5.1-3

(a) $$\varphi_{\text{PM}}(t) = A\cos\left[\omega_c t + k_p m(t)\right] = 10\cos[10{,}000t + k_p m(t)]$$

We are given that $\varphi_{\text{PM}}(t) = 10\cos(13{,}000t)$ with $k_p = 1000$. Clearly, $m(t) = 3t$ over the interval $|t| \le 1$.

(b) $$\varphi_{\text{FM}}(t) = A\cos\left[\omega_c t + k_f \int^t m(\alpha)\,d\alpha\right] = 10\cos\left[10{,}000t + k_f \int^t m(\alpha)\,d\alpha\right]$$

Therefore $$k_f \int^t m(\alpha)\,d\alpha = 1000 \int^t m(\alpha)\,d\alpha = 3000t$$

Hence $$3t = \int^t m(\alpha)\,d\alpha \qquad \Longrightarrow m(t) = 3$$

5.2-2 $\varphi_{\text{EM}}(t) = 10\cos(\omega_c t + 0.1\sin 2000\pi t)$. Here, the baseband signal bandwidth $B = 2000\pi/2\pi = 1000$ Hz. Also,

$$\omega_i(t) = \omega_c + 200\pi\cos 2000\pi t$$

Therefore, $\Delta\omega = 200\pi$ and $\Delta f = 100$ Hz and $B_{\text{EM}} = 2(\Delta f + B) = 2(100 + 1000) = 2.2$ kHz.

5.2-3 $\varphi_{\text{EM}}(t) = 5\cos(\omega_c t + 20\sin 1000\pi t + 10\sin 2000\pi t)$.
Here, the baseband signal bandwidth $B = 2000\pi/2\pi = 1000$ Hz. Also,

$$\omega_i(t) = \omega_c + 20{,}000\pi\cos 1000\pi t + 20{,}000\pi\cos 2000\pi t$$

Therefore, $\Delta\omega = 20{,}000\pi + 20{,}000\pi = 40{,}000\pi$ and $\Delta f = 20$ kHz and $B_{\text{EM}} = 2(\Delta f + B) = 2(20{,}000 + 1000) = 42$ kHz.

5.2-7 From pair 22(Table 3.1), we obtain $e^{-t^2} \Longleftrightarrow \sqrt{\pi}\,e^{-\omega^2/4}$. The spectrum $M(\omega) = \sqrt{\pi}\,e^{-\omega^2/4}$ is a Gaussian pulse, which decays rapidly. Its 3 dB bandwidth is 1.178 rad/s=0.187 Hz. This is an extremely small bandwidth compared to Δf.

Also $\dot{m}(t) = -2te^{-t^2/2}$. The spectrum of $\dot{m}(t)$ is $M'(\omega) = j\omega M(\omega) = j\sqrt{\pi}\omega\,e^{-\omega^2/4}$. This spectrum also decays rapidly away from the origin, and its bandwidth can also be assumed to be negligible compared to Δf.

For FM : $\Delta f = \frac{k_f m_p}{2\pi} = \frac{6000\pi \times 1}{2\pi} = 3$ kHz and $B_{\text{FM}} \approx 2\Delta f = 2 \times 3 = 6$ kHz.

For PM : To find m'_p, we set the derivative of $\dot{m}(t) = -2te^{-t^2/2}$ equal to zero. This yields

$$\ddot{m}(t) = -2e^{-t^2/2} + 4t^2e^{-t^2/2} = 0 \qquad \Longrightarrow t = \frac{1}{\sqrt{2}}$$

and $m'_p = \dot{m}(\frac{1}{\sqrt{2}}) = 0.858$, and

$\Delta f = \frac{k_p m'_p}{2\pi} = \frac{8000\pi \times 0.858}{2\pi} = 3.432$ kHz and $B_{\text{PM}} \approx 2(\Delta f) = 2(3.432) = 6.864$ kHz.

5.3-1 The block diagram of the design is shown in Fig. S5.3-1.

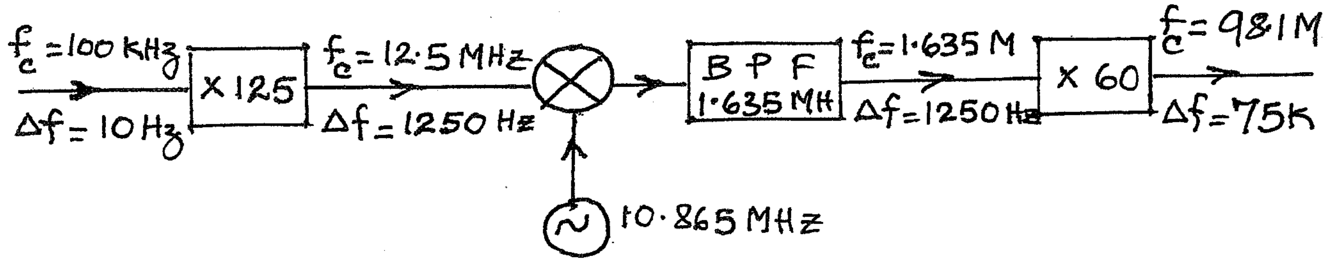

Fig. S5.3-1

5.4-2 Figure S5.4-2 shows the waveforms at points b, c, d, and e. The figure is self explanatory.