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A Better Constant-Factor Approximation for Selected-Internal Steiner Minimum Tree

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Abstract The selected-internal Steiner minimum tree problem is a generalization of original Steiner minimum tree problem. Given a weighted complete graph $G = (V, E)$ with weight function c, and two subsets $R' \subsetneq R \subseteq V$ with $|R - R'| \geq 2$, selected-internal Steiner minimum tree problem is to find a minimum subtree T of G interconnecting R such that any leaf of T does not belong to R'. In this paper, suppose c is metric, we obtain a $(1 + \rho)$-approximation algorithm for this problem, where ρ is the best-known approximation ratio for the Steiner minimum tree problem.

Keywords Selected-internal Steiner tree · Approximation algorithm

1 Introduction

Given a weighted complete graph $G = (V, E)$ with a weight function c and a subset R, Steiner minimum tree problem is to find a minimum subtree of G interconnecting R. Steiner tree can be applied in many fields such as VLSI routing [14], network

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routing [15], phylogeny [6, 13], et al. [2, 5, 10]. In the past years, some generalizations of Steiner minimum tree problem have arisen, such as Steiner minimum tree problem on some special metric spaces [8, 9], the full Steiner tree problem [4, 17] and the k-size Steiner tree problem [1].

In many practical applications, all vertices of R should be the leaves of the Steiner tree. For example, the global routing in VLSI-Design, all vertices of R must be leaves of the Steiner tree. From this motivation, Lin and Xue [16] introduced terminal Steiner minimum tree problem. Given a weighted complete graph $G=(V,E)$ with weight function c, and a subsets $R \subseteq V$, terminal Steiner minimum tree problem is to find a minimum subtree T of G interconnecting R such that any vertex of R is a leaf of T. Simultaneously, they gave a $(2+\rho)$-approximation algorithm for terminal Steiner tree minimum problem when c is metric, where $\rho = 1 + \frac{\ln 3}{2} \approx 1.55$ is the best-known approximation ratio for Steiner minimum tree problem [19]. Fuchs [7] and Drake and Hougardy [3] improved the approximation ratio from $2+\rho$ to 2ρ separately. In 2007, Martinez et al. [18] improved the ratio to $2\rho - \rho/(3\rho - 2)$, which is the best-known approximation ratio.

Recently, Hsieh and Gao [11] and Hsieh and Yang [12] investigated two variants of the terminal Steiner minimum problem: partial-terminal Steiner minimum tree problem and selected-internal Steiner minimum tree problem. Given a weighted complete graph $G=(V,E)$ with weight function c, and two subsets $R' \subseteq R \subseteq V$, partial-terminal Steiner minimum tree problem is to find a minimum subtree T of G interconnecting R such that all vertices of R' must be leaves of T. Selected-internal Steiner minimum tree problem is a contrary problem of partial-terminal Steiner minimum tree problem. Given a weighted complete graph $G=(V,E)$ with weight function c, and two subsets $R' \subsetneq R \subseteq V$ with $|R - R'| \geq 2$, selected-internal Steiner minimum tree problem is to find a minimum subtree T of G interconnecting R such that any leaf of T does not belong to R'. Since Steiner minimum tree problem is a special case ($R' = \emptyset$) of these two problems, the NP-completeness and MAX SNP-hardness of these two problems can immediately follow. In these papers [11, 12], they gave 2ρ-approximation algorithms for these problems when c is metric. In this paper, we study selected-internal Steiner minimum tree problem and present a $(1+\rho)$-approximation algorithm for this problem when c is metric.

The rest of this paper is organized as follows. In Sect. 2, we first give the definition of metric and some useful notations. Then we present a $(1+\rho)$-approximation algorithm for selected-internal Steiner minimum tree problem by 3 subsections. In Sect. 2.1, we firstly divide R to a pairwise disjoint tree sequence $\mathcal{T} = \{T_1, \ldots\}$. Then, we contract every T_i to v_{T_i} to obtain a new graph G_1 and set $R_1 = \{v_{T_i}\}$. Final, we construct a Steiner tree T' of G_1 on R_1 and a Steiner tree T of G on R. In Sect. 2.2, we modify T' to a new Steiner tree T'' of G_1 on R_1 with approximation ratio $(1+\rho)$. In final subsection, we obtain a selected-internal Steiner tree T''' of G on R with approximation ratio $(1+\rho)$. Finally, we conclude our results and discuss a special case when $R = V$, which is the selected-internal minimum spanning tree problem.

