Papell, 2009 and Molodtsova et al., 2008, 2011). Although there are institutional reasons to favor the Taylor-Rule approach, Engel, Mark and West (2007) conclude that while such models have some power to beat the random walk at long horizons, the results appear to be the strongest under PPP fundamentals.

Alternatively, we can ignore empirical identification of factors with economic variables and used principal components to estimate factors F_t for forecasting. This was the strategy of Engel et al. (2015). So how does the multilateral model compare with a pure statistical factor model? To address this question, we compare our forecasts to a principal components forecasting model.[11] Using quarterly data beginning in 1973, Engel et al. find that predictions of the factor-based forecasts significantly dominate random walk forecasts in mean-square error when forecasting from 1999 to 2007. For this model, recursively estimate the unobserved vector of factors by principal components $\hat{F}_t^{pc}$ to be used in the forecasting model

$$s_{it+h} - s_{it} = \alpha_i + \beta_{i1}\hat{F}_{1t}^{pc} + \beta_{i2}\hat{F}_{2t}^{pc} + \beta_{i3}s_{it} + \epsilon_{it+h} \tag{28}$$

We note that Engel et al., used the 'restricted' version of the forecasting which includes an extra-round of estimation. They forecasted by recursively estimating both the principal components and factor loadings which were inputted into the forecasting model $s_{it+h} - s_{it} = \alpha_i + \beta_i\hat{s}_{it}^o + \epsilon_{it+h}$ where $\hat{s}_{it}^o = s_{it} - \hat{\delta}_{i1}\hat{F}_{1t} - \hat{\delta}_{i2}\hat{F}_{2t}$. Here, we use principal components in the 'unrestricted' forecasting model. This eliminates the estimation of factor loadings, which gives more accurate forecasts than the restricted forecasts.

The fourth model that we consider is the dollar-carry factor combination. This formulation replaces the euro factor with the carry exchange rate (in levels) in (26),

$$s_{it+h} - s_{it} = \alpha_i + \beta_{i1}s_{it} + \beta_{i2}\bar{s}_t + \beta_{i2}\bar{s}_t^c + \beta_{i3}\bar{S}_t^i + \epsilon_{i,t+h}. \tag{29}$$

The principal components model and the dollar-carry formulations are also multilateral exchange rate models.

We reserve the period form 2005.01 to 2012.12 for forecasting. Forecasts are generated at horizons $h = 1, 2, \ldots, 24$ and for each month from 2005.01 through 2012.12. The 1-month forecast of 2005.01 is generated using data up to 2004.12 for model estimation, while the 24-month forecast of 2005.01 is generated using data up to 2003.01. Thus the number of forecasts we make is independent of the forecast horizon.

To summarize, we are comparing our identified multilateral dollar-euro factor exchange rate model to the bilateral PPP fundamentals model (Bi-PPP), the two-principal components model (PC) and a dollar-carry factor model. We list them as,

$$\begin{aligned}
\text{Dollar-Euro:} \quad & s_{it+h} - s_{it} = \alpha_i + \beta_{i1}s_{it} + \beta_{i2}\bar{s}_t + \beta_{i2}\bar{S}_t^{\text{EUR}} + \beta_{i3}\bar{S}_t^i + \epsilon_{i,t+h} \\
\text{Bi-PPP:} \quad & s_{it+h} - s_{it} = \alpha_i + \beta_i\left(s_{it} - (p_{it} - p_{\text{USA}t})\right) + \epsilon_{it+h} \\
\text{PC:} \quad & s_{it+h} - s_{it} = \alpha_i + \beta_{i1}\hat{F}_{1t} + \beta_{i2}\hat{F}_{2t} + \beta_{i3}s_{it} + \epsilon_{it+h} \\
\text{Dollar-Carry:} \quad & s_{it+h} - s_{it} = \alpha_i + \beta_{i1}s_{it} + \beta_{i2}\bar{s}_t + \beta_{i2}\bar{s}_t^c + \beta_{i3}\bar{S}_t^i + \epsilon_{i,t+h}
\end{aligned}$$

Results comparing the bilateral PPP-based model to our dollar-euro exchange rate model are shown in Table 4. The table reports Theil's U for (27) at horizons $h = 1, 6, 12, 24$ months. Forecast

[11] Engel et al. considered 1,2, and 3 factor models. The forecasting ability of the 2 and 3 factor models were nearly identical and dominated that of the 1 factor model.

performance for bilateral PPP fundamentals is shown in panel A. As is typically found, forecasting accuracy improves with length of horizon. The model is generally able to forecast over this time period. Clark-West rejection frequencies are non zero for 20 of the 27 exchange rates, indicative of significant forecasting ability for those currencies. Panel B shows ratios of Theil's U from the multilateral model to the bilateral model. Values below 1 indicate that the multilateral model forecasts more accurately than the bilateral PPP model. At the 24 month horizon, multilateral model forecasts dominate bilateral forecasts for all exchange rates. Both models can forecast better than the random walk. The dollar-euro multilateral model forecasts more accurately than bilateral PPP.

