Constructing Connected Dominating Sets with Bounded Diameters in Wireless Networks

Yingshu Li *
Department of Computer Science
Georgia State University
Atlanta, GA 30303
yli@cs.gsu.edu

Donghyun Kim Feng Zou Ding-Zhu Du
Department of Computer Science
University of Texas at Dallas
Richardson, TX 75083
{donghyunkim,phenix.zou}@student.utdallas.edu
dzdu@utdallas.edu

Abstract

In wireless networks, due to the lack of fixed infrastructure or centralized management, a Connected Dominating Set (CDS) of the graph representing the network is an optimum candidate to serve as the virtual backbone of a wireless network. However, constructing a minimum CDS is NP-hard. Furthermore, almost all of the existing CDS construction algorithms neglect the diameter of a CDS, which is an important factor. In this paper, we investigate the problem of constructing a CDS with a bounded diameter in wireless networks and propose a heuristic algorithm, Connected Dominating Sets with Bounded Diameters (CDS-BD), with constant performance ratios for both the size and diameter of the constructed CDS.

1 Introduction

Wireless networks including wireless ad hoc networks and wireless sensor networks have been attracting more and more attentions in the recent years and they are being used in a variety of military and civil applications such as battlefields, disaster recoveries, conferences, concerts, environmental detections, health applications and *etc*. It is widely believed that wireless networks would be an ideal and important part of the next generation network to provide flexible deployment and mobile connectivity.

*This work was supported in part by NSF Grant CCF-0545667.

A wireless network consists of a collection of static/mobile nodes (hosts). These nodes dynamically form a temporary network without the use of any existing or predefined network infrastructure. Due to the above fact, a Connected Dominating Set (CDS) of the graph representing a network is usually used as the *virtual backbone* of the network and plays an important role in routing. However, the problem of constructing a minimum CDS whose size is the smallest is NP-hard [6]. Nodes within a wireless network communicate over a shared, scarce wireless channel. Compared with cables in wired networks, wireless links have much less available bandwidth. Each node has an omini-directional antenna and messages are transmitted in all directions. The transmission can be heard by all the other nodes within the transmission range. Since the nodes in a CDS may have heavy load working as the central management agents, minimizing the size of the CDS can greatly help with reducing transmission interference and the number of control messages. If the receiver is not within the transmission range of the sender, they need to communicate through multi-hop links by using some intermediate nodes to relay the messages. We call this multi-hop routing. This characteristic of wireless networks induces us to take into account another factor of a CDS, the diameter, which is the longest shortest path between any pair of nodes in the CDS. With the help of a CDS with small size and diameter, routing is easier and can adapt quickly to topology changes of a network. Only the CDS nodes, rather than every node in a network, need to maintain the routing information. Furthermore, if there is no topology changes in the subgraph induced by the CDS, there is no need to update the routing information, which reduces both storage space and message complexities. If a non-CDS node wants to deliver a message to another non-CDS node, it first sends the message to a neighboring CDS node. Then the search space for the route is reduced to the CDS. After the message is relayed to the destination's neighboring CDS node, this node can deliver the message to the destination. In this paper, we consider how to construct a CDS with the smallest size and diameter.

The rest of this paper is organized as follows. Section 2 reviews the related work. Section 3 illustrates the communication model and some preliminaries. In Section 4, the CDS-BD algorithm as well as the analysis of the algorithm are presented. Section 5 concludes the paper.

2 Related work

The idea of using a CDS as a virtual backbone for routing was proposed in [7]. Since then many efforts [11] - [3] have been made to design approximations or heuristics for MCBS (or weakly MCDS). Guha and Khuller first proposed a two-stage greedy $(\ln \Delta + 3)$-approximation [8] for MCDS in general graphs where Δ is the maximum node degree in the graph, as well as a lower bound $(\ln \Delta + 1)$ for any polynomial-time approximation for MCDS provided $NP \not\subseteq 2^{polylog(n)}$. In the first algorithm, a CDS is built from one node, then the searching space for the next dominator(s) is restricted to the current set of dominatees and the CDS expands until there is no white nodes exist. In the second algorithm, all the possible dominators are determined in the first phase, then they are connected through some intermediate nodes in the second phase. Das et al. [12] implemented the algorithms in [8]. The result was then improved in [11] by giving a one-stage greedy $(\ln \Delta + 2)$-approximation where Δ is the maximum node degree in the graph. A polynomial-time approximation scheme was given in [5] for MCDS in unit disk graphs. This means that theoretically, the performance ratio for polynomial-time approximation can be as small as $1+\varepsilon$ for any positive number ε. However, its running time grows very rapidly as ε goes to 0 and hence it is not worth implementing in practice.