2 A $(1+\rho)$-Approximation Algorithm

At the beginning of this section, we introduce some useful notations. Given a complete graph $G=(V,E)$ with a weight function c on its edges and a set $R\subseteq V$. A function c is *metric* if it satisfying the following conditions:

1. $c(u,v)\geq 0$ and the equality holds if and only if $u=v$;
2. $c(u,v)=c(v,u)$ for any u and v;
3. triangular inequality, that is, $c(u,w)+c(w,v)\geq c(u,v)$ for any u, v and w.

In the following study, we assume that the weight function c is metric. Let T be a Steiner tree of G on R. We call an edge as an R^+*-edge* if both its endpoints belong to R, otherwise call it an R^-*-edge*. Denote E^T_{R+} and E^T_{R-} as the sets of the R^+-edge and R^--edge of T, respectively. For two subsets V_1 and V_2 of $V(G)$, denote $\text{dist}(V_1,V_2)=\min_{v_1\in V_1,v_2\in V_2} c(v_1v_2)$ as the distance between V_1 and V_2.

Then, we will give a $(1+\rho)$-approximation algorithm for selected-internal Steiner minimum tree problem in the following three steps. In the first step, we give Algorithm 1 to divide R to a pairwise disjoint tree sequence $\mathcal{T}=\{T_1,\ldots\}$. Next, we contract every T_i to v_{T_i} to obtain a new graph G_1 and set $R_1=\{v_{T_i}\}$. Based on Algorithm 1, we present Algorithm 2 to construct a Steiner tree T' of G_1 on R_1 with approximation ratio ρ and a Steiner tree T of G on R such that there is a Steiner minimum tree T_{opt} of G on R, satisfying

$$\begin{aligned} &E(T)=E(T')\cup E(\mathcal{T}), \quad E(\mathcal{T})\subseteq E^T_{R+} \quad \text{and} \\ &c(E(T)\setminus E(\mathcal{T}))\leq \rho\, c(E(T_{opt})\setminus E(\mathcal{T})). \end{aligned} \tag{1}$$

In the second step, we modify T' to a new Steiner tree T'' of G_1 on R_1 with approximation ratio $(1+\rho)$. In T'', every v_{T_i} has degree no less than 2 unless T_i has vertex belonging to $R-R'$. In the final, we "blossom" every v_{T_i} of T'' to T_i and modify it to a selected-internal Steiner tree T''' of G on R with approximation ratio $(1+\rho)$.

2.1 Construction of T' and T

In this subsection, we first give Algorithm 1 (in next page) to divide R to a disjoint tree sequence $\mathcal{T}$.

Note that in the line 17, if $u\in V-R$, then a new tree T_i will be included in $\mathcal{T}$ and T_i satisfies the following property:

$$\begin{aligned} &\text{there is a vertex } w\in V-R \text{ such that } \text{dist}(w,T_i)<\text{dist}(v,T_i) \\ &\quad \text{for any } v\in R-V(T_i). \end{aligned} \tag{2}$$

We claim that in the line 13, when $u\in R-S-V(T_i)$ is true, if the old tree T_j follows property (2), the modified tree $T'_j:=T_j\cup T_i\cup\{e'\}$ also follows, where e' is a shortest edge between $V(T_i)$ and $V-V(T_i)$ and one of its endpoint is in T_j. For this purpose, since T_j has property (2), there is $w\in V-R$ such that $\text{dist}(w,T_j)<\text{dist}(v,T_j)$ for any other $v\in R-T_j$. Denote the corresponding edge as e. It implies that $c(e)<c(e')$. For any edge e'' between $V(T_i)$ and $R-V(T'_j)$, $c(e'')\geq c(e')>c(e)$. Hence, the modified tree T'_j also has the property (2).

By recurrence, we obtain the following result.