Results comparing our dollar-euro factor model to the unrestricted principal components model are shown in Table 5. Panel A shows Theil's U and Clark-West rejection frequencies for the Engel et al. statistical factors model. The model is able to forecast exchange rates about as well as the bilateral PPP based model. Significant forecasting ability is detected for 21 of 27 currencies at the 6 month horizon and 27 of 27 at the 12 and 24 month horizons for the principal components model. Panel B shows ratios of Theil's U from the multilateral model to the principal components model. Values below 1 indicate that the multilateral model forecasts more accurately. At the 24 month horizon, multilateral model forecasts dominate principal components forecasts for 19 of 27 exchange rates. The unrestricted principal components model is also able to forecast. Prediction accuracy of the dollar-euro model is approximately the same as the principal components model. Overall, multilateral model forecasts dominate principal components forecasts in 60 percent of the forecast horizons.

Results comparing our dollar-euro factor model to the dollar-carry model are shown in Table 6. Although the carry was not identified in our sample as an empirical factor, the dollar-carry model is able to forecast. It is because the carry factor can be thought as a weighted average 14 bilateral exchange rates. By including the dollar factor with the weighted average, the Euro factor can be identified. The Clark-West test rejects the null hypothesis (of no forecasting ability) for every currency, and Theil's U is less than 1 for every currency at 12 and 24 months forecast horizons. The dollar-euro factor model dominates the dollar-carry model at the 12 month horizon for 15 of 27 exchange rates and at the 24 month horizon in 18 of 27. The dollar-carry model forecasts generally as accurately as the dollar-euro model.

Table 4: Bilateral PPP and Dollar-Euro Factor Models: Theil's U and Clark-West Rejection Frequencies

	A. U(Bi-PPP)				CW	B. U(Dollar-Euro)				CW
	Horizon				reject	Horizon				reject
	1	6	12	24	freq	1	6	12	24	freq
AUS	1.012	0.982	0.929	0.955	0.625	1.035	**0.783**	**0.477**	**0.321**	0.875
BRA	1.069	1.164	1.334	2.192	0	1.180	1.355	**0.985**	**0.344**	0.542
CAN	1.021	0.968	0.899	0.628	0.708	1.045	**0.893**	**0.626**	**0.278**	0.708
CHL	1.028	1.062	1.090	1.812	0	1.113	1.173	**0.799**	**0.496**	0.708
COL	1.064	1.258	1.584	2.336	0	**1.059**	**0.741**	**0.642**	**0.462**	0.958
CZE	1.018	0.965	0.891	0.569	0.542	1.289	1.016	**0.884**	0.646	0.708
EUR	1.014	0.955	0.868	0.928	0.792	1.087	0.959	**0.808**	**0.874**	0.75
GBR	1.020	0.980	0.924	0.733	0.667	1.069	**0.776**	**0.617**	**0.576**	0.917
HUN	0.999	0.877	0.699	0.482	0.958	1.129	0.903	**0.661**	**0.319**	0.833
ICE	1.012	0.914	0.787	0.668	0.875	1.061	**0.745**	**0.347**	**0.403**	0.958
IND	1.017	0.891	0.761	0.735	0.833	1.017	**0.827**	**0.651**	**0.374**	0.958
ISR	0.987	0.867	0.825	0.581	0.958	1.057	0.977	**0.625**	**0.200**	0.958
JPN	1.008	0.774	0.610	0.477	0.958	1.014	**0.526**	**0.324**	0.527	0.958
KOR	1.013	0.969	0.900	0.861	0.75	1.103	**0.768**	**0.417**	**0.391**	0.875
MEX	1.011	0.936	0.836	0.709	0.875	1.072	**0.788**	**0.546**	**0.611**	0.917
NOR	1.116	1.422	1.711	2.025	0	1.157	**0.994**	**0.669**	**0.322**	0.708
NZL	1.020	0.946	0.885	0.760	0.75	1.094	**0.894**	**0.768**	**0.562**	0.833
PHI	1.087	1.225	1.657	1.976	0	**1.036**	**0.841**	**0.824**	**0.552**	0.917
POL	1.036	0.964	0.891	0.722	0.625	**0.983**	**0.767**	**0.569**	**0.295**	1
ROM	0.999	0.879	0.742	0.964	0.958	1.103	0.896	0.800	**0.546**	1
RSA	1.011	0.896	0.749	0.591	0.958	1.112	0.955	**0.672**	**0.348**	0.708
SIN	1.004	0.879	0.728	0.747	0.958	1.078	**0.815**	**0.541**	**0.350**	0.875
SUI	1.025	0.952	0.853	0.699	0.833	1.076	**0.830**	**0.405**	**0.562**	0.958
SWE	0.989	0.850	0.696	0.482	0.958	1.059	0.905	0.844	**0.254**	0.958
TAI	1.107	1.346	1.320	1.360	0	**1.104**	**1.181**	**0.772**	**0.370**	0.625
THL	1.068	1.170	1.325	3.560	0	**1.001**	**1.036**	**0.680**	**0.533**	1
TUR	1.004	0.873	0.756	0.608	0.917	1.025	**0.723**	**0.397**	**0.264**	0.875

Note: Theil's U values less than 1 are underlined to signify forecast accuracy relative to the random walk. Dollar-Euro forecasts that beat Bi-PPP in bold.