Besides the above centralized approximations, some distributed ones have also been developed and they can be classified into two families. In the first family [15], a trivial connected dominating set is constructed at the beginning and then redundant nodes are deleted based on some certain rules. The proposed algorithm is a distributed algorithm where each node knows the connectivity information within the 2-hop neighborhood. If a node has two unconnected neighbors, it becomes a dominator. The generated CDS is easy to maintain. However, the size of the CDS is large. The authors did not give the performance ratio of the proposed algorithm. Later on in [14], the authors pointed out the performance ratio of Wu and Li's algorithm which is O(n). In the second family [2, 13, 1, 14, 9, 4], a dominating set is constructed at the first step and all the nodes in this dominating set are then connected to form a CDS. In these algorithms, it is popular to construct an Maximal Independent Set (MIS) to serve as a dominating set. Then by connecting the nodes in the MIS, a CDS is then obtained. Wan *et. al.* [14] proposed two 2-phase distributed algorithms. In these two algorithms, a spanning tree is first constructed. Every node in the spanning tree is then labelled as either a dominator or a dominatee based on a ranking scheme. The algorithms are employed upon Unit Disk Graphs (UDG) to obtain a constant performance ratio of 8. Cardei *et. al.* [2] also designed a 2-phase distributed algorithm. This algorithm requires a leader to be selected at the beginning of the first phase. The improvement is that it is not necessary for the root to wait for the COMPLETE messages from the furthest nodes. The performance ratio of this algorithm is also 8. Alzoubi *et. al.* [1] noticed that it is difficult to maintain the CDS constructed using the algorithm in [14] and designed a localized distributed 2-phase algorithm where only local information is needed at each node. An MIS is generated in a distributed manner without building a tree or selecting a leader. If a node has the smallest ID within its 1-hop neighborhood, it becomes a dominator. After there are no white nodes, the dominators are responsible for identifying paths to connect all the dominators. In this algorithm, no network connectivity information is utilized and the performance ratio is 192. In [9], the authors gave another localized distributed algorithm with the performance ratio of 172. The authors in [4] proposed two distributed algorithms to approximate a minimum CDS. These algorithms take linear time. All of the above works only consider to minimize the size of a CDS. The diameter of a CDS which is also an important parameter should be another primary concern when constructing a CDS. In [10], the diameter of a CDS is taken into account. However, the performance ratio of the algorithm is not given. In this paper, we proposed an algorithm which constructs a CDS with the purpose of minimizing both the size and the diameter of the CDS. Furthermore, we analyzed the performance ratio of this algorithm. The time complexity of our algorithm is smaller than the algorithm in [10] which is $O(n^3)$. The performance ratios of our algorithm for both the size and diameter of the constructed CDS are constant.

3 Wireless communication model and preliminaries

We use a graph $G = (V, E)$ to represent a wireless network. The node set V is the set of nodes in the network and the edge set E represents all the links in the network. There is an edge between a pair of nodes if they are within the transmission range of each other, that is, they can communicate. We assume that all the nodes are deployed in a 2-D plane. If the transmission ranges of all the nodes are the same, G is modelled as a Unit Disk Graph (UDG) [6]. Otherwise, G is a general graph. A Dominating Set (DS) of a graph $G = (V, E)$ is a subset $C \subset V$ such that each node either belongs to C or is adjacent to at least one node in C. A CDS is a DS which induces a connected subgraph. The nodes in a CDS are called the *dominators*, otherwise, *dominatees*. The size of a CDS s(CDS) is the number of dominators. Denote d_{ij} as the distance between node i and node j. Then the diameter of a CDS $d(CDS) = max(d_{ij})$, where i and j are dominators. Fig.1 illustrates a UDG with its CDS.

