

SOL 1.1 :- The differential form of continuity eqn is

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

we know from Maxwells eqns :

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking divergence on both sides,

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} + \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

∴ scalar triple product with 2 same vectors

$$\Rightarrow \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho$$

Sol 1.2:-

$$\nabla_{\rightarrow} \times \underline{E}_{\rightarrow} = - \frac{\partial \underline{B}_{\rightarrow}}{\partial t}$$

$$\Rightarrow \iint \nabla_{\rightarrow} \times \underline{E}_{\rightarrow} \cdot \hat{n} \, ds = \iint - \frac{\partial \underline{B}_{\rightarrow}}{\partial t} \cdot d\underline{s}_{\rightarrow}$$

$$\Rightarrow \oint \underline{E}_{\rightarrow} \cdot d\underline{l}_{\rightarrow} = - \frac{\partial}{\partial t} \iint \underline{B}_{\rightarrow} \cdot d\underline{s}_{\rightarrow}$$

$$\nabla_{\rightarrow} \times \underline{H}_{\rightarrow} = \underline{J}_{\rightarrow} + \frac{\partial \underline{D}_{\rightarrow}}{\partial t}$$

$$\Rightarrow \iint \nabla_{\rightarrow} \times \underline{H}_{\rightarrow} \cdot d\underline{s}_{\rightarrow} = \iint \underline{J}_{\rightarrow} \cdot d\underline{s}_{\rightarrow} + \iint \frac{\partial \underline{D}_{\rightarrow}}{\partial t} \cdot d\underline{s}_{\rightarrow}$$

$$\Rightarrow \oint \underline{H}_{\rightarrow} \cdot d\underline{l}_{\rightarrow} = \iint \underline{J}_{\rightarrow} \cdot d\underline{s}_{\rightarrow} + \frac{d}{dt} \iint \underline{D}_{\rightarrow} \cdot d\underline{s}_{\rightarrow}$$

$$\nabla_{\rightarrow} \cdot \underline{D}_{\rightarrow} = \rho$$

$$\iiint \nabla_{\rightarrow} \cdot \underline{D}_{\rightarrow} \, dV = \iiint \rho \, dV \Rightarrow \oint \underline{D}_{\rightarrow} \cdot d\underline{s}_{\rightarrow} = q_{\text{tot}}$$

$$\nabla_{\rightarrow} \cdot \underline{B}_{\rightarrow} = 0$$

$$\Rightarrow \iiint \nabla_{\rightarrow} \cdot \underline{B}_{\rightarrow} \, dV = 0$$

$$\Rightarrow \oint \underline{B}_{\rightarrow} \cdot d\underline{s}_{\rightarrow} = 0$$

Sol 1.3 :-

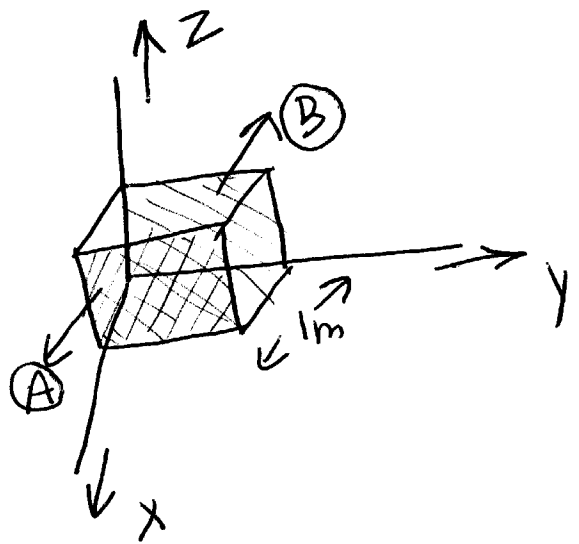
$$(a) \vec{D} = \hat{a}_x (3+x)$$

we know :

$$\oint_S \vec{D} \cdot d\vec{s} = q$$

Let's calculate  $\vec{D} \cdot d\vec{s}$  on each face.

