2.1-4 This problem is identical to Example 2.2b. except that $\omega_1 \neq \omega_2$. In this case, the third integral in P_g (see p. 19 is not zero. This integral is given by

$$I_3 = \lim_{T\to\infty} \frac{2C_1C_2}{T} \int_{-T/2}^{T/2} \cos(\omega_1 t + \theta_1)\cos(\omega_1 t + \theta_2)\, dt$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T} \left[\int_{-T/2}^{T/2} \cos(\theta_1 - \theta_2)\, dt + \int_{-T/2}^{T/2} \cos(2\omega_1 t + \theta_1 + \theta_2)\, dt \right]$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T} [T \cos(\theta_1 - \theta_2)] + 0 = C_1C_2\cos(\theta_1 - \theta_2)$$

Therefore

$$P_g = \frac{C_1^2}{2} + \frac{C_2^2}{2} + C_1C_2\cos(\theta_1 - \theta_2)$$

2.1-8 (a) Power of a sinusoid of amplitude C is $C^2/2$ [Eq. (2.6a)] regardless of its frequency ($\omega \neq 0$) and phase. Therefore, in this case $P = (10)^2/2 = 50$.
(b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids [Eq. (2.6b)]. Therefore, in this case $P = \frac{(10)^2}{2} + \frac{(16)^2}{2} = 178$.
(c) $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. Hence from Eq. (2.6b) $P = \frac{(10)^2}{2} + \frac{1}{2} + \frac{1}{2} = 51$.
(d) $10\cos 5t\cos 10t = 5(\cos 5t + \cos 15t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25$.
(e) $10\sin 5t\cos 10t = 5(\sin 15t - \sin 5t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(-5)^2}{2} = 25$.
(f) $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. Using the result in Prob. 2.1-7, we obtain $P = (1/4) + (1/4) = 1/2$.

2.3-2 All the signals are shown in Fig. S2.3-2.

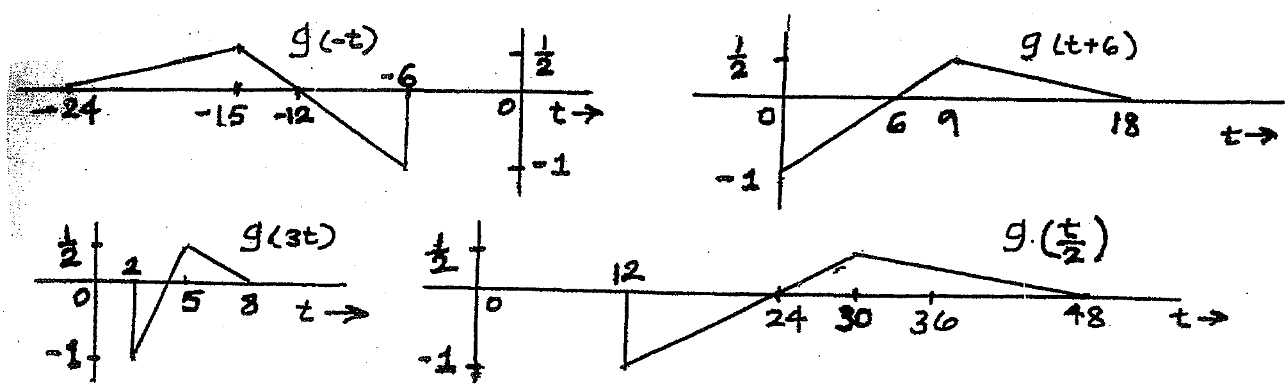

Fig. S2.3-2

2.5-4 (a) In this case $E_x = \int_0^1 \sin^2 2\pi t\, dt = 0.5$, and

$$c = \frac{1}{E_x}\int_0^1 g(t)x(t)\, dt = \frac{1}{0.5}\int_0^1 t\sin 2\pi t\, dt = -1/\pi$$

(b) Thus, $g(t) \approx -(1/\pi)x(t)$, and the error $e(t) = t + (1/\pi)\sin 2\pi t$ over $(0 \leq t \leq 1)$, and zero outside this interval. Also E_g and E_e (the energy of the error) are

$$E_g = \int_0^1 g^2(t)\, dt = \int_0^1 t^2\, dt = 1/3 \quad \text{and} \quad E_e = \int_0^1 [t - (1/\pi)\sin 2\pi t]^2\, dt = \frac{1}{3} - \frac{1}{2\pi^2}$$