Table 5: Principal Components and Dollar-Euro Factor Models: Theil's U and Clark-West Rejection Frequencies

	A. U(PC)				CW	B. U(Dollar-Euro)/U(PC)			
	Horizon				reject	Horizon			
	1	6	12	24	freq	1	6	12	24
AUS	**0.979**	**0.736**	**0.583**	**0.461**	1	1.057	1.064	**0.818**	**0.696**
BRA	1.096	1.410	1.238	**0.490**	0.375	1.077	**0.961**	**0.796**	**0.702**
CAN	**0.993**	**0.844**	**0.628**	**0.295**	0.958	1.052	1.058	**0.997**	**0.942**
CHL	1.028	1.139	**0.791**	**0.363**	0.708	1.083	1.030	1.010	1.366
COL	1.018	**0.917**	**0.919**	**0.538**	0.917	1.040	**0.808**	**0.699**	**0.859**
CZE	1.148	**0.954**	**0.802**	**0.623**	0.75	1.123	1.065	1.102	1.037
EUR	1.061	**0.877**	**0.785**	**0.858**	0.75	1.025	1.094	1.029	1.019
GBR	1.035	**0.746**	**0.623**	**0.588**	0.917	1.033	1.040	**0.990**	**0.980**
HUN	1.105	**0.831**	**0.597**	**0.285**	0.875	1.022	1.087	1.107	1.119
ICE	1.049	**0.814**	**0.671**	**0.576**	0.917	1.011	**0.915**	**0.517**	**0.700**
IND	**0.966**	**0.854**	**0.834**	**0.560**	1	1.053	**0.968**	**0.781**	**0.668**
ISR	1.060	1.286	1.124	**0.778**	0.5	**0.997**	**0.760**	**0.556**	**0.257**
JPN	1.084	**0.823**	**0.654**	**0.664**	0.875	**0.935**	**0.639**	**0.495**	**0.794**
KOR	1.003	**0.694**	**0.459**	**0.476**	0.958	1.100	1.107	**0.908**	**0.821**
MEX	1.025	**0.728**	**0.557**	**0.631**	0.917	1.046	1.082	**0.980**	**0.968**
NOR	1.004	**0.948**	**0.834**	**0.474**	0.708	1.152	1.049	**0.802**	**0.679**
NZL	1.044	**0.865**	**0.748**	**0.542**	0.875	1.048	1.034	1.027	1.037
PHI	1.085	**0.908**	**0.803**	**0.659**	0.833	**0.955**	**0.926**	1.026	**0.838**
POL	1.101	**0.985**	**0.682**	**0.294**	0.792	**0.893**	**0.779**	**0.834**	1.003
ROM	1.015	**0.963**	**0.993**	**0.472**	0.917	1.087	**0.930**	**0.806**	1.157
RSA	1.063	**0.895**	**0.693**	**0.371**	0.708	1.046	1.067	**0.970**	**0.938**
SIN	1.017	**0.779**	**0.517**	**0.330**	0.917	1.060	1.046	1.046	1.061
SUI	1.144	1.045	**0.998**	**0.824**	0.583	**0.941**	**0.794**	**0.406**	**0.682**
SWE	1.037	1.002	**0.930**	**0.464**	0.917	1.021	**0.903**	**0.908**	**0.547**
TAI	1.014	**0.898**	**0.742**	**0.566**	0.833	1.089	1.315	1.040	**0.654**
THL	1.045	1.010	**0.799**	**0.731**	0.75	**0.958**	1.026	**0.851**	**0.729**
TUR	1.106	**0.835**	**0.591**	**0.396**	0.792	**0.927**	**0.866**	**0.672**	**0.667**

Note: Panel A: Theil's U and Clark-West rejection frequencies for principal components model forecasts. Bolded values indicate forecast ability relative to the random walk. Panel B: Ratio of Theil's U for Dollar-Euro model to principal components model. Bolded values indicate forecast ability relative to the principal components model.