Algorithm 1

1: Input: A complete graph $G = (V, E)$ with a weight function c and a set $R \subseteq V$.
2: Output: A sequence of pairwise disjoint subtrees $\mathcal{T} = \{T_1, T_2, \ldots\}$ such that $\bigcup V(T_i) = R$.
3: Set $S = R$, $i = 1$ and $\mathcal{T} = \emptyset$. /*S records the vertices in R not covered by $\mathcal{T}$ */
4: **if** $S = \emptyset$ **then**
5:     output $\mathcal{T}$ and stop;
6: **else** Choosing a vertex v of S and creating a new tree T_i, set $V(T_i) := \{v\}$, $E(T_i) := \emptyset$
7:     and $S := S - \{v\}$.
8: **end if**
9: Search $V - V(T_i)$ with the vertex ordering $S, R - S - V(T_i), V - R$, to find the first vertex u such that $\text{dist}(u, T_i) = \min_{w \in V(G - T_i)} \text{dist}(w, T_i)$. Let the corresponding edge be ut.
10: **if** $u \in S$ **then**
11:     set $V(T_i) := V(T_i) \cup \{u\}$, $E(T_i) := E(T_i) \cup \{ut\}$ and $S := S - \{u\}$. Goto line 9.
12: **end if**
13: **if** $u \in R - S - V(T_i)$ **then**
14:     u is contained in some T_j with $1 \leq j < i$. Set $T_j := T_j \cup T_i + ut$ and delete T_i.
15:     Goto line 4.
16: **end if**
17: **if** $u \in V - R$ or $u = \text{NULL}$ **then**
18:     set $\mathcal{T} := \mathcal{T} \cup T_i$ and $i := i + 1$. Goto line 4.
19: **end if**

Lemma 1 *Let $T_1, T_2, \ldots, T_m$ be the output tree sequence of Algorithm 1. If $R \neq V$, for any T_i, there is vertex $w \in V - R$ such that* $\text{dist}(w, T_i) < \text{dist}(v, T_i)$ *for any $v \in R - V(T_i)$.*

Lemma 2 *For a complete graph $G = (V, E)$ with a weight function c and a set $R \subseteq V$, let $T_1, T_2, \ldots, T_m$ be the output of Algorithm 1, there exists a Steiner minimum tree T of G on R such that every T_i is a subtree of T.*

Proof Firstly, order edges of $E' = \bigcup_{i=1}^{m} E(T_i)$ as $\{e_1, e_2, \ldots\}$, according to their order of appearance in Algorithm 1. Among all Steiner minimum trees of G on R, choose T satisfying the following conditions: (1) $|E(T) \cap E'|$ as large as possible; (2) under the condition (1), choosing T such that the index of the first edge in $E' \setminus E(T)$ as large as possible. We shall show that T contains $T_1, \ldots, T_m$ as subtrees.

Suppose to the contrary that the first edge in $E' \setminus E(T)$ is $e_j = uv$. By Algorithm 1, there exists some stage at which there is a subtree T' (which is a subtree of some T_i) such that e_j is a shortest edge between $V(T')$ and $V - V(T')$. Since T is a tree, there is a unique path P between u and v on T. Let e be the unique edge on P between $V(T')$ and $V - V(T')$. Then $c(e_j) \leq c(e)$ and $\widetilde{T} = T + e_j - e$ is also a

Steiner tree. Since T is a Steiner minimum tree, we have that $c(e_j) = c(e)$ and $c(\widetilde{T}) = c(T)$. By the choice of T, e must be in E', otherwise $|E(\widetilde{T}) \bigcap E'| > |E(T) \cap E'|$. Let $e = e_t$. By the structure of T', we have $t > j$. But then, the index of the first edge in $E' \setminus E(\widetilde{T})$ is larger than that of $E' \setminus E(T)$, contradicting to condition (2). So, T contains all edges of $T_1, \dots, T_m$. □

Now, we construct a new graph G_1 by contracting every T_i to a new vertex, denoted it by v_{T_i}. Let $R_1 = \{v_{T_1}, \dots, v_{T_m}\}$. By Lemma 2, we obtain the following corollary.

Corollary 3 *For any Steiner minimum tree T_{R_1} of G_1 on R_1, if we "blossom" every v_{T_i} to T_i, then the new tree T_R is a Steiner minimum tree of G on R.*

Proof Suppose to the contrary that the new tree T_R is not a Steiner minimum tree of G on R. By Lemma 2, there is a Steiner minimum tree T'_R on R such that every T_i is a subtree of T'_R and $c(T'_R) < c(T_R)$. Contract every T_i to a new vertex in T'_R. The resulting tree is a Steiner tree of G_1 on R_1 which has less weight than T_{R_1}, a contradiction. □

Based on Algorithm 1, we present the following Algorithm 2 which constructs two Steiner trees T' and T. By Corollary 3, there exists a Steiner minimum tree T_{opt} such that T satisfying (1).

