3.1-1

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} g(t)\cos\omega t\,dt - j\int_{-\infty}^{\infty} g(t)\sin\omega t\,dt$$

If $g(t)$ is an even function of t, $g(t)\sin\omega t$ is an odd function of t, and the second integral vanishes. Moreover, $g(t)\cos\omega t$ is an even function of t, and the first integral is twice the integral over the interval 0 to ∞. Thus when $g(t)$ is even

$$G(\omega) = 2\int_0^{\infty} g(t)\cos\omega t\,dt \tag{1}$$

Similar argument shows that when $g(t)$ is odd

$$G(\omega) = -2j\int_0^{\infty} g(t)\sin\omega t\,dt \tag{2}$$

If $g(t)$ is also real (in addition to being even), the integral (1) is real. Moreover from (1)

$$G(-\omega) = 2\int_0^{\infty} g(t)\cos\omega t\,dt = G(\omega)$$

Hence $G(\omega)$ is real and even function of ω. Similar arguments can be used to prove the rest of the properties.

3.2-4 Observe that $1 + \text{sgn}(t) = 2u(t)$. Adding pairs 7 and 12 in Table 3.1 and then dividing by 2 yields the desired

$$\delta(t+T) - \delta(t-T) \Longleftrightarrow 2j\sin T\omega$$

3.3-2 **Fig. (b)** $g_1(t) = g(-t)$ and

$$G_1(\omega) = G(-\omega) = \frac{1}{\omega^2}[e^{-j\omega} + j\omega e^{-j\omega} - 1]$$

Fig. (c) $g_2(t) = g(t-1) + g_1(t-1)$. Therefore

$$\begin{aligned} G_3(\omega) &= [G(\omega) + G_1(\omega)]e^{-j\omega} = [G(\omega) + G(-\omega)]e^{-j\omega} \\ &= \frac{2e^{-j\omega}}{\omega^2}(\cos\omega + \omega\sin\omega - 1) \end{aligned}$$

Fig. (d) $g_3(t) = g(t-1) + g_1(t+1)$

$$\begin{aligned} G_4(\omega) &= G(\omega)e^{-j\omega} + G(-\omega)e^{j\omega} \\ &= \frac{1}{\omega^2}[2 - 2\cos\omega] = \frac{4}{\omega^2}\sin^2\frac{\omega}{2} = \text{sinc}^2\left(\frac{\omega}{2}\right) \end{aligned}$$

Fig. (e) $g_4(t) = g(t - \frac{1}{2}) + g_1(t + \frac{1}{2})$, and

$$\begin{aligned} G_4(\omega) &= G(\omega)e^{-j\omega/2} + G_1(\omega)e^{j\omega/2} \\ &= \frac{e^{-j\omega/2}}{\omega^2}[e^{j\omega} - j\omega e^{j\omega} - 1] + \frac{e^{j\omega/2}}{\omega^2}[e^{-j\omega} + j\omega e^{-j\omega} - 1] \\ &= \frac{1}{\omega^2}[2\omega\sin\frac{\omega}{2}] = \text{sinc}\left(\frac{\omega}{2}\right) \end{aligned}$$

Fig. (f) $g_5(t)$ can be obtained in three steps: (i) time-expanding $g(t)$ by a factor 2 (ii) then delaying it by 2 seconds. (iii) and multiplying it by 1.5 [we may interchange the sequence for steps (i) and (ii)]. The first step (time-expansion by a factor 2) yields

$$f\left(\frac{t}{2}\right) \Longleftrightarrow 2G(2\omega) = \frac{1}{2\omega^2}(e^{j2\omega} - j2\omega e^{j2\omega} - 1)$$

Second step of time delay of 2 secs. yields

$$f\left(\frac{t-2}{2}\right) \Longleftrightarrow \frac{1}{2\omega^2}(e^{j2\omega} - j2\omega e^{j2\omega} - 1)e^{-j2\omega} = \frac{1}{2\omega^2}(1 - j2\omega - e^{-j2\omega})$$

The third step of multiplying the resulting signal by 1.5 yields

$$g_5(t) = 1.5f\left(\frac{t-2}{2}\right) \Longleftrightarrow \frac{3}{4\omega^2}(1 - j2\omega - e^{-j2\omega})$$

3.7-3 Recall that

$$g_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_2(\omega)e^{j\omega t}\,d\omega \quad \text{and} \quad \int_{-\infty}^{\infty} g_1(t)e^{j\omega t}\,dt = G_1(-\omega)$$

Therefore

$$\int_{-\infty}^{\infty} g_1(t)g_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} g_1(t)\left[\int_{-\infty}^{\infty} G_2(\omega)e^{j\omega t}\,d\omega\right]dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} G_2(\omega)\left[\int_{-\infty}^{\infty} g_1(t)e^{j\omega t}\,dt\right]d\omega = \frac{1}{2\pi}\int G_1(-\omega)G_2(\omega)\,d\omega$$

Interchanging the roles of $g_1(t)$ and $g_2(t)$ in the above development, we can show that

$$\int_{-\infty}^{\infty} g_1(t)g_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(\omega)G_2(-\omega)\,d\omega$$