Table 6: Dollar-Carry and Dollar-Euro Factor Models: Theil's U and Clark-West Rejection Frequencies

	A. U(Dollar-Carry)				CW	B. U(Dollar-Euro)/U(Dollar-Carry)			
	Horizon				reject	Horizon			
	1	6	12	24	freq	1	6	12	24
AUS	1.052	**0.771**	**0.493**	**0.435**	0.917	**0.984**	1.016	**0.968**	**0.738**
BRA	1.103	1.115	**0.979**	**0.594**	0.542	1.070	1.215	1.006	**0.579**
CAN	1.031	**0.902**	**0.618**	**0.282**	0.75	1.014	**0.990**	1.013	**0.986**
CHL	1.148	1.094	**0.917**	**0.757**	0.708	**0.970**	1.072	**0.871**	**0.655**
COL	1.068	**0.722**	**0.644**	**0.493**	0.958	**0.992**	1.026	**0.997**	**0.937**
CZE	1.159	1.003	**0.894**	**0.583**	0.667	1.112	1.013	**0.989**	1.108
EUR	1.075	**0.955**	**0.779**	**0.735**	0.833	1.011	1.004	1.037	1.189
GBR	1.024	**0.747**	**0.615**	**0.422**	0.958	1.044	1.039	1.003	1.365
HUN	1.084	**0.790**	**0.527**	**0.318**	0.917	1.042	1.143	1.254	1.003
ICE	1.068	**0.807**	**0.370**	**0.477**	0.958	**0.993**	**0.923**	**0.938**	**0.845**
IND	**0.945**	**0.666**	**0.631**	**0.386**	1	1.076	1.242	1.032	**0.969**
ISR	1.039	**0.951**	**0.645**	**0.273**	0.917	1.017	1.027	**0.969**	**0.733**
JPN	1.023	**0.649**	**0.430**	**0.530**	0.958	**0.991**	**0.810**	**0.753**	**0.994**
KOR	1.091	**0.768**	**0.431**	**0.511**	0.875	1.011	1.000	**0.968**	**0.765**
MEX	1.069	**0.809**	**0.613**	**0.696**	0.917	1.003	**0.974**	**0.891**	**0.878**
NOR	1.086	**0.896**	**0.623**	**0.322**	0.792	1.065	1.109	1.074	1.000
NZL	1.014	**0.845**	**0.745**	**0.471**	0.958	1.079	1.058	1.031	1.193
PHI	**0.986**	**0.967**	**0.959**	**0.594**	0.958	1.051	**0.870**	**0.859**	**0.929**
POL	1.063	**0.866**	**0.588**	**0.283**	0.875	**0.925**	**0.886**	**0.968**	1.042
ROM	1.156	**0.959**	**0.739**	**0.763**	1	**0.954**	**0.934**	1.083	**0.716**
RSA	1.064	**0.818**	**0.599**	**0.354**	0.875	1.045	1.167	1.122	**0.983**
SIN	1.052	**0.829**	**0.587**	**0.335**	0.833	1.025	**0.983**	**0.922**	1.045
SUI	1.094	**0.872**	**0.595**	**0.625**	0.792	**0.984**	**0.952**	**0.681**	**0.899**
SWE	1.102	**0.909**	**0.786**	**0.417**	0.875	**0.961**	**0.996**	1.074	**0.609**
TAI	1.208	1.124	**0.707**	**0.343**	0.708	**0.914**	1.051	1.092	1.079
THL	1.008	1.037	**0.821**	**0.552**	1	**0.993**	**0.999**	**0.828**	**0.966**
TUR	1.045	**0.723**	**0.443**	**0.290**	0.917	**0.981**	1.000	**0.896**	**0.910**

Note: Panel A: Theil's U and Clark-West rejection frequencies for dollar-carry model forecasts. Bolded values indicate forecast ability relative to the random walk. Panel B: Ratio of Theil's U for Dollar-Euro model to dollar-carry model. Bolded values indicate forecast ability relative to dollar-carry model.

Table 7: Theil's U and Clark-West Rejection Frequencies for Daily Forecasts. Principal Components and Dollar-Euro Factor Models

	A. Principal Components					CW	B. Dollar-Euro to Principal Components					CW
	Horizon					reject	Horizon					reject
	1 day	1 week	2 weeks	3 weeks	4 weeks	Rate	1 day	1 week	2 weeks	3 weeks	4 weeks	Rate
AUS	1.016	0.958	0.870	0.709	0.582	0.875	**0.993**	**0.930**	**0.978**	1.047	1.120	0.958
BRA	0.983	0.810	0.714	0.594	0.508	1	1.005	**0.953**	**0.866**	**0.783**	**0.774**	1
CAN	1.008	0.974	0.936	0.792	0.664	0.875	**0.993**	**0.926**	**0.912**	**0.896**	**0.873**	0.958
DEN	1.033	1.019	0.983	0.870	0.719	0.667	1.018	1.026	1.044	1.018	1.013	0.583
EUR	1.034	1.027	0.996	0.884	0.731	0.667	1.009	1.042	1.054	1.050	1.081	0.625
GBR	0.997	0.890	0.853	0.776	0.620	0.958	1.029	1.113	1.199	1.290	1.368	0.5
IND	1.052	1.135	1.262	1.375	1.467	0.333	1.005	1.091	1.091	1.049	1.035	0.333
JPN	1.059	1.164	1.229	1.229	1.149	0	**0.987**	**0.973**	**0.888**	**0.792**	**0.728**	0.917
KOR	1.006	0.919	0.817	0.699	0.564	0.958	1.003	**0.949**	**0.857**	**0.755**	**0.668**	0.958
MEX	1.047	1.093	0.991	0.794	0.658	0.625	1.051	1.198	1.128	1.013	**0.954**	0.542
NOR	1.051	1.086	1.063	0.914	0.794	0.583	1.001	**0.979**	**0.960**	**0.899**	**0.812**	0.625
NZL	1.016	0.964	0.858	0.739	0.672	0.833	**0.985**	**0.897**	**0.938**	1.018	1.052	0.958
RSA	1.008	0.920	0.864	0.812	0.786	0.958	**0.990**	**0.914**	**0.853**	**0.798**	**0.733**	0.958
SIN	0.996	0.855	0.735	0.512	0.328	1	1.012	1.004	1.024	1.066	1.165	0.958
SUI	1.080	0.873	0.849	0.826	0.794	0.958	1.105	1.034	1.026	**0.977**	**0.858**	0.958
SWE	1.081	1.304	1.086	0.817	0.625	0.708	**0.994**	**0.982**	**0.995**	1.022	1.056	0.708
TAI	1.032	1.040	1.031	0.998	0.948	0.417	1.020	1.048	1.040	1.042	**0.969**	0.458
THL	1.082	1.371	1.524	1.675	1.749	0.417	**0.973**	**0.874**	**0.856**	**0.780**	**0.742**	0.792

Note: Theil's U values less than 1 are underlined to signify forecast accuracy relative to the random walk. Dollar-Euro forecast accuracy relative to the principal components in bold.

Daily forecasting. Can the dollar-euro model forecast at daily horizons? Here, we consider forecasting with daily exchange rates at 1-day, 1, 2, 3, and 4 week horizons. Daily observations on exchange rates were not available for several countries. Forecasting is done for the 18 currencies that were available to us from IHS Global Insight and FRED. The daily sample contains 565 observations beginning 01/02/2013 and ending 04/03/2015. The sample from 02/18/2014 to 04/03/2015 is reserved for out-of-sample forecasting. Results for the principal components model and the dollar-euro model are shown in Table 6. Compared to the random walk, both models are able to forecast daily exchange rates. The dollar-euro model dominates principal components in RMFE (and is dominated by) in nearly half (45 of 90) of the currency and forecast horizons shown. Statistically significant forecasting ability is found by positive Clark-West rejection frequencies in 17 of 18 currencies for the principal components model (it cannot forecast the yen) and for all 18 currencies for the dollar-euro model. Forecasting accuracy as measured by Theil's U is also seen to improve at the longer horizons with daily data.

Does the Crisis Drive Forecasting Results? While the dollar-euro multilateral model can forecast, one concern may be that cross-sectional exchange rate correlation was unusually high during the global financial crisis and that forecasting performance is driven primarily by the crisis period. The U.S. was perceived as a 'safe haven' and the demand for U.S. Treasury debt, viewed as a safe asset, was particularly strong with international investors.

To address this issue, we generate monthly forecasts with the bilateral PPP fundamentals and our multilateral model using 1995.01 to 1998.08 for the out-of-sample forecasting period. The bilateral PPP model does not forecast well in this period. Clark-West tests do not reject equal forecast accuracy (with the random walk) for 11 of 27 currencies. The three multilateral models, (dollar-euro, principal components, dollar-carry) are able to forecast in the pre-crisis period. Clark-West rejections are obtained for every exchange rate for all three multilateral models. The pre-crisis forecasting results are contained in the appendix.

5 Conclusion

This paper studies the source of comovments across exchange rates. We identified a dollar factor and a euro factor as the pair of common empirical factors driving a panel of exchange rates. Drawing on the SDF approach to the exchange rate, our identification can be interpreted as evidence that a global, a US and a euro-zone stochastic discount factor exhibit dominance in exchange rate movements. More generally, these represent global factors that have relevance for understanding asset prices in the international context.

Exchange rate factors emerge from economically large regions. Economic size evidently matters in exchange rate determination, and possibly in other international asset pricing settings. For example, Hassan (2013) argues that economic size matters in national interest rate determination.

On the empirical side, the identification implies a multilateral dollar-euro factor model for bilateral exchange rates. In out-of-sample forecasting, the multilateral model outperforms the random walk and the bilateral purchasing-power parity fundamentals model. Forecast performance was in line with the pure statistical (principal components) factor forecasting model in terms of mean-square forecast error. The alternative multilateral model consisting of a dollar and carry factor generates similarly accurate forecasts.

