2.1-1 Both \( \varphi(t) \) and \( w_0(t) \) are periodic.

The average power of \( \varphi(t) \) is

\[
P_\varphi = \frac{1}{T} \int_0^T \varphi^2(t) \, dt = \frac{1}{\pi} \int_0^\pi (\sin^2 t)^2 \, dt = \frac{1-\cos^2 \pi}{\pi}.
\]

The average power of \( w_0(t) \) is

\[
P_w = \frac{1}{T} \int_0^T w_0^2(t) \, dt = \frac{1}{\pi} \int_0^\pi 1 \, dt = 1.
\]

2.1-2

(a) Since \( x(t) \) is a real signal, \( E_x = \int_0^2 x^2(t) \, dt \).

Solving for Fig. S2.1-2(a), we have

\[
E_x = \int_0^1 (1)^2 \, dt = 2, \quad E_y = \int_0^1 (1)^2 \, dt + \int_1^2 (-1)^2 \, dt = 2
\]

\[
E_{x+y} = \int_0^1 (2)^2 \, dt = 4, \quad E_{x-y} = \int_1^2 (2)^2 \, dt = 4
\]

Therefore, \( E_{x+y} = E_x + E_y \).

Solving for Fig. S2.1-2(b), we have

\[
E_x = \int_0^\pi (1)^2 \, dt + \int_{\pi/2}^{2\pi} (-1)^2 \, dt = 2\pi, \quad E_y = \int_0^{\pi/2} (1)^2 \, dt + \int_{\pi/2}^{3\pi/2} (-1)^2 \, dt + \int_{3\pi/2}^{2\pi} (1)^2 \, dt + \int_{2\pi}^{3\pi/2} (-1)^2 \, dt = 2\pi
\]

\[
E_{x+y} = \int_0^{\pi/2} (2)^2 \, dt + \int_{\pi/2}^{3\pi/2} (0)^2 \, dt + \int_{3\pi/2}^{2\pi} (-2)^2 \, dt = 4\pi
\]

\[
E_{x-y} = \int_0^{\pi/2} (0)^2 \, dt + \int_{\pi/2}^{3\pi/2} (2)^2 \, dt + \int_{3\pi/2}^{2\pi} (-2)^2 \, dt + \int_{2\pi}^{3\pi/2} (0)^2 \, dt = 4\pi
\]

Therefore, \( E_{x+y} = E_x + E_y \).

(b) \( E_x = \int_0^{\pi/4} (1)^2 \, dt + \int_{\pi/4}^\pi (-1)^2 \, dt = \pi \), \( E_y = \int_0^{\pi} (1)^2 \, dt = \pi \)
\[ E_{x+y} = \int_0^{\pi/4} (2)^2 \, dt + \int_0^{\pi/4} (0)^2 \, dt = \pi, \quad E_{x-y} = \int_0^{\pi/4} (0)^2 \, dt + \int_0^{\pi/4} (-2)^2 \, dt = 3\pi \] Therefore, \( E_{x+y} \neq E_x + E_y \), and \( E_{x-y} = E_x \pm E_y \) are not true in general.

2.1-3

\[
P_g \quad = \quad \frac{1}{T_0} \int_0^{T_0} C^2 \cos^2 (\omega_0 t + \theta) \, dt = \frac{C^2}{2 T_0} \int_0^{T_0} \left[ 1 + \cos (2\omega_0 t + 2\theta) \right] \, dt
\]

\[
= \frac{C^2}{2 T_0} \left[ \int_0^{T_0} \, dt + \int_0^{T_0} \cos (2\omega_0 t + 2\theta) \, dt \right] = \frac{C^2}{2 T_0} \left[ T_0 + 0 \right] = \frac{C^2}{2}
\]

2.1-4 If \( \omega_1 = \omega_2 \), then

\[
g^2(t) \quad = \quad (C_1 \cos (\omega_1 t + \theta_1) + C_2 \cos (\omega_1 t + \theta_2))^2
\]

\[
\quad = \quad C_1^2 \cos^2 (\omega_1 t + \theta_1) + C_2^2 \cos^2 (\omega_1 t + \theta_2) + 2C_1C_2 \cos (\omega_1 t + \theta_1) \cos (\omega_1 t + \theta_2)
\]

\[
P_g \quad = \quad \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^{T_0} \left( C_1 \cos (\omega_1 t + \theta_1) + C_2 \cos (\omega_1 t + \theta_2) \right)^2 \, dt
\]

\[
= \quad \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T_0 \to \infty} 2C_1 C_2 \frac{1}{T_0} \int_0^{T_0} \cos (\omega_1 t + \theta_1) \cos (\omega_1 t + \theta_2) \, dt
\]

\[
= \quad \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T_0 \to \infty} 2C_1 C_2 \frac{1}{T_0} \int_0^{T_0} \frac{1}{2} \left[ \cos (2\omega_1 t + \theta_1 + \theta_2) + \cos (\theta_1 - \theta_2) \right] \, dt
\]

\[
= \quad \frac{C_1^2}{2} + \frac{C_2^2}{2} + 0 + \frac{2C_1 C_2}{2} \cos (\theta_1 - \theta_2)
\]

\[
= \quad \frac{C_1^2 + C_2^2 + 2C_1 C_2 \cos (\theta_1 - \theta_2)}{2}
\]

2.1-5

\[
P_g \quad = \quad \frac{1}{4} \int_{-2}^{2} (t^3)^2 \, dt = 64/7
\]

(a) \( P_{-g} \quad = \quad \frac{1}{4} \int_{-2}^{2} (-t^3)^2 \, dt = 64/7 \)

(b) \( P_{2g} \quad = \quad \frac{1}{4} \int_{-2}^{2} (2t^3)^2 \, dt = 4(64/7) = 256/7 \)

(c) \( P_{cg} \quad = \quad \frac{1}{4} \int_{-2}^{2} (ct^3)^2 \, dt = 64c^2/7 \)

Changing the sign of a signal does not affect its power. Multiplication of a signal by a constant \( c \) increases the power by a factor of \( c^2 \).

2.1-6 Let us denote the signal in question by \( g(t) \) and its energy by \( E_g \).

(a), (b) For parts (a) and (b), we write

\[
E_g = \int_0^{2\pi} \sin^2 t \, dt = \frac{1}{2} \int_0^{2\pi} dt - \frac{1}{2} \int_0^{2\pi} \cos 2t \, dt = \pi + 0 = \pi
\]
\[ E_g = \int_{2\pi}^{4\pi} \sin^2 t \, dt = \frac{1}{2} \int_{2\pi}^{4\pi} dt - \frac{1}{2} \int_{2\pi}^{4\pi} \cos 2t \, dt = \pi + 0 = \pi \]

\[ E_g = \int_{0}^{2\pi} (2\sin t)^2 \, dt = 4 \left[ \frac{1}{2} \int_{0}^{2\pi} dt - \frac{1}{2} \int_{0}^{2\pi} \cos 2t \, dt \right] = 4[\pi + 0] = 4\pi \]

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way, we can show that the energy of \( k\mathbb{g}(t) \) is \( k^2 E_g \).

**2.1.7**

\[ P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)g^*(t) \, dt \]

\[ = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{m=1}^{n} \sum_{r=1}^{n} D_k^* D_r e^{i(\omega_k - \omega_r)t} \, dt \]

\[ = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{m=1}^{n} \sum_{r=1}^{n} D_k^* D_r e^{i(\omega_k - \omega_r)t} + \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=1}^{n} |D_k|^2 \, dt \]

The integrals of the cross-product terms (when \( k \neq r \)) are finite because the integrands (functions to be integrated) are periodic signals (made up of sinusoids). These terms, when divided by \( T \to \infty \), yield zero. The remaining terms (\( k = r \)) yield

\[ P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=1}^{n} |D_k|^2 \, dt = \sum_{k=1}^{n} |D_k|^2 \]

**2.1.8**

(a) From Eq. (2.5a), the power of a signal of amplitude \( C \) is \( P_g = \frac{C^2}{2} \), regardless of phase and frequency; therefore, \( P_g = 100/2 = 50 \); the rms value is \( \sqrt{P_g} = 5\sqrt{2} \).

(b) From Eq. (2.5b), the power of the sum of two sinusoids of different frequencies is the sum of the power of individual sinusoids, regardless of the phase, \( \frac{C_1^2}{2} + \frac{C_2^2}{2} \), therefore, \( P_g = 100/2 + 256/2 = 50 + 128 = 178 \); the rms value is \( \sqrt{P_g} = \sqrt{178} \).

(c) \( g(t) = (10 + 2 \sin (3t)) \cos (10t) = 10 \cos (10t) + 2 \sin (3t) \cos (10t) = 10 \cos (10t) + \sin (13t) - \cos (7t) \)

Therefore, \( P_g = 100/2 + 1/2 + 1/2 = 50 + 0.5 + 0.5 = 51 \); the rms value is \( \sqrt{P_g} = \sqrt{51} \).