To construct a CDS, we employ an Maximal Independent Set (MIS) which is also a subset of all the nodes in the network. The nodes in an MIS are pairwise nonadjacent and no more nodes can be added to preserve this property. Fig.2(b) shows an MIS which contains all the black nodes. Therefore, each node which is not in the MIS is adjacent to at least one node in the MIS. Thus an MIS is a DS. If the nodes in an MIS are connected by adding more nodes to the MIS, a CDS can be obtained.

4 The CDS-BD Algorithm

The CDS with Bounded Diameter (CDS-BD) construction algorithm is illustrated in Algorithm 1. We use hop count to measure the distance between a pair of nodes. Initially, all the nodes are white. CDS-BD first identifies some MIS nodes and color them black. All the neighbors of black nodes are then colored grey. Thereafter, the connectivity of the current black node set is taken care of by changing more nodes into black.

Algorithm 1 CDS-BD($G = (V, E)$)

1: Choose a root $r \in V$. Let $V_k = \{y \in V \mid HopCount(r, y) = k, 0 \leq k \leq \infty\}$ and G_k the subgraph of G induced by V_k.
2: Find an maximal independent set (MIS) I_k of G_k for all even k. Let I be the union of I_k for all even k.
3: Color all nodes of I in black and color every node adjacent to a black in grey.
4: **while** a white node x exists **do**
5:     color x in black and color its adjacent write nodes in grey. /*Note: black nodes not in I must belong to I_k for some odd k.*/
6: **end while**
7: Let B be the set of black nodes.
8: /*Connect B into a CDS in the following way:*/
9: Suppose k^* is the maximum k. Let $i^* = \lfloor k^*/2 \rfloor$.
10: **for** $i = 1$ **to** i^* **do**
11:     **for** every black node x in I_{2i} **do**
12:         suppose node y in V_{2i-1} is adjacent to x and node z in V_{2i-2} is adjacent to y;
13:         color y and z in black if they are not.
14:     **end for**
15: **end for**
16: /*Note: i) If z is not black, then z must have a black neighbor in I_{2i-2}. ii) Through Line 10 to Line 15, all nodes in I are connected together by adding at most $2(|I| - 1)$ nodes. */
17: Let C be the black connected components containing I. Note that other than C, each black connected component is still a singleton x in I_k for some odd k. There must exist a node y in I_{k-1} adjacent to x. Color y in black if y is not. /*Note: If y is not black, then y must have a black neighbor.*/

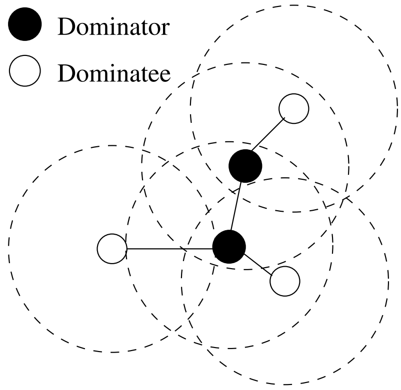

Figure 1. A UDG containing a CDS.

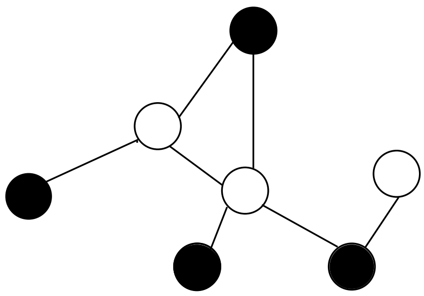

Figure 2. An MIS formed by all the black nodes.

