1. Design algorithms for the following operations for a binary tree:
   (a) preorderNext(v): return the node visited after node v in a preorder traversal of T (10 points)
   (b) inorderNext(v): return the node visited after node v in an inorder traversal of T (10 points).
   (c) postorderNext(v): return the node visited after node v in a postorder traversal of T (10 points).

2. Let T be a binary tree with n nodes. The distance between two nodes v and w in T is the number of nodes on the path from v to w in T. The diameter of T is the maximum distance between two nodes in T. Describe an efficient algorithm for finding the diameter of T (10 points).

3. Illustrate the execution of the heap-sort algorithm on the following input sequence:
   (20,6,17,5,11,24,3,19,25,16,4).
   (10 points).

4. Tamarindo Airlines wants to give a first-class upgrade coupon to their top log n frequent flyers, based on the number of miles accumulated, where n is the total number of frequent flyers. The algorithm they currently use, which runs in O(n log n) time, sorts the flyers by the number of miles flown and then scans the sorted list to pick the top log n flyers. Describe an algorithm that identifies the top log n flyers in O(n) time (10 points).

5. Consider a hash table T with 11 entries and the hash function h(x) = x mod 11. Show the result of hashing the keys 10,16,11,43,33,13,14,12,3, and 22, assuming collisions are handled by
   (a) open addressing with linear probing (10 points).
   (b) double hashing using a secondary hash function h'(x) = 5 − (x mod 5) (10 points).

6. Insert, into an initially empty binary search tree, entries with keys 29, 41, 23,55, 48,26,10, and 14, in this order. Then remove the keys 41 and 23, in this order. Draw the tree after each operation (insert and remove). (20 points).