The error $[t + (1/\pi)\sin 2\pi t]$ is orthogonal to $x(t)$ because

$$\int_0^1 \sin 2\pi t[t + (1/\pi)\sin 2\pi t]\, dt = 0$$

Note that $E_g = c^2E_x + E_e$. To explain these results in terms of vector concepts we observe from Fig. 2.15 that the error vector **e** is orthogonal to the component c**x**. Because of this orthogonality, the length of **f** [energy of $g(t)$] is equal to the sum of the square of the lengths of c**x** and **e** [sum of the energies of $cx(t)$ and $e(t)$]

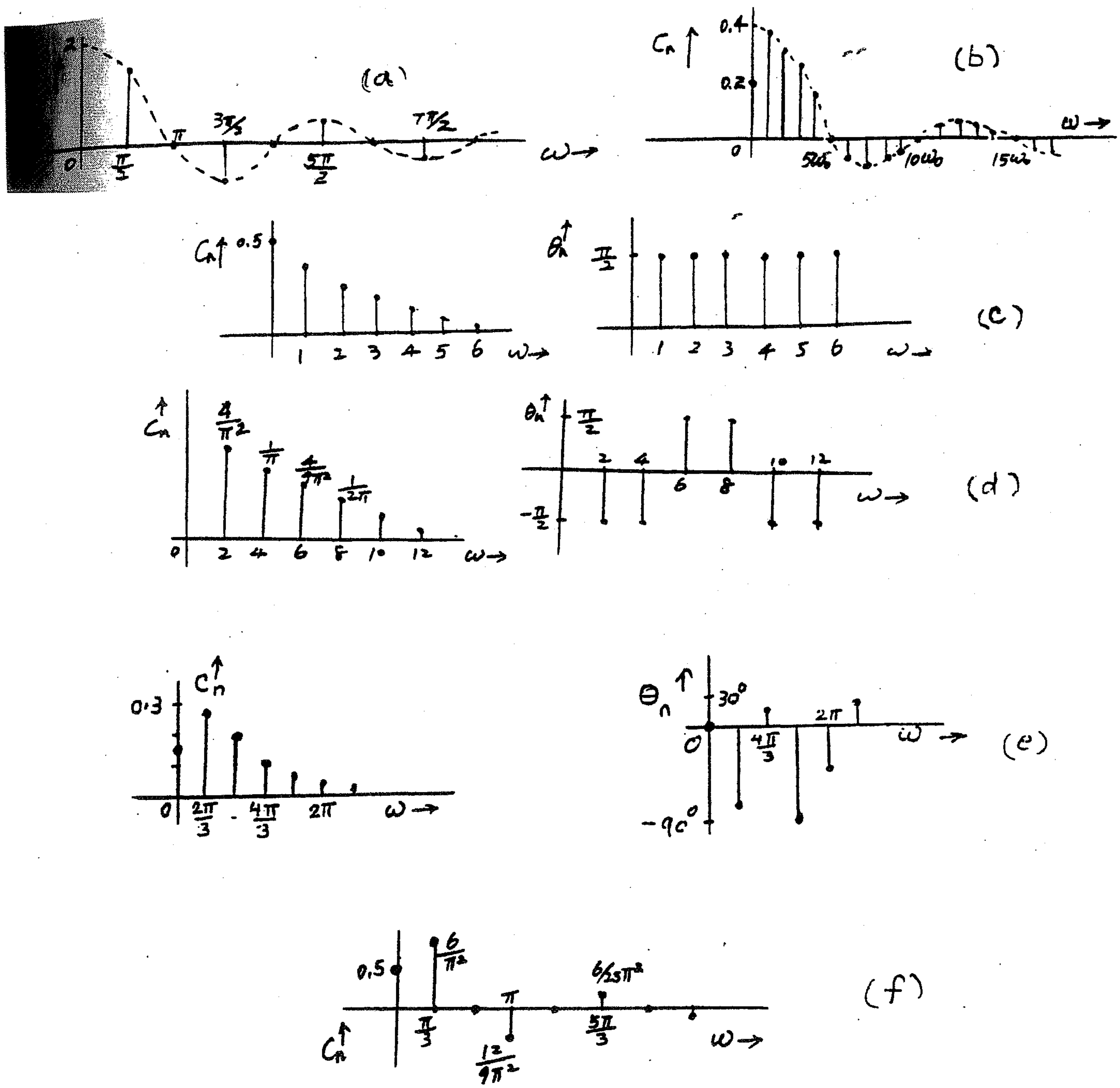

Fig. S2.8-4

The reason for vanishing of the cosines terms is that when 0.5 (the dc component) is subtracted from $g(t)$, the remaining function has odd symmetry. Hence, the Fourier series would contain dc and sine terms only. Figure S2.8-4c shows the plot of C_n and θ_n.

(d) $T_0 = \pi$, $\omega_0 = 2$ and $g(t) = \frac{4}{\pi}t$.

$a_0 = 0$ (by inspection).

$a_n = 0 \quad (n > 0)$ because of odd symmetry.

$$b_n = \frac{4}{\pi}\int_0^{\pi/4} \frac{4}{\pi} t \sin 2nt \, dt = \frac{2}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

$$\begin{aligned} g(t) &= \frac{4}{\pi^2}\sin 2t + \frac{1}{\pi}\sin 4t - \frac{4}{9\pi^2}\sin 6t - \frac{1}{2\pi}\sin 8t + \cdots \\ &= \frac{4}{\pi^2}\cos\left(2t - \frac{\pi}{2}\right) + \frac{1}{\pi}\cos\left(4t - \frac{\pi}{2}\right) + \frac{4}{9\pi^2}\cos\left(6t + \frac{\pi}{2}\right) + \frac{1}{\pi}\cos\left(8t + \frac{\pi}{2}\right) + \cdots \end{aligned}$$

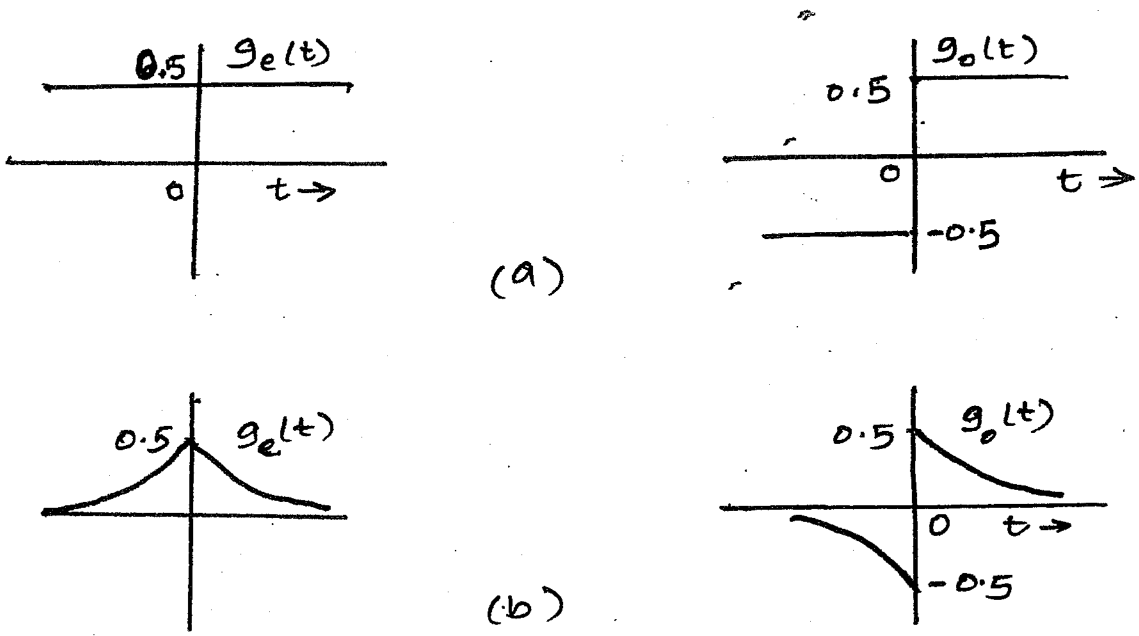

Fig. S2.8-5

$$g_o(t) = \frac{1}{2}\left[e^{-at}u(t) - e^{at}u(-t)\right]$$

The even and odd components of the signal $e^{-at}u(t)$ are shown in Fig. S2.8-5b.
For $g(t) = e^{jt}$, we have

$$e^{jt} = g_e(t) + g_o(t)$$

where

$$g_e(t) = \frac{1}{2}\left[e^{jt} + e^{-jt}\right] = \cos t$$

and

$$g_o(t) = \frac{1}{2}\left[e^{jt} - e^{-jt}\right] = j \sin t$$