Fig.3 shows an example. The number beside each node is its ID. In this example, node 0 is the root and $I = \{0, 3, 5, 7, 15\}$. The constructed MIS $B = \{0, 3, 5, 7, 14, 15\}$ which has one more node 14 than set I. Node 14 is a singleton discussed at Line 17 in CDS-BD. Nodes 2, 11, 12, 13 are then changed to black to connect the nodes in B. Therefore, the final CDS is $A = \{0, 2, 3, 5, 7, 11, 12, 13, 14, 15\}$. This CDS has a size of 10 and a diameter of 7.

Lemma 1 *B is an MIS.*

Proof. Denote E as the set of black nodes which are turned into black by Line 4 through Line 6 in CDS-BD. Then $B = I \cup E$. From Line 2 of CDS-BD, we know any pair of the nodes in I are independent. From Line 4 through Line 6, we know any pair of the nodes in E are independent. Suppose $u \in E, v \in I$, and u and v are adjacent. Before Line 4, u is a white node. However, u is a neighbor of v and should have been colored grey. This is a contradiction. Thus, any node in E is not adjacent to any node in I. We can no longer change any node to black after Line 6. Therefore, B is an MIS as well as a dominating set. □

Theorem 1 *All the black nodes form a CDS.*

Proof. B is a dominating set from Lemma 1. Line 9 through Line 17 are devoted to connect all the nodes in B. Therefore, after the termination of CDS-BD, all the black nodes form a CDS. □

Since the CDS construction problem is NP-hard, we derive the performance ratio of our algorithm to evaluate it as following.

Theorem 2 *Let A be the set of all black nodes obtained from the above algorithm. Then $|A| \leq 11.4mcds + 1.6$, where $mcds$ is a CDS with the smallest size. Let D^* be the minimum diameter of a CDS. Then $diameter(A) \leq 3D^* + 7$.*

Proof. It is known that for an MIS in a UDG, $|MIS| \leq 3.8mcds + 1.2$ [16]. So, $|B| \leq 3.8mcds + 1.2$. Hence,

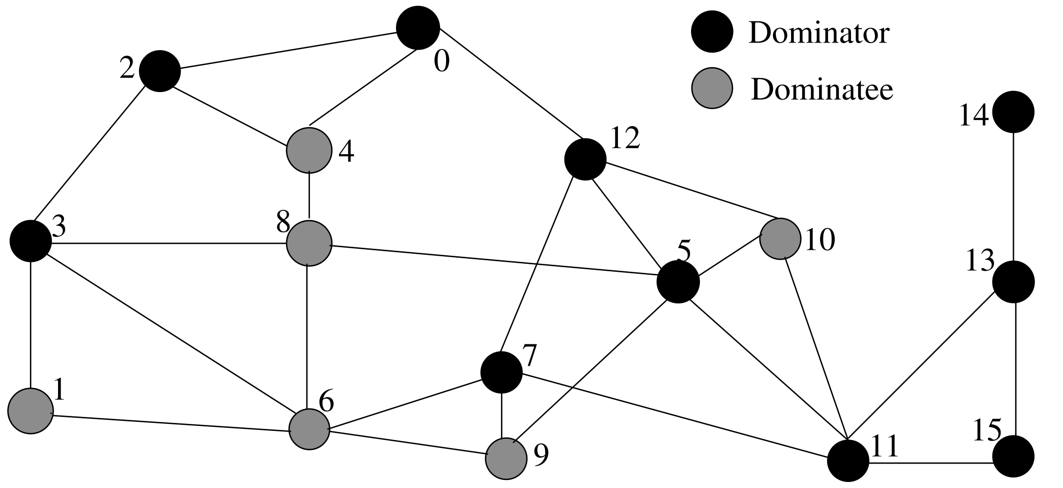

Figure 3. An example CDS construction.

$|A| \leq 3|B|-2 \leq 11.4mcds+1.6$. From Line 10 to Line 17, it is easy to know that every black node in V_k for even k is away from r within a distance at most $1.5k$ and every black node in V_k for odd k is away from r within a distance at most $1.5k+0.5$. Suppose G has diameter D. Then $D \geq k^*$ and the diameter of a connected dominating set is at least $D-2$. Therefore, $diameter(A) \leq 2(1.5k^*+0.5) \leq 3(D-2)+7 = 3D^*+7$. □

Theorem 3 *The time complexity of CDS-BD is $O(n^2)$, where n is the number of the nodes in a network.*

Proof. The MIS constructions for G_k (k is even) takes $O(n^2)$ time. Connecting the nodes in B can be completed in $O(n)$ time. Therefore, the time complexity of CDS-BD is $O(n^2)$. □

5 Conclusion

In this paper, we investigate the problem of constructing a CDS with a bounded diameter in wireless networks and propose a heuristic algorithm, CDS-BD, with constant performance ratios for both the size and diameter of the constructed CDS. It is our further interest to design a distributed algorithm for this problem considering some other network factors such as energy, bandwidth, fault tolerance and etc.

References

[1] K. M. Alzoubi, W. P, J, and O. Frieder. Message-optimal connected dominating sets in mobile ad hoc networks. *Proceedings of the 3rd ACM international symposium on mobile ad hoc networking and computing*, June 2002.

[2] M. Cardei, M. X. Cheng, X. Cheng, and D.-Z. Du. Connected domination in ad hoc wireless networks. *International Conference on Computer Science and Informatics*, 2002.

[3] Y. P. Chen and A. L. Liestman. Approximating minimum size weakly-connected dominating sets for clustering mobile ad hoc networks. *Proceedings of the third ACM international symposium on Mobile ad hoc networking and computing*, June 2002.

[4] X. Cheng, M. Ding, H. Du, , and X. Jia. Virtual backbone construction in multihop ad hoc wireless networks. *Wireless Communications and Mobile Computing*, 6(2):183–190, 2006.

[5] X. Cheng, X. Huang, D. Li, W. Wu, and D.-Z. Du. A polynomial-time approximation scheme for minimum connected dominating set in ad hoc wireless networks. *Networks*, 42:202–208, 2003.

[6] B. N. Clark, C. J. Colbourn, and D. S. Johnson. Unit disk graphs. *Discrete Mathematics*, 86:165–177, 1990.

[7] A. Ephremides, J. Wieselthier, and D. Baker. A design concept for reliable mobile radio networks with frequency hopping signaling. *Proc. IEEE*, 75(1):56–7, 1987.

[8] S. Guha and S. Khuller. Approximation algorithms for connected dominating sets. *Algorithmica*, 20:374–387, 1998.

[9] Y. Li, S. Zhu, M. Thai, and D.-Z. Du. Localized construction of connected dominating set in wireless networks. *NSF International Workshop on Theoretical Aspects of Wireless Ad Hoc, Sensor and Peer-to-Peer Networks*, June 2004.

[10] K. Mohammed, L. Gewali, and V. Muthukumar. Generating quality dominating sets for sensor network. *Proceedings of the Sixth International Conference on Computational Intelligence and Multimedia Applications*, pages 204–211, August 2005.

[11] L. Ruan, H. Du, X. Jia, W. Wu, Y. Li, and K.-I. Ko. A greedy approximation for minimum connected dominating sets. *Theoretical Computer Science*, 329(1-3):325–330, December 2004.

[12] R. Sivakumar, B. Das, and V. Bharghavan. An improved spine-based infrastructure for routing in ad hoc networks. *IEEE Symposium on Computer and Communications*, June 1998.

[13] I. Stojmenovic, M. Seddigh, and J. Zunic. Dominating sets and neighbor elimination based broadcasting algorithms in wireless networks. *Proc. IEEE Hawaii Int. Conf. on System Sciences*, January 2001.

[14] P.-J. Wan, K. M. Alzoubi, and O. Frieder. Distributed construction of connected dominating set in wireless ad hoc networks. *IEEE Infocom*, 2002.

[15] J. Wu and H. L. Li. On calculating connected dominating set for efficient routing in ad hoc wireless networks. *Proceedings of the 3rd ACM International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, pages 7–14, 1999.

[16] W. Wu, H. Du, X. Jia, Y. Li, and S. C.-H.Huang. Minimum connected dominating sets and maximal independent sets in unit disk graphs. *Theoretical Computer Science*, 352(1-3):1–7, 2006.