The point of the forecasting analysis was not to find the best forecasting model but to demonstrate value of identification. Instead of looking at bilateral determinants on a case-by-case basis, our identification says that empirical researchers need only understand the determinants the dollar and euro factors to understand most of the variation in any bilateral exchange rate.

Our findings suggests future directions for research. First, macro-modeling should recognize the potential importance of multi-country models for exchange rate determination. In empirical modeling, one should pay special attention to the role of the US and the euro zones on bilateral exchange rates. Consideration of multilateral factors can potentially solve the Obstfeld-Rogoff (2000) exchange rate disconnect puzzle. New directions for international asset pricing might emphasize a heightened role for global, US and euro stochastic discount factors.

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Appendix

Clark-West test

Interpreting MSFE as an estimator of the true (or population) MSFE of the model, Clark and West (2007) argue that this leads to greater bias in the MSFE of larger models than smaller models due to the fact that the larger model has more parameters to be estimated with the same amount of data. Clark and West (2007) therefore propose an adjusted MSFE to account for this bias. This adjustment is particularly appropriate when using out-of-sample loss as a basis for model evaluation (as it is in the current application).To test whether model a has a lower MSFE than model b we employ Clark and West's (2007) test of equal MSFEs from nested models.

The Clark and West test of the null hypothesis that $\ddot{U}_h^{(a,b)} < 1$ is based on testing whether the mean of

$$J_{i,s,h}^{(a,b)} = \left(\hat{s}_{i,s+h}^{a} - s_{i,s+h}\right)^2 - \left(\hat{s}_{i,s+h}^{a} - s_{i,s+h}^{b}\right)^2 - P^{-1}\textstyle\sum_{s=1}^{P}\left(\hat{s}_{i,s+h}^{b} - s_{i,s+h}\right)^2$$

is less than zero. Clark and West (2007) show that

$$P^{-1}\textstyle\sum_{s=1}^{P} J_{i,s,h}^{(a,b)} / \sqrt{V\left(P^{-1}\textstyle\sum_{s=1}^{P} J_{i,s,h}^{(a,b)}\right)} \overset{a}{\sim} N\left(0,1\right)$$

under the null hypothesis that $\ddot{U}_h^{(a,b)} = 1$. To estimate $V\left(J_{i,h}^{(a,b)}\right)$ they suggest using the Newey-West estimator. We use the estimator with the truncation lag set to be $h-1$ since the forecast errors overlap $h-1$ periods.

Forecasting in the Pre-Crisis Period

Here we report forecasting results for the bilateral PPP model and our multilateral model when the forecasting period ends before the onset of the global financial crisis. The out-of-sample forecasting period begins 2003.10 and ends 2008.08.

Table 8: Pre-Crisis Bilateral PPP and Dollar-Euro Factor Models: Theil's U and Clark-West Rejection Frequencies

	A. U(Bi-PPP)				CW	B. U(Dollar-Euro)				CW
	Horizon				reject	Horizon				reject
	1	6	12	24	freq	1	6	12	24	freq
AUS	1.026	0.922	1.241	1.189	0.125	**0.967**	**0.617**	**0.382**	**0.227**	1
BRA	1.367	2.125	1.855	2.067	0	**0.933**	**0.569**	**0.252**	**0.258**	1
CAN	1.031	0.847	0.635	0.633	0.917	1.034	**0.558**	**0.246**	**0.089**	0.958
CHL	1.098	1.281	1.432	1.294	0	1.076	**0.478**	**0.235**	**0.388**	0.958
COL	1.238	2.031	2.156	1.745	0	1.002	**0.578**	**0.329**	**0.266**	0.958
CZE	0.955	0.550	0.384	0.506	0.958	1.267	0.713	0.521	**0.215**	0.875
EUR	1.054	0.996	1.397	0.924	0	1.123	**0.943**	**0.693**	**0.305**	0.75
GBR	1.045	1.027	1.125	0.771	0.25	1.061	**0.644**	**0.367**	**0.122**	0.958
HUN	1.007	0.833	0.780	1.080	0.667	1.125	0.935	**0.728**	**0.180**	0.75
ICE	1.030	0.993	1.507	1.734	0	1.203	**0.819**	**0.527**	**0.217**	0.958
IND	1.064	1.139	1.217	1.635	0	1.093	**0.631**	**0.213**	**0.624**	0.917
ISR	1.031	0.964	0.738	0.436	0.75	1.109	**0.874**	**0.519**	**0.394**	0.792
JPN	1.017	0.878	0.755	0.450	0.792	1.203	1.091	**0.556**	**0.159**	0.792
KOR	1.058	1.140	1.363	1.065	0	1.040	**0.873**	**0.643**	**0.180**	0.958
MEX	1.037	0.971	0.726	0.194	0.708	1.140	**0.770**	**0.489**	0.214	0.875
NOR	1.018	0.874	1.183	1.102	0.208	1.125	0.939	**1.021**	**0.549**	0.792
NZL	1.034	0.998	1.316	1.052	0	1.057	**0.496**	**0.272**	**0.194**	1
PHI	1.210	1.513	1.836	1.186	0	1.056	0.785	**0.627**	**0.200**	0.917
POL	1.028	0.675	0.495	0.321	0.875	1.238	**0.800**	0.579	**0.230**	0.833
ROM	1.189	1.422	1.812	3.244	0.917	1.009	**0.697**	**0.490**	**0.340**	0.958
RSA	1.018	1.142	1.860	0.861	0.208	1.225	1.291	**1.165**	**0.082**	0.833
SIN	1.044	0.880	0.725	0.357	0.792	1.015	**0.545**	**0.241**	**0.203**	0.958
SUI	1.046	0.991	1.216	0.735	0.042	1.131	**0.675**	**0.393**	**0.349**	0.917
SWE	1.031	1.026	1.376	0.945	0.042	1.189	**0.937**	**0.559**	**0.189**	0.792
TAI	0.993	0.832	0.609	0.318	0.958	**0.985**	**0.549**	**0.452**	**0.286**	1
THL	1.104	1.294	1.457	1.094	0	**0.986**	**0.726**	**0.462**	**0.193**	1
TUR	1.203	4.195	7.935	32.463	0	1.191	3.499	1.530	**0.499**	0.792

Note: Theil's U values less than 1 are underlined to signify forecast accuracy relative to the random walk. Dollar-Euro forecasts that beat Bi-PPP in bold.

Table 9: Pre-Crisis Principal Components and Dollar-Euro Factor Models: Theil's U and Clark-West Rejection Frequencies

	A. U(PC)				CW	B. U(Dollar-Euro)/U(PC)			
	Horizon				reject	Horizon			
	1	6	12	24	freq	1	6	12	24
AUS	**0.994**	**0.717**	**0.409**	**0.270**	0.958	**0.973**	**0.861**	**0.934**	**0.841**
BRA	1.277	1.264	**0.363**	**0.167**	0.75	**0.731**	**0.450**	**0.694**	1.545
CAN	1.058	**0.739**	**0.340**	**0.109**	0.875	**0.977**	**0.755**	**0.724**	**0.817**
CHL	1.123	**0.621**	**0.283**	**0.161**	0.917	**0.958**	**0.770**	**0.830**	2.410
COL	**0.988**	**0.634**	**0.382**	**0.217**	0.958	1.014	**0.912**	**0.861**	1.226
CZE	1.202	**0.645**	**0.602**	**0.369**	0.917	1.054	1.105	**0.865**	**0.583**
EUR	1.065	**0.981**	**0.707**	**0.248**	0.708	1.054	**0.961**	**0.980**	1.230
GBR	1.057	**0.690**	**0.445**	**0.191**	0.958	1.004	**0.933**	**0.825**	**0.639**
HUN	1.114	1.072	**0.848**	**0.273**	0.625	1.010	**0.872**	**0.858**	**0.659**
ICE	1.003	**0.898**	**0.634**	**0.198**	0.958	1.199	**0.912**	**0.831**	1.096
IND	1.083	**0.755**	**0.353**	**0.513**	0.917	1.009	**0.836**	**0.603**	1.216
ISR	1.050	**0.838**	**0.694**	**0.452**	0.917	1.056	1.043	**0.748**	**0.872**
JPN	1.084	1.159	**0.764**	**0.118**	0.792	1.110	**0.941**	**0.728**	1.347
KOR	1.088	1.052	**0.727**	**0.147**	0.667	**0.956**	**0.830**	**0.884**	1.224
MEX	1.191	**0.895**	**0.502**	**0.158**	0.75	**0.957**	**0.860**	**0.974**	1.354
NOR	1.127	1.237	1.115	**0.542**	0.458	**0.998**	**0.759**	**0.916**	1.013
NZL	**0.971**	**0.583**	**0.431**	**0.223**	0.958	1.089	**0.851**	**0.631**	**0.870**
PHI	1.011	**0.900**	**0.706**	**0.277**	0.833	1.045	**0.872**	**0.888**	**0.722**
POL	1.192	**0.800**	**0.594**	**0.251**	0.833	1.039	1.000	**0.975**	**0.916**
ROM	1.004	**0.779**	**0.604**	**0.273**	0.958	1.005	**0.895**	**0.811**	1.245
RSA	1.196	2.069	1.452	**0.096**	0.625	1.024	**0.624**	**0.802**	**0.854**
SIN	1.147	**0.846**	**0.488**	**0.317**	0.833	**0.885**	**0.644**	**0.494**	**0.640**
SUI	1.044	**0.907**	**0.651**	**0.294**	0.917	1.083	**0.744**	**0.604**	1.187
SWE	1.100	**0.936**	**0.578**	**0.110**	0.875	1.081	1.001	**0.967**	1.718
TAI	**0.957**	**0.532**	**0.419**	**0.309**	1	1.029	1.032	1.079	**0.926**
THL	1.058	**0.931**	**0.763**	**0.398**	0.792	**0.932**	**0.780**	**0.606**	**0.485**
TUR	1.121	2.893	1.651	**0.810**	0.708	1.062	1.209	**0.927**	**0.616**

Note: Panel A: Theil's U and Clark-West rejection frequencies for principal components model forecasts. Bolded values indicate forecast ability relative to the random walk. Panel B: Ratio of Theil's U for Dollar-Euro model to principal components model. Bolded values indicate forecast abilty relative to the principal components model.

Table 10: Pre-Crisis Dollar-Carry and Dollar-Euro Factor Models: Theil's U and Clark-West Rejection Frequencies

	A. U(Dollar-Carry)				CW	B. U(Dollar-Euro)/U(Dollar-Carry)			
	Horizon				reject	Horizon			
	1	6	12	24	freq	1	6	12	24
AUS	**0.962**	**0.605**	**0.376**	**0.315**	1	1.005	1.020	1.016	0.721
BRA	**0.918**	**0.803**	**0.268**	**0.298**	1	1.016	**0.709**	**0.940**	**0.866**
CAN	1.070	**0.677**	**0.265**	**0.094**	0.917	**0.966**	**0.824**	**0.928**	**0.947**
CHL	1.047	**0.491**	**0.290**	**0.638**	0.958	1.028	**0.974**	**0.810**	**0.608**
COL	1.044	**0.573**	**0.376**	**0.274**	0.958	**0.960**	1.009	**0.875**	**0.971**
CZE	1.249	**0.763**	**0.587**	**0.248**	0.833	1.014	**0.934**	**0.888**	**0.867**
EUR	1.137	**0.955**	**0.700**	**0.299**	0.75	**0.988**	**0.987**	**0.990**	1.020
GBR	1.033	**0.627**	**0.390**	**0.215**	0.958	1.027	1.027	**0.941**	**0.567**
HUN	1.091	**0.945**	**0.756**	**0.236**	0.75	1.031	**0.989**	**0.963**	**0.763**
ICE	**0.978**	**0.758**	**0.538**	**0.219**	1	1.230	1.080	**0.980**	**0.991**
IND	1.060	**0.599**	**0.208**	**0.626**	0.958	1.031	1.053	1.024	**0.997**
ISR	1.059	**0.812**	**0.519**	**0.412**	0.833	1.047	1.076	1.000	**0.956**
JPN	1.067	1.035	**0.758**	**0.145**	0.875	1.127	1.054	**0.734**	1.097
KOR	1.073	**0.888**	**0.674**	**0.184**	0.875	**0.969**	**0.983**	**0.954**	**0.978**
MEX	1.169	**0.818**	**0.891**	**0.216**	0.833	**0.975**	**0.941**	**0.549**	**0.991**
NOR	1.122	1.110	1.241	**0.651**	0.417	1.003	**0.846**	**0.823**	**0.843**
NZL	1.012	**0.465**	**0.286**	**0.261**	1	1.044	1.067	**0.951**	**0.743**
PHI	1.102	**0.804**	**0.664**	**0.211**	0.917	**0.958**	**0.976**	**0.944**	**0.948**
POL	1.218	**0.821**	**0.617**	**0.231**	0.833	1.016	**0.974**	**0.938**	**0.996**
ROM	1.022	**0.719**	**0.668**	**0.412**	0.958	**0.987**	**0.969**	**0.734**	**0.825**
RSA	1.151	1.516	1.209	**0.109**	0.5	1.064	**0.852**	**0.964**	**0.752**
SIN	1.074	**0.710**	**0.387**	**0.276**	0.875	**0.945**	**0.768**	**0.623**	**0.736**
SUI	1.119	**0.646**	**0.368**	**0.347**	0.958	1.011	1.045	1.068	1.006
SWE	1.097	**0.885**	**0.519**	**0.167**	0.833	1.084	1.059	1.077	1.132
TAI	1.011	0.574	**0.429**	**0.349**	0.958	**0.974**	**0.956**	1.054	**0.819**
THL	1.115	**0.859**	**0.518**	**0.316**	0.875	**0.884**	**0.845**	**0.892**	**0.611**
TUR	1.168	3.484	2.178	**0.596**	0.75	1.020	1.004	**0.702**	**0.837**

Note: Panel A: Theil's U and Clark-West rejection frequencies for dollar-carry model forecasts. Bolded values indicate forecast ability relative to the random walk. Panel B: Ratio of Theil's U for Dollar-Euro model to dollar-carry model. Bolded values indicate forecast abilty relative to dollar-carry model.