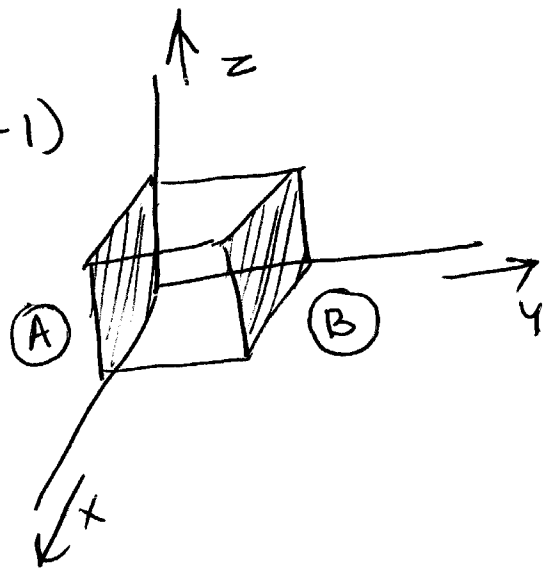
$$\begin{aligned} \oint_S \vec{D} \cdot d\vec{s} &= (3+1) \cdot 1 + (3+0) \cdot (-1) \\ &= 4 - 3 = 1 \text{ C} \end{aligned}$$



Other contributions are zero.

$$(b) \vec{D} = \hat{a}_y (4+y^2)$$

$$\begin{aligned} \oint_S \vec{D} \cdot d\vec{s} &= (4+1)(1) + (4+0)(-1) \\ &= 5 - 4 = 1 \text{ C} \end{aligned}$$



sol 1.5 :-

Source free  $\Rightarrow \nabla \cdot \vec{D} = 0$ 

Given:  $\vec{E} = [\hat{a}_x A(x+y) + \hat{a}_y B(x-y)] \cos \omega t$

$$\Rightarrow \nabla \cdot \vec{D} = 0$$

$$\Rightarrow A \frac{\partial}{\partial x} (x+y) + B \frac{\partial}{\partial y} (x-y) = 0$$

$$\Rightarrow A - B = 0$$

$$\Rightarrow A = B$$

sol 1.8 :-

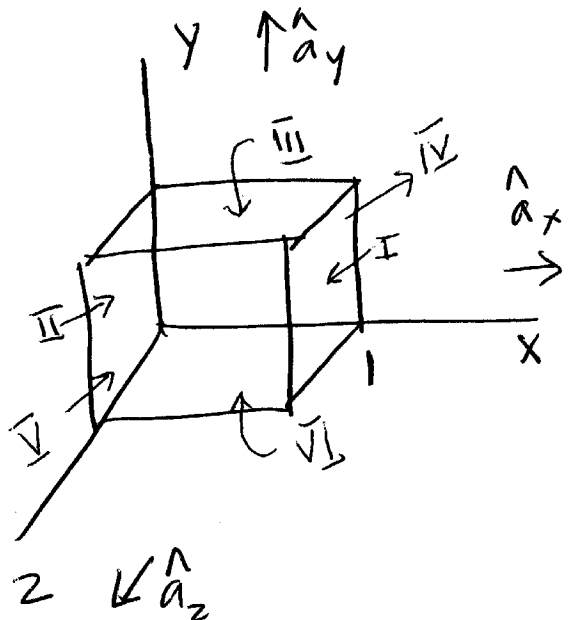
$$\vec{J}_d = \hat{a}_x yz + \hat{a}_y y^2 + \hat{a}_z xyz$$

Calculate  $\oiint \vec{J}_d \cdot d\vec{s}$  on

each face.

$$I: \int_0^1 \int_0^1 yz \, dy \, dz$$

$$= \frac{yz}{2} \Big|_0^1 \frac{z^2}{2} \Big|_0^1 = \frac{1}{4}$$



(5)

$$\text{II} : - \int_0^1 \int_0^1 yz \, dy \, dz = -\frac{1}{4}$$

$$\text{III} : \int_0^1 \int_0^1 y^2 \, dx \, dz = 1$$

$$\text{IV} : \int_0^1 \int_0^1 xy(0) \, dx \, dz = 0$$

$$\begin{aligned} \text{V} : \int_0^1 \int_0^1 xyz \, dx \, dy \\ = \frac{x^2}{2} \Big|_0^1 \frac{y^2}{2} \Big|_0^1 = \frac{1}{4} \end{aligned}$$

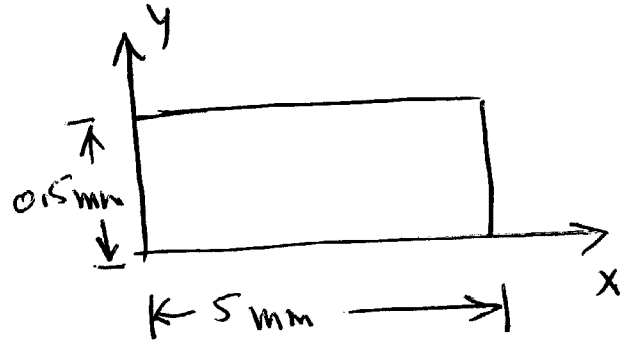
$$\text{VI} : \int_0^1 \int_0^1 y^2 \, dx \, dz = \int_0^1 \int_0^1 (0) \, dx \, dz = 0$$

$$\begin{aligned} \therefore \text{net flux of } \vec{J}_d &= \oiint \vec{J}_d \cdot \vec{ds} = \frac{1}{4} - \frac{1}{4} + 1 + 0 + \frac{1}{4} + 0 \\ &= \frac{5}{4} \end{aligned}$$

(6)

Sol 1.13:-  $J = \hat{a}_z 10^4 \cos(2\pi 10^9 t)$

current decays by  $e^{-10^6 y}$  exponentially.



$$\therefore \iint_S \vec{J} \cdot d\vec{S}$$

$$= \int_0^{0.125} \int_0^5 10^4 e^{-10^6 y} \cos(2\pi 10^9 t) dx dy$$

$$= 10^4 \cos(2\pi 10^9 t) \times 5 \times \int_0^{0.125} e^{-10^6 y} dy$$

$$= 10^4 \cos(2\pi 10^9 t) 5 \left[ \frac{e^{-10^6 \times 0.125} - 1}{-10^6} \right]$$

$$= -\frac{5}{100} \cos(2\pi 10^9 t) \left[ e^{-2.5 \times 10^5} - 1 \right]$$

$$= \frac{\cos 2\pi 10^9 t}{20}$$

- Total Current =  $2 \times \cos 2\pi 10^9 t = \cos 2\pi 10^9 t$

Sol 1.18:-

$$\underline{E}(x, y, z, t) = \text{Re} \left[ \underline{E}_{\vec{r}}(x, y, z) e^{j\omega t} \right]$$

$$\underline{E}_{\vec{r}}(x, y, z, t) = \frac{1}{2} \left[ \underline{E}_{\vec{r}} e^{j\omega t} + (\underline{E}_{\vec{r}} e^{j\omega t})^* \right]$$

$$= \frac{1}{2} \left[ \text{Re} \left[ \underline{E}_{\vec{r}} e^{j\omega t} \right] + \text{Im} \left[ \underline{E}_{\vec{r}} e^{j\omega t} \right] - \text{Im} \left[ \underline{E}_{\vec{r}} e^{j\omega t} \right] + \text{Re} \left[ \underline{E}_{\vec{r}} e^{j\omega t} \right] \right]$$

$$= \text{Re} \left[ \underline{E}_{\vec{r}}(x, y, z) e^{j\omega t} \right]$$

Similarly for  $\underline{B}_{\vec{r}}$ .

Sol 1.29:- Given :  $\underline{E}^i = \hat{a}_x E_0 e^{-j\beta_0 z}$

$$\underline{E}^r = \hat{a}_x \Gamma_0 E_0 e^{+j\beta_0 z}$$

$$\underline{E}^t = \hat{a}_x \tau_0 E_0 e^{-j\beta_0 z}$$

$$E_0 = \text{constant} \quad \beta_0 = \omega \sqrt{\mu_0 \epsilon_0}$$

$$\beta = \omega \sqrt{\mu \epsilon}$$

8

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \eta_2 = \sqrt{\frac{\mu_0}{\epsilon}}$$

$$\Gamma_0 = \frac{\sqrt{\frac{\mu_0}{\epsilon}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{\epsilon_0}} + \sqrt{\frac{\mu_0}{\epsilon}}}$$

; Like in RF

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$= \frac{\sqrt{\epsilon_0} - \sqrt{\epsilon}}{\sqrt{\epsilon} + \sqrt{\epsilon_0}}$$

$$\Pi_0 = \frac{2 \sqrt{\frac{\mu_0}{\epsilon}}}{\sqrt{\frac{\mu_0}{\epsilon}} + \sqrt{\frac{\mu_0}{\epsilon_0}}}$$

$$; \Pi = \frac{2Z_2}{Z_1 + Z_2}$$

$$= \frac{2}{\sqrt{\epsilon}} \times \frac{\sqrt{\epsilon} \sqrt{\epsