EE6316 Fields and Waves

Homework Assignment #3 Solutions

Due on: Tue, April 11, 2006
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1. Balanis, Chapter 5, page 244, #5.1

The wave impedance of a dielectric medium

$$\eta_1 = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{\eta_2}{2} \approx 189\ (\Omega) \tag{1.1}$$

, where $\eta_2 \approx 377\ (\Omega)$ is the wave impedance of air.

The reflection coefficient

$$\Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.33 \tag{1.2}$$

The transmission coefficient

$$T^b = \frac{2\eta_2}{\eta_2 + \eta_1} = 1.33 \tag{1.3}$$

The incident E-field

$$\mathbf{E}^i = (\hat{\mathbf{a}}_\mathbf{y}) E_0 e^{-j\beta z} = (\hat{\mathbf{a}}_\mathbf{y})(2\times 10^{-3}) e^{-j\beta z}\ \text{(V/m)} \tag{1.4}$$

The reflected E-field

$$\mathbf{E}^r = (\hat{\mathbf{a}}_\mathbf{y}) \Gamma^b E_0 e^{-j\beta z} = (\hat{\mathbf{a}}_\mathbf{y})(6.6\times 10^{-4}) e^{-j\beta z}\ \text{(V/m)} \tag{1.5}$$

The transmitted E-field

$$\mathbf{E}^t = (\hat{\mathbf{a}}_\mathbf{y}) T^b E_0 e^{-j\beta z} = (\hat{\mathbf{a}}_\mathbf{y})(2.66\times 10^{-3}) e^{-j\beta z}\ \text{(V/m)} \tag{1.6}$$

, where $E_0 = 2\times 10^{-3}$ (V/m)

Since in general

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E} \tag{1.7}$$

, where $\hat{\mathbf{k}}$ is the unit vector in the direction of propagation, the H-fields come out to be

$$\mathbf{H}^i = \frac{1}{\eta_1}\hat{\mathbf{a}}_\mathbf{z} \times \mathbf{E}^i = (-\hat{\mathbf{a}}_\mathbf{x})\frac{1}{\eta_1}\left|\mathbf{E}^i\right| = (-\hat{\mathbf{a}}_\mathbf{x})(1.06\times 10^{-5}) e^{-j\beta z}\ \text{(A/m)} \tag{1.8}$$

$$\mathbf{H}^r = \frac{1}{\eta_1}(-\hat{\mathbf{a}}_\mathbf{z}) \times \mathbf{E}^r = \frac{1}{\eta_1}(\hat{\mathbf{a}}_\mathbf{x})\left|\mathbf{E}^r\right| = (\hat{\mathbf{a}}_\mathbf{x})(3.49\times 10^{-6}) e^{-j\beta z}\ \text{(A/m)} \tag{1.9}$$

$$\mathbf{H}^t = \frac{1}{\eta_2}\hat{\mathbf{a}}_\mathbf{z} \times \mathbf{E}^t = \frac{1}{\eta_2}(-\hat{\mathbf{a}}_\mathbf{x})\left|\mathbf{E}^t\right| = (-\hat{\mathbf{a}}_\mathbf{x})\left(7.06\times10^{-6}\right)e^{-j\beta z} \text{ (A/m)} \tag{1.10}$$

And finally, the power densities

$$\mathbf{S}^i = (\hat{\mathbf{a}}_\mathbf{z})\frac{|E_0|^2}{2\eta_1} = (\hat{\mathbf{a}}_\mathbf{z})\left(1.06\times10^{-8}\right) \text{ (W/m}^2\text{)} \tag{1.11}$$

$$\mathbf{S}^r = (-\hat{\mathbf{a}}_\mathbf{z})|\Gamma_b|^2\mathbf{S}^i = (-\hat{\mathbf{a}}_\mathbf{z})\left(1.15\times10^{-9}\right) \text{ (W/m}^2\text{)} \tag{1.12}$$

$$\mathbf{S}^t = (\hat{\mathbf{a}}_\mathbf{z})\left(1-|\Gamma_b|^2\right)\mathbf{S}^i = (\hat{\mathbf{a}}_\mathbf{z})\left(9.45\times10^{-9}\right) \text{ (W/m}^2\text{)} \tag{1.13}$$

2. Balanis, Chapter 5, page 244, #5.2

The wave impedance of water

$$\eta_2 = \sqrt{\frac{\mu_0}{81\varepsilon_0}} = \frac{\eta_1}{9} \tag{1.14}$$

, where η_1 is the wave impedance of air.

The reflection coefficient

$$\Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.8 \tag{1.15}$$

The transmission coefficient

$$T^b = \frac{2\eta_2}{\eta_2 + \eta_1} = 0.2 \tag{1.16}$$

The ratio of the reflected power density to the incident power density

$$\frac{S^r}{S^i} = \left|\Gamma^b\right|^2 = 64\% \tag{1.17}$$

The ratio of the transmitted power density to the incident power density

$$\frac{S^t}{S^i} = \left|T^b\right|^2\frac{\eta_1}{\eta_2} = (0.04)(9) = 36\% \tag{1.18}$$

3. Balanis, Chapter 5, page 248, #5.17

Interface	θ_B
(a) water to air	6.34°
(b) air to water	83.6°
(c) glass to air	18.4°

4. Balanis, Chapter 5, page 248, #5.19

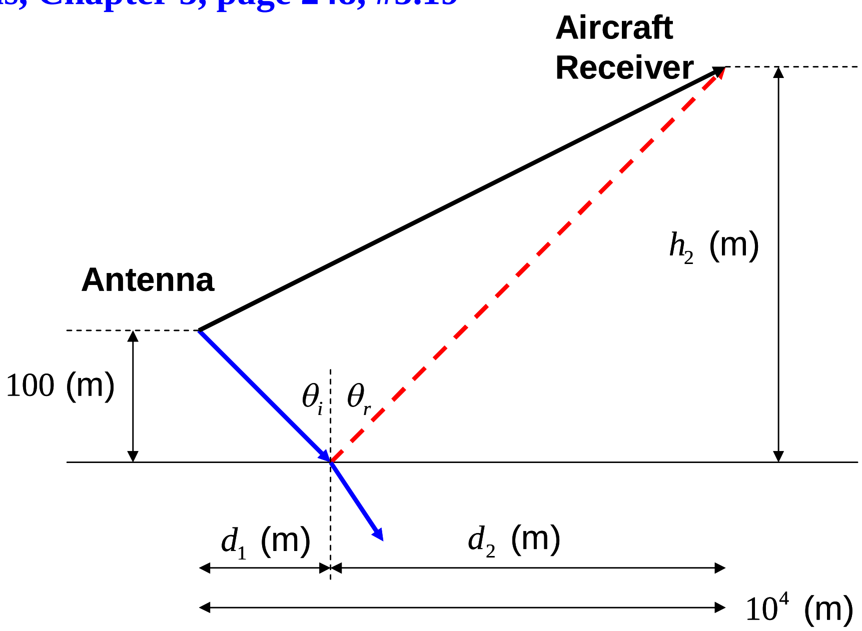

Figure 1. Total transmission at the air-water interface.

In Figure 1 is shown the total transmission occurring at the air-water interface, where the reflected wave has no parallel polarized component included. This happens, according to the results of Example 5-3 on page 195 of the book, when

$$\theta_i = \theta_{Baw} = 83.66° \tag{1.19}$$

So,

$$\tan(90 - \theta_{Baw}) = \frac{10}{d_1} \tag{1.20}$$

$$\tan(90 - \theta_{Baw}) = \frac{h_2}{d_2} \tag{1.21}$$

$$d_1 + d_2 = 10^4 \tag{1.22}$$

Solving (1.20) through (1.22) for the unknowns

$$\begin{aligned} d_1 &= 9.0 \times 10^1 \text{(m)} \\ d_2 &= 9.9 \times 10^3 \text{(m)} \\ h_2 &= 1.1 \times 10^3 \text{(m)} \end{aligned} \tag{1.23}$$

5. Slater, page 166, #4

The total power radiated, according to (4.2) on page 159 of the book, is

$$P = \frac{\mu_0 \sqrt{\varepsilon_0 \mu_0}\, \omega^4 M^2}{12\pi} \tag{1.24}$$

Let this be rewritten as

$$P = \frac{1}{2} R |I|^2 \tag{1.25}$$

, where R is the equivalent resistance, so

$$R = \frac{\mu_0 \sqrt{\varepsilon_0 \mu_0}\, \omega^4 M^2}{6\pi} \frac{1}{|I|^2} \tag{1.26}$$

The dipole moment

$$M = qL \tag{1.27}$$

The current

$$I = \frac{dq}{dt} = j\omega q \tag{1.28}$$

Plug (1.27) and (1.28) into (1.26), and then

$$R = \frac{\mu_0 \sqrt{\varepsilon_0 \mu_0}\, \omega^2 L^2}{6\pi} \tag{1.29}$$

The wavelength is

$$\lambda = \frac{c}{f} = \frac{2\pi}{\omega} \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{1.30}$$

, so the frequency comes out to be

$$\omega = \frac{2\pi}{\lambda} \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{1.31}$$

Plug this into (1.29)

$$R = \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{L^2}{\lambda^2} \tag{1.32}$$

6. **Balanis, Chapter 6, page 306, #6.5**
7. **Balanis, Chapter 6, page 306, #6.18**
8. **Balanis, Chapter 6, page 307, #6.20**