Algorithm 2

1: Input: A complete graph G, a cost function c on $E(G)$ and a set $R \subseteq V$.
2: Output: Two Steiner trees $T'$and T.
3: Use Algorithm 1 to obtain $T_1, \dots, T_m$.
4: Contract T_i to v_{T_i} $(i = 1, \dots, m)$ to construct G_1. Set $R_1 = \{v_{T_i}\}_{i=1}^m$.
5: Use a ρ-approximation algorithm to obtain a Steiner tree T' of G_1 on R_1.
6: "Blossom" v_{T_i} to T_i $(i = 1, \dots, m)$ to form a Steiner tree T of G on R.

2.2 Modification of T'

In Sect. 2.1, we obtain a Steiner Tree T' of G_1 on R_1 with approximation ratio ρ to the Steiner minimum tree of G_1 on R_1. In this subsection we modify T' to a new Steiner tree T'' with approximation ratio $(1 + \rho)$. In T'', every v_{T_i} has degree no less than 2 unless T_i has vertex belonging to $R - R'$.

If $m = 1$, obviously v_{T_1} is the Steiner minimum tree T' of G_1 on R_1. Set $T'' = T'$. If $m = 2$, let e be the edge connecting v_{T_1} and v_{T_2}. Then $T' = (\{v_{T_1}, v_{T_2}\}, \{e\})$ is a Steiner minimum tree. Since $|R - R'| \geq 2$, one of T_1 and T_2, say T_1 should contain some vertex of $R - R'$. By Lemma 1, there is a vertex $w \in V - R$ such that $c(wv_{T_2}) = \text{dist}(w, T_2) < \text{dist}(T_1, T_2) = c(e)$. Set $T'' = T' + \{wv_{T_2}\}$. Then

$$c(T'') = c(T') + c(w, v_{T_2}) < 2c(e) = 2c(T').$$

In the following, suppose $m \geq 3$. Firstly, we give some useful results.

Lemma 4 *Let G be a weighted complete graph on at least two vertices with metric weight function c and T a minimum spanning tree of G. Then, for any two vertices u and v, G has a Hamilton cycle C such that edge $uv \in E(C)$ and $c(C) \leq 2c(T)$.*

Proof We use induction on $|G|$. If $|G| = 2$, the digon through u and v is as desired. Suppose $|G| = k \geq 3$ and the assertion is true for all graphs with less than k vertices. Considering the minimum spanning tree T, if there is a leaf w different from u and v, let $e = ws$ be the unique edge incident with w and $\widetilde{T} = T - w$. Then $\widetilde{T}$ is a minimum spanning tree of $G[V(\widetilde{T})]$ and $\{u, v\} \subseteq V(\widetilde{T})$. By inductive hypothesis, there is a Hamilton cycle C' of $G[V(\widetilde{T})]$ with $c(C') \leq 2c(\widetilde{T})$ and $uv \in E(C')$. Let t be a neighbor of s on C' with $st \neq uv$. Set $C = C' - st + sw + wt$. Then

$$\begin{aligned} c(C) &= c(C') - c(st) + c(sw) + c(wt) \\ &\leq 2c(\widetilde{T}) + 2c(ws) \\ &= 2c(T). \end{aligned}$$

If such a vertex w does not exist, then T is a Hamilton path from u and v, and $C = T + uv$ is as desired. □

As a consequence, we have

Corollary 5 *Let G be a weighted complete graph with metric weight function c and T a minimum spanning tree of G. For any two vertices u and v, there exists a Hamilton path P from u to v such that $c(P) \leq 2c(T)$.*

In the following, we modify T' for the case $m \geq 3$. Let Q be the set of all leaves of T' in R_1. Then $Q \subseteq R_1$. If $|Q|$ is odd, we modify Q as follows. If $Q = R_1$, then there exists a vertex v_{T_i} such that T_i contains some vertex of $R - R'$. In this case, set $Q := Q - v_{T_i}$. If $Q \neq R_1$, then there exists a vertex $v \in R_1 - Q$ and set $Q := Q \cup \{v\}$. Now, Q is a subset of R_1 and $|Q|$ is even. Find a minimum perfect matching M of Q in polynomial time. We claim that $c(M) \leq c(T_{opt})$, where T_{opt} is a Steiner minimum tree of G_1 on R_1. For this purpose, let C be a Hamilton cycle as in Lemma 4. "Short cut" C to a cycle C' containing only vertices of Q. Since c is a metric weight function, we have $c(C') \leq c(C) \leq 2c(T_{opt})$. Then, one of the two perfect matchings of C', say M_1, satisfies $c(M_1) \leq c(T_{opt})$. The claim follows from $c(M) \leq c(M_1)$.

Now, add the edges of M one by one to modify T' such that every v_{T_i}, for which T_i contains only vertices in R', has degree at least 2.

Let e be an edge in M. Since T' is a tree, $T' + e$ has the unique cycle C. If C contains all vertices of T', then there exists a vertex $v \in R_1 \subseteq V(C)$ such that T_v contains some vertex of $R - R'$. If some neighbor u of v on C with $u \in V - R$, let $T' := T' + e - uv$ (see Fig. 1(a)). Else, the both neighbors of v on C belong to R_1. Let u be one of them. By Lemma 1, there exists a vertex $w \in V(G_1) - R_1$ such

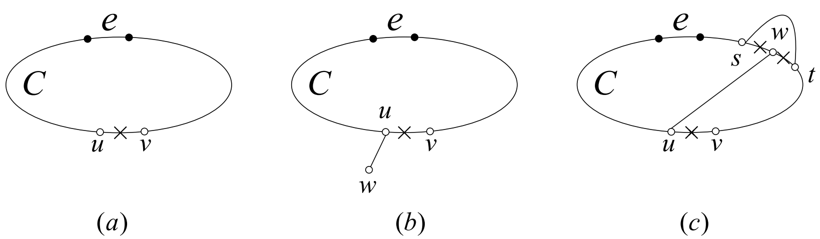

Fig. 1 The three types of new tree

that $c(wu) < c(uv)$. If w is not on C, let $T' := T' + e + uw - uv$ (see Fig. 1(b)). If $w \in V(C)$, let s and t be the neighbors of w on C and $T' := T' + e + uw - uv + st - sw - wt$ (see Fig. 1(c)). Then the new tree T' is a Steiner tree of G_1 on R_1 such that the degree of any vertex of R_1 is not less than 2 except v (but it does not matter since T_v contains some vertices in $R - R'$), and the weight of new tree is less than the weight of the old tree plus $c(e)$ (by $c(uw) < c(uv)$ and the assumption that c is metric).

If C does not contain all vertices of T', there exists a vertex v of $V(C)$ such that $d_{T'}(v) \geq 3$. If some neighbor u of v on C has degree more than 2 or $u \in V(G_1) - R_1$, let $T' := T' + e - uv$. Otherwise, let u be a neighbor of v on C. Then $u \in R_1$ with degree 2 in T'. By Lemma 1, there exists a vertex $w \in V(G_1) - R_1$ such that $c(wu) < c(uv)$. If $w \notin V(T')$ or $w \in C$, then T' can be modified similarly to the above case. Hence, suppose $w \in V(T' - C)$. In this case, the graph $H_1 = T' + e + uw - uv$ has the unique cycle C_1. Clearly, $c(H_1) \leq c(T') + c(e)$. If $d_{T'}(w) \geq 2$, let $s \neq u$ be the other neighbor of w on C_1, and t be another neighbor of w on T'. Set $T' := H_1 - sw - wt + st$ (see Fig. 2(a)). In all above case, the new tree T' is a Steiner tree satisfying the following conditions:

(1) all vertices of $V(C) \cap R_1$ have degree at least 2,
(2) the degree of any other vertex of R_1 does not alter, and
(3) the weight of the new tree is less than the weight of the old tree plus $c(e)$.

Next, we consider the final case $w \in V(T' - C)$ with $d_{T'}(w) = 1$. Let s be the other neighbor of w on H_1. Set $H_2 := H_1 + us - \{w\}$ (see Fig. 2(b)). Then H_1 contains a Steiner tree of G_1 on R_1, and $c(H_2) \leq c(H_1) \leq c(T') + c(e)$. Furthermore, H_2 has unique cycle C_2 and $|V(H_2) - R_1| < |V(H_1) - R_1|$. Repeat the above procedure. Since $|V(G_1) - R_1|$ is finite, there is a stage where we come across a case different from the final one, at which a modified tree T' satisfying conditions (1)–(3) is obtained.

After all edges of M having been added and all corresponding modifications having been made, let the final tree be T''. Then T'' is a Steiner tree of G_1 on R_1 such that v_{T_i} has degree 1 only when T_i contains vertex in $R - R'$. Furthermore,

$$c(T'') \leq c(T') + c(M) \leq (1 + \rho)c(T_{opt}).$$

Lemma 6 *We can obtain a Steiner tree T'' of G_1 on R_1 with approximation ratio $(1 + \rho)$ in polynomial time. In T'', every v_{T_i} has degree no less than 2 unless T_i contains vertex in $R - R'$.*

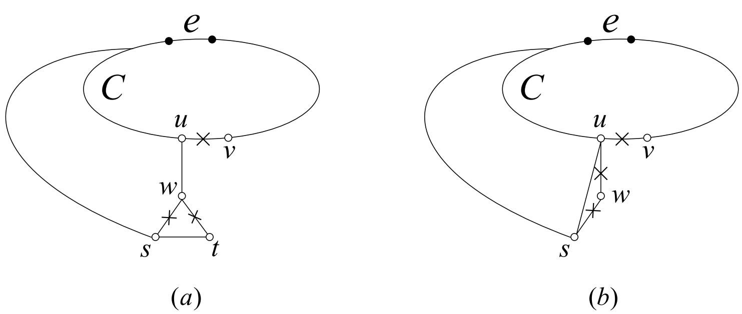

Fig. 2 The two cases of $w \in V(T' - C)$

2.3 "Blossom" Every v_{T_i} of T'' and Modification

In this subsection, we "blossom" every v_{T_i} to T_i and modify it to a new Steiner tree T''' of G on R such that every vertex of R' is not a leaf.

If $d_{T''}(v_{T_i}) \geq 2$ and there are at least two vertices u and v of T_i connected to vertices outside of T_i. By Corollary 5, there is a path P from u and v containing all vertices of T_i with $c(P) \leq 2c(T_i)$. Replace T_i by P. If $d_{T''}(v_{T_i}) \geq 2$ and there is only one vertex u of T_i connected to vertices outside of T_i. Choose two neighbors of u in $V - V(T_i)$, saying v and w. By Lemma 4, there is a cycle C containing all vertices of T_i with $c(C) \leq 2c(T_i)$. Let t be a neighbor of u on C. Replace T_i by $C - ut$ and uw by wt. If $d_{T''}(v_{T_i}) = 1$, then T_i has a vertex $u \in R - R'$ by Lemma 6. Let v be the unique vertex connected to some vertex outside of T_i. If $u \neq v$, replace T_i by P, where P is a path from v to u containing all vertices of T_i with $c(P) \leq 2c(T_i)$. If $u = v$, let w be the unique neighbor of u in $V - V(T_i)$ and t another vertex of T_i. By Lemma 4, there is a cycle C containing all vertices of T_i with $c(C) \leq 2c(T_i)$ and $ut \in C$. Replace T_i by $C - vt$ and uw by wt.

After modifying all T_i as above, the resulting graph T''' is a Steiner tree of G on R such that every vertex of R' is not a leaf. By Corollary 3, let T_{R_1} be a Steiner minimum tree of G_1 on R_1 and T_R be a Steiner minimum tree of G on R with $\bigcup_{i=1}^{m} E(T_i) \subseteq E(T_R)$. Then

$$\begin{aligned} c(T''') &\leq (1+\rho)c(T_{R_1}) + 2c(T_1) + \cdots + 2c(T_m) \\ &\leq (1+\rho)(c(T_{R_1}) + c(T_1) + \cdots + c(T_m)) \\ &\leq (1+\rho)c(T_R). \end{aligned}$$

Hence, we obtain the following theorem.

Theorem 7 *Given a complete graph $G = (V, E)$ with a metric weight function c on its edges. Let $R' \subset R \subseteq V$ with $|R - R'| \geq 2$. We can obtain a selected-internal Steiner tree with approximation ratio $(1 + \rho)$ in polynomial time.*

3 Conclusion and Discussion

In this paper, we present an $(1+\rho)$-approximation algorithm for selected-internal Steiner minimum tree problem. It would be interesting and challenging to design a better approximation algorithm or design a approximation algorithm for general weight function c.

Then, we discuss a special case when $R = V$, i.e., selected-internal minimum spanning tree problem. This problem is NP-complete since when $|R - R'| = 2$, this problem is equivalent to the minimum Hamilton path problem. Similarly, by Corollary 5, we can obtain a 2-approximation algorithm for selected-internal minimum spanning tree problem.

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