(d) \( g(t) = 10 \cos (5t) \cos (10t) = \frac{10(\cos (15t) + \cos (5t))}{2} = 5 \cos (15t) + 5 \cos (5t) \)

Therefore, \( P_g = 25/2 + 25/2 = 25 \); the rms value is \( \sqrt{P_g} = 5 \).

(e) \( g(t) = 10 \sin (5t) \cos (10t) = 5(\cos (15t) - \cos (5t)) = 5 \cos (15t) - 5 \cos (5t) \)

Therefore, \( P_g = 25/2 + 25/2 = 25 \); the rms value is \( \sqrt{P_g} = 5 \).

(f) \( |g(t)|^2 = \cos^2 (\omega_0 t) \)

Therefore, \( P_g = 1/2 = 0.5 \); the rms value is \( \sqrt{P_g} = \sqrt{0.5} \).
2.1-9

(a) Power \( P_g = \frac{1}{T} \int_0^T 1 \cdot dt = 1 \), and the rms value is \( \sqrt{P_g} = \sqrt{1} = 1 \)

(b) Power \( P_g = \frac{1}{10\pi} \left[ \int_0^{\pi} 1 \cdot dt + \int_0^{2\pi} 0 \cdot dt + \int_0^{10\pi} 1 \cdot dt \right] = \frac{1}{10\pi} \left[ \pi + 0 + 0 \right] = \frac{\pi}{10} \); and the rms value is \( \sqrt{1/5} \).

(c) Power

\[
P_g = \frac{1}{T} \int_0^T g^2(t) \, dt = \frac{1}{6} \left[ \int_0^1 g^2(t) \, dt + \int_1^2 g^2(t) \, dt + \int_2^4 g^2(t) \, dt + \int_4^5 g^2(t) \, dt + \int_5^6 g^2(t) \, dt \right]
\]

\[
= \frac{1}{6} \left[ 1 + \int_1^2 (t^2 - 2) \, dt + 0 + \int_4^5 (t - 4)^2 \, dt + 1 \right]
\]

\[
= \frac{1}{6} \left[ 1 + \frac{1}{3} + 0 + \frac{1}{3} + 1 \right] = \frac{4}{9}
\]

and the rms value is \( \sqrt{4/9} = 2/3 \)

\[2.2-1\] If \( a \) is complex with real part 0, \( a = \alpha \); then, \( g(t) = e^{-\alpha t} \) and \( |g(t)|^2 = 1 \)

\[
P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} 1 \cdot dt = \lim_{T \to \infty} \frac{1}{T} T = 1.
\]

Hence it is a power signal. It is not an energy signal since \( E_g = \int_{-\infty}^{\infty} |g(t)|^2 \cdot dt = \infty \). If \( a \) is real, then both \( E_g = \int_{-\infty}^{\infty} |e^{-\alpha t}|^2 \cdot dt = \infty \) and \( P_g = \infty \).

\[2.2-2\] Let \( c = a + jb \), where \( a, b \) are real valued. Therefore, \( |e^{-ct}| = |e^{-(a+jb)t}| = |e^{-at} \cdot e^{-jbt}| = |e^{jbt}| \cdot |e^{-at}| = 1 = |e^{-at}| \)

\[
E_g = \int_{-\infty}^{\infty} |e^{-ct}|^2 \cdot dt = \int_{-\infty}^{\infty} e^{-2at} \cdot dt = \infty
\]

\[
P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |e^{-ct}|^2 \cdot dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-2at} \cdot dt = \infty
\]

Therefore, \( e^{-ct} \) is neither energy nor a power signal for a complex value of \( c \) with nonzero real part.

2.3-1

\( g_2(t) = g(t - 1) + g_1(t - 1), \quad g_3(t) = g(t - 1) + g_1(t + 1), \quad g_4(t) = g(t - 0.5) + g_1(t + 0.5) \)

The signal \( g_6(t) \) can be obtained by (i) delaying \( g(t) \) by 1 second (replace \( t \) with \( t - 1 \)), (ii) then time-expanding by a factor 2 (replace \( t \) with \( t/2 \)), (iii) then multiplying by 1.5. Thus \( g_6(t) = 1.5g(\frac{t}{2} - 1) \).

\[2.3-2\] (a) See Fig. S2.3-2a.
(b) Energy of $g(t)$,

$$E_g = \int_{-\infty}^{\infty} g^2(t) \, dt = \int_{6}^{15} \left[ \frac{1}{6} (t - 12) \right]^2 \, dt + \int_{15}^{24} \left[ -\frac{1}{18} (t - 24) \right]^2 \, dt = \frac{9}{4} + \frac{3}{4} = 3$$

Since [see Problem 2.3-5], time shifting or time inversion does not change the signal energy,

$$E_{g(-t)} = E_{g(t+12)} = E_{g(t)} = 3$$

On the other hand, a scaling of $g(at)$ will change the signal energy to $E_g/a$ [Problem 2.3-5], the energy of $g(3t)$ is 1 and energy of $g(6-2t)$ is $\frac{3}{2}$.

2.3-3 See Fig. S2.3.3.

2.3-4 Denote $g(at) = f(t)$. Since $g(t)$ is periodic with period $T$,

$$g(t) = g(t + T)$$

$$g(a t) = g(a(t + T)) = g\left( a\left(t + \frac{T}{a}\right) \right)$$

$$f(t) = f\left(t + \frac{T}{a}\right)$$

Therefore, $g(at)$ is periodic with period $T/a$.

The average power of $g(at)$ is

$$P_{g(at)} = \lim_{T \to \infty} \frac{a}{T} \int_{-T/2a}^{T/2a} g^2(at) \, dt = \lim_{T \to \infty} \frac{a}{T} \int_{-T/2}^{T/2} g^2(zt) \, dz = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(z) \, dz = P_g$$

Therefore, the average power remains the same.
\[
\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{|a|} \phi(0) = \frac{1}{|a|} \int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt
\]
\[
\delta(at) = \frac{1}{|a|} \delta(t)
\]

Therefore,
\[
\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi} \delta(f)
\]

2.4-3 Using the fact that \(\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi} \delta(f)\), and the equality \(\int_{-\infty}^{\infty} \phi(t)\delta(t-T)\,dt = \phi(T)\), we get

(a) \(\int_{-\infty}^{\infty} g(\tau + a)\delta(t-\tau)\,d\tau = g(t + a)\)

(b) \(\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)\,d\tau = g(t)\)

(c) 1

(d) 0

(e) 0

(f) 5

(g) \(g(-1)\)

(h)
\[
\int_{-\infty}^{\infty} \cos \frac{\pi}{2}(x-5)\delta(2x-3)\,dx = \int_{-\infty}^{\infty} \cos \frac{\pi}{2}(x-5)\delta \left(2\left(x - \frac{3}{2}\right)\right)\,dx = \frac{1}{2} \int_{-\infty}^{\infty} \cos \frac{\pi}{2}(x-5)\delta \left(x - \frac{3}{2}\right)\,dx = \frac{1}{2} \cos \frac{\pi}{2} \left(\frac{3}{2} - 5\right) = \sqrt{3}/4
\]

Here, we used the fact \(\delta(at) = \frac{1}{|a|} \delta(t)\) (see Problem 2.3-2)

2.5-1
\[
|e|^2 = |g|^2 + c^2 |x|^2 - 2c \cdot x
\]

To minimize error, set \(\frac{d|e|^2}{dc} = 0\):
\[
2c|x|^2 - 2c \cdot x = 0
\]
\[
c = \frac{g \cdot x}{|x|^2} = \frac{\langle g, x \rangle}{|x|^2}
\]

2.5-2

(a) In this case \(E_x = \int_0^1 dt = 1\), and
\[
c = \frac{1}{E_x} \int_0^1 g(t)x(t)\,dt = \frac{1}{1} \int_0^1 t\,dt = 0.5
\]
(a) If \( x(t) \) and \( y(t) \) are orthogonal, then we can show that the energy of \( x(t) \pm y(t) \) is \( E_x + E_y \).

\[
\int \left| x(t) \pm y(t) \right|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt + \int_{-\infty}^{\infty} |y(t)|^2 dt \pm \int_{-\infty}^{\infty} x(t)y^*(t) dt \pm \int_{-\infty}^{\infty} x^*(t)y(t) dt \\
= \int_{-\infty}^{\infty} |x(t)|^2 dt + \int_{-\infty}^{\infty} |y(t)|^2 dt
\]

The last result follows from the fact that because of orthogonality, the two integrals of the cross products \( x(t)y^*(t) \) and \( x^*(t)y(t) \) are zero [see Eq. (2.40)]. Thus the energy of \( x(t) + y(t) \) is equal to that of \( x(t) - y(t) \) if \( x(t) \) and \( y(t) \) are orthogonal.

(b) We can use a similar argument to show that the energy of \( c_1 x(t) + c_2 y(t) \) is equal to that of \( c_1 x(t) - c_2 y(t) \) if \( x(t) \) and \( y(t) \) are orthogonal. This energy is given by \( |c_1|^2 E_x + |c_2|^2 E_y \).

(c) If \( z(t) = x(t) \pm y(t) \), then it follows from part (a) in the preceding derivation that

\[
E_z = E_x + E_y \pm (E_{xy} + E_{yx})
\]

2.6-1 We shall use Eq. (2.51) to compute \( \rho_n \) for each of the four cases. Let us first compute the energies of all the signals:

\[
E_x = \int_0^1 \sin^2 2\pi t dt = 0.5
\]

In the same way, we find \( E_{g_1} = E_{g_2} = E_{g_3} = E_{g_4} = 0.5 \).

From Eq. (2.51), the correlation coefficients for four cases are found as follows:

1. \[
\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 \sin 2\pi t \sin 4\pi t dt = 0
\]

2. \[
\frac{1}{\sqrt{(0.5)(0.5)}} \int (\sin 2\pi t)(-\sin 2\pi t) dt = -1
\]

3. \[
\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 0.707 \sin 2\pi t dt = 0
\]

4. \[
\frac{1}{\sqrt{(0.5)(0.5)}} \left[ \int_0^{0.5} 0.707 \sin 2\pi t dt - \int_{0.5}^1 0.707 \sin 2\pi t dt \right] = 2.828/\pi = 0.9
\]

Signals \( x(t) \) and \( g_2(t) \) provide the maximum protection against noise.

2.6-2 Since

\[
g(t) = u(t) - u(t - 2)
\]

it is a rectangular function that exists only from \( t = 0 \) to \( t = 2 \). Thus, if \( \tau > 2 \) or if \( \tau < -2 \), then \( g(t + \tau) \) and \( g(t) \) does not overlap. Thus,

\[
\psi_g(\tau) = \int 0 \cdot dt = 0 \quad \text{for} \quad |\tau| \geq 2
\]

If \( 0 \leq \tau < 2 \), then

\[
\psi_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t + \tau) dt = \int_0^{2-\tau} 1 \cdot dt = 2 - \tau \quad 0 \leq \tau < 2
\]

Because \( \psi_g(-\tau) = \psi_g(\tau) \), we have

\[
\psi_g(\tau) = \begin{cases} 
2 - |\tau| & \text{for} \quad |\tau| < 2 \\
0 & \text{for} \quad |\tau| \geq 2
\end{cases}
\]
Similarly, if $g(t)$ is an odd function of $t$, then $g(t) \cos n\omega_0 t$ is an odd function of $t$ and $g(t) \sin n\omega_0 t$ is an even function of $t$. Therefore

$$a_0 = a_n = 0$$

$$b_n = \frac{4}{T_0} \int_0^{T_0/2} g(t) \sin n\omega_0 t \, dt$$

Observe that because of symmetry, the integration required to compute the coefficients need be performed over only half the period.

(a) $T_0 = 4$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$. Because of even symmetry, all sine terms are zero.

\[
g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \left( \frac{n\pi}{2} t \right)\]

\[
a_0 = 0 \text{ (by inspection of its lack of dc)}
\]

\[
a_n = \frac{4}{n\pi} \left[ \int_0^1 \cos \left( \frac{n\pi}{2} t \right) \, dt - \int_1^2 \cos \left( \frac{n\pi}{2} t \right) \, dt \right]
\]

\[
= \frac{4}{n\pi} \sin \frac{n\pi}{2}
\]
Therefore, the Fourier series for \( g(t) \) is

\[
g(t) = \frac{4}{\pi} \left( \cos \frac{\pi t}{2} - \frac{1}{3} \cos \frac{3\pi t}{2} + \frac{1}{5} \cos \frac{5\pi t}{2} - \frac{1}{7} \cos \frac{7\pi t}{2} + \cdots \right)
\]

Here \( b_n = 0 \), and we allow \( C_n \) to take negative values. Figure S2.8-2(a) shows the plot of \( C_n \).

(b) \( T_0 = 10\pi \), \( \omega_0 = \frac{2\pi}{T_0} = \frac{1}{5} \). Because of even symmetry, all the sine terms are zero.

\[
g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \left( \frac{n}{5} t \right) + b_n \sin \left( \frac{n}{5} t \right)
\]

\[
a_0 = \frac{1}{5} \quad \text{(by inspection)}.
\]

\[
a_n = \frac{2}{10\pi} \int_{-\pi}^{\pi} \cos \left( \frac{n}{5} t \right) \, dt
\]

\[= \frac{1}{5\pi} \left[ \sin \left( \frac{n}{5} t \right) \right]_{-\pi}^{\pi} = \frac{2}{\pi n} \sin \left( \frac{n\pi}{5} \right)
\]

\[
b_n = \frac{2}{10\pi} \int_{-\pi}^{\pi} \sin \left( \frac{n}{5} t \right) \, dt
\]

\[= 0 \quad \text{(integrand is an odd function of} t)\]

Here \( b_n = 0 \), and we allow \( C_n \) to take negative values. Note that \( C_n = a_n \) for \( n = 0, 1, 2, 3, \ldots \). Fig. S2.8-2(b) shows the plot of \( C_n \).

(c) \( T_0 = 2\pi \), \( \omega_0 = 1 \), and

\[
g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt
\]

with

\[
a_0 = 0.5 \quad \text{(by inspection of the dc or average)}
\]

\[
a_n = \frac{1}{\pi} \int_{0}^{2\pi} \frac{t}{2\pi} \cos nt \, dt = 0, \quad \text{and} \quad b_n = \frac{1}{\pi} \int_{0}^{2\pi} \frac{t}{2\pi} \sin nt \, dt = -\frac{1}{\pi n}
\]

and

\[
g(t) = 0.5 - \frac{1}{\pi} \left( \sin t + \frac{1}{2} \sin 2t + \frac{1}{3} \sin 3t + \frac{1}{4} \sin 4t + \cdots \right)
\]

\[= 0.5 + \frac{1}{\pi} \left[ \cos \left( t + \frac{\pi}{2} \right) + \frac{1}{2} \cos \left( 2t + \frac{\pi}{2} \right) + \frac{1}{3} \cos \left( 3t + \frac{\pi}{2} \right) + \cdots \right]
\]

The cosine terms vanish because when 0.5 (the dc component) is subtracted from \( g(t) \), the remaining function has odd symmetry. Hence, the Fourier series would contain dc and sine terms only. Figure S2.8-2(c) shows the plots of \( C_n \) and \( \theta_n \).
(d) \( T_0 = \pi, \omega_0 = 2 \) and \( g(t) = \frac{4}{\pi} t \).
\[ a_0 = 0 \quad \text{(by inspection)} \]
\[ a_n = 0 \quad (n > 0) \quad \text{(because of odd symmetry)} \]
\[ b_n = \frac{4}{\pi} \int_0^{\pi/4} \frac{4}{\pi} t \sin 2nt \, dt = \frac{2}{\pi n} \left( \frac{2}{\pi n} \sin \frac{\pi n}{2} - \cos \frac{\pi n}{2} \right) \]
\[ g(t) = \frac{4}{\pi^2} \sin 2t + \frac{1}{\pi} \sin 4t - \frac{4}{9\pi^2} \sin 6t - \frac{1}{2\pi} \sin 8t + \cdots \]
\[ = \frac{4}{\pi^2} \cos \left( 2t - \frac{\pi}{2} \right) + \frac{1}{\pi} \cos \left( 4t - \frac{\pi}{2} \right) + \frac{4}{9\pi^2} \cos \left( 6t + \frac{\pi}{2} \right) + \frac{1}{\pi} \cos \left( 8t + \frac{\pi}{2} \right) + \cdots \]

Figure S2.8-2(d) shows the plots of \( C_n \) and \( \theta_n \).

(e) \( T_0 = 3, \omega_0 = 2\pi/3, \) and
\[ a_0 = \frac{1}{3} \int_0^1 t \, dt = \frac{1}{6} \]
\[ a_n = \frac{2}{3} \int_0^1 t \cos \frac{2n\pi}{3} t \, dt = \frac{3}{2\pi^2 n^2} \left[ \cos \frac{2n\pi}{3} + \frac{2n\pi}{3} \sin \frac{2n\pi}{3} \right] \]
\[ b_n = \frac{2}{3} \int_0^1 t \sin \frac{2n\pi}{3} t \, dt = \frac{3}{2\pi^2 n^2} \left[ \sin \frac{2n\pi}{3} - \frac{2n\pi}{3} \cos \frac{2n\pi}{3} \right] \]

Therefore, \( C_0 = \frac{1}{6} \) and
\[ C_n = \frac{3}{2\pi^2 n^2} \left[ \sqrt{2 + \frac{4\pi^2 n^2}{9} - 2 \cos \frac{2n\pi}{3} - \frac{4n\pi}{3} \sin \frac{2n\pi}{3}} \right] \]
and
\[ \theta_n = \tan^{-1} \left( \frac{\frac{2n\pi}{3} \cos \frac{2n\pi}{3} - \sin \frac{2n\pi}{3}}{\cos \frac{2n\pi}{3} + \frac{2n\pi}{3} \sin \frac{2n\pi}{3} - 1} \right) \]

Figure S2.8-2(e) shows the plots of \( C_n \) and \( \theta_n \).

(f) \( T_0 = 6, \omega_0 = \pi/3, a_0 = 0.5 \) (by inspection of the dc value). There is even symmetry, and \( b_n = 0 \).
\[ a_n = \frac{4}{6} \int_0^3 g(t) \cos \frac{n\pi}{3} t \, dt = \frac{2}{3} \left[ \int_0^1 \cos \frac{n\pi}{3} t \, dt + \int_1^2 (2 - t) \cos \frac{n\pi}{3} t \, dt \right] = \frac{6}{\pi^2 n^2} \left[ \cos \frac{n\pi}{3} - \cos \frac{2n\pi}{3} \right] \]
\[ g(t) = 0.5 + \frac{6}{\pi^2} \left( \cos \frac{\pi}{3} t - \frac{2}{9} \cos \frac{5\pi}{3} t + \frac{1}{25} \cos \frac{7\pi}{3} t + \cdots \right) \]

Observe that even harmonics vanish. This is because if the dc (0.5) is subtracted from \( g(t) \), the resulting function has half-wave symmetry. Figure S2.8-2(f) shows the plot of \( C_n \).
(a) An even function \( g_e(t) \) and an odd function \( g_o(t) \) have the properties that

\[
ge_e(t) = g_e(-t)
\]

and

\[
ge_o(t) = -g_o(-t)
\]

Every signal \( g(t) \) can be expressed as a sum of even and odd components because

\[
g(t) = \frac{1}{2} \left[ g(t) + g(-t) \right] + \frac{1}{2} \left[ g(t) - g(-t) \right]
\]

From the definitions of the Eq. (1), it can be seen that the first component on the right-hand side is an even function, while the second component is odd. This is readily seen from the fact that replacing \( t \) by \(-t\) in the first component yields the same function. The same maneuver in the second component yields the negative of that component.

(b)(i) To find the odd and the even components of \( g(t) = u(t) \), we have

\[
g(t) = g_e(t) + g_o(t)
\]

where from Eq. (1), we write

\[
\begin{align*}
ge_e(t) & = \frac{1}{2} [u(t) + u(-t)] = \frac{1}{2} \\
ge_o(t) & = \frac{1}{2} [u(t) - u(-t)] = \frac{1}{2} \text{sgn}(t)
\end{align*}
\]

The even and odd components of function \( u(t) \) are shown in Fig. S2.8-3(a).

(ii) Similarly, to find the odd and the even components of \( g(t) = e^{-at}u(t) \), we have

\[
g(t) = g_e(t) + g_o(t)
\]

where

\[
\begin{align*}
ge_e(t) & = \frac{1}{2} [e^{-at}u(t) + e^{at}u(-t)] \\
ge_o(t) & = \frac{1}{2} [e^{-at}u(t) - e^{at}u(-t)]
\end{align*}
\]

The even and odd components of function \( e^{-at}u(t) \) are shown in Fig. S2.8-3(b).

(iii) For \( g(t) = e^{jt} \), we have

\[
e^{jt} = g_e(t) + g_o(t)
\]

where

\[
ge_e(t) = \frac{1}{2} [e^{jt} + e^{-jt}] = \cos t
\]

and

\[
ge_o(t) = \frac{1}{2} [e^{jt} - e^{-jt}] = j \sin t
\]

The even and odd components of function \( e^{jt} \) are shown in Fig. S2.8-3(c).
(f) \( \omega_0 = 1 \), therefore, \( T_0 = \frac{2\pi}{\omega_0} = 2\pi \). We can use

\[
g(t) = \begin{cases} 
t, & 0 \leq t \leq \frac{\pi}{2} \\
0, & \frac{\pi}{2} \leq t \leq \pi \\
t + \pi, & \pi \leq t \leq \frac{3\pi}{2} \\
0, & \frac{3\pi}{2} \leq t \leq 2\pi
\end{cases}
\]

Using formula from 2.8-4 (b)(i), we get

\[
a_n = \frac{4}{T_0} \int_0^{T_0/2} g(t) \cos nt \, dt = \frac{2}{\pi} \int_0^{\pi/2} t \cos nt \, dt = 2 \frac{\cos \frac{\pi n}{2} + \frac{\pi n}{2} \sin \frac{\pi n}{2}}{n^2} - \frac{2}{\pi} \frac{1}{n^2} = \begin{cases} 0, & n \to \text{even} \\
\frac{2}{\pi n^2} \left( \frac{\pi n}{2} \sin \frac{\pi n}{2} - 1 \right), & n \to \text{odd}
\end{cases}
\]

Similarly,

\[
b_n = \frac{4}{T_0} \int_0^{T_0/2} g(t) \sin nt \, dt = \frac{2}{\pi} \int_0^{\pi/2} t \sin nt \, dt = 2 \frac{\sin \frac{\pi n}{2} - \frac{\pi n}{2} \cos \frac{\pi n}{2}}{n^2} = \begin{cases} 0, & n \to \text{odd} \\
\frac{2}{\pi n} \sin \left( \frac{\pi n}{2} \right), & n \to \text{even}
\end{cases}
\]

We now have the Fourier series

\[
f(t) = \frac{\pi}{2} + \sum_{n \text{ odd}} \left[ \frac{2}{\pi n^2} \left( \frac{\pi n}{2} \sin \frac{\pi n}{2} - 1 \right) \cos \left( \frac{\pi n}{2} t \right) + \frac{2}{\pi n} \sin \left( \frac{\pi n}{2} \right) \sin \left( \frac{\pi n}{2} t \right) \right]
\]

(2.9-1) See Fig. S.2.9-1.

(a) \( T_0 = 4, \omega_0 = \pi/2 \). Also \( D_0 = 0 \) (by inspection):

\[
D_n = \frac{1}{2\pi} \int_{-1}^{1} e^{-j(\pi n/2)t} \, dt - \int_{-1}^{1} e^{-j(n\pi/2)t} \, dt = \frac{2}{\pi n} \sin \frac{n\pi}{2}, \quad |n| \geq 1
\]

(b) \( T_0 = 10\pi, \omega_0 = 2\pi/10\pi = 1/5 \). Also \( D_0 = 1/5 \) (by inspection):

\[
g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n\pi}{5} t}
\]

where

\[
D_n = \frac{1}{10\pi} \int_{-\pi}^{\pi} e^{-j\frac{n\pi}{5} t} \, dt = \frac{j}{2\pi n} \left( -2j \sin \frac{n\pi}{5} \right) = \frac{1}{\pi n} \sin \left( \frac{n\pi}{5} \right)
\]

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(c) 
\[ g(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jnt} \]
where, by inspection,
\[ D_0 = 0.5 \]
\[ D_n = \frac{1}{2\pi} \int_{0}^{2\pi} \frac{t}{2\pi} e^{-jnt} dt = \frac{j}{2\pi n} \]
so that
\[ |D_n| = \frac{1}{2\pi n} \]
and
\[ \angle D_n = \begin{cases} \frac{\pi}{2}, & n > 0 \\ -\frac{\pi}{2}, & n < 0 \end{cases} \]

(d) \( T_0 = \pi, \omega_0 = 2 \) and \( D_n = 0 \),
\[ g(t) = \sum_{n=-\infty}^{\infty} D_ne^{j2nt} \]
where
\[ D_n = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} e^{-j2nt} dt = \frac{-j}{\pi n} \left( \frac{2}{\pi n} \sin \frac{\pi n}{2} - \cos \frac{\pi n}{2} \right) \]

(e) \( T_0 = 3, \omega_0 = \frac{2\pi}{3} \). Also, \( D_0 = 1/6 \) (by inspection):
\[ g(t) = \sum_{n=0}^{\infty} D_n e^{j2\pi nt/3} \]
where
\[ D_n = \frac{1}{3} \int_{0}^{1} t e^{-j2\pi nt/3} dt = \frac{3}{4\pi^2 n^2} \left[ e^{-j2\pi n/3} \left( \frac{j2\pi n}{3} + 1 \right) - 1 \right] \]
Therefore
\[ |D_n| = \frac{3}{4\pi^2 n^2} \left[ \sqrt{2 + \frac{4\pi^2 n^2}{9} - 2\cos \frac{2\pi n}{3} - \frac{4\pi n}{3} \sin \frac{2\pi n}{3}} \right] \]
and
\[ \angle D_n = \tan^{-1} \left( \frac{\frac{2\pi n}{3} \cos \frac{2\pi n}{3} - \sin \frac{2\pi n}{3}}{\cos \frac{2\pi n}{3} + \frac{2\pi n}{3} \sin \frac{2\pi n}{3} - 1} \right) \]

(f) \( T_0 = 6, \omega_0 = \pi/3 \) \( D_0 = 0.5 \)
\[ g(t) = 0.5 + \sum_{n=-\infty}^{\infty} D_n e^{j\pi nt/3} \]
\[ D_n = \frac{1}{6} \left[ \int_{-2}^{-1} (t+2) e^{-j\pi nt/3} dt + \int_{-1}^{1} e^{-j\pi nt/3} dt + \int_{1}^{2} (-t+2) e^{-j\pi nt/3} dt \right] = \frac{3}{\pi^2 n^2} \left( \cos \frac{n\pi}{3} - \cos \frac{2n\pi}{3} \right) \]
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Fig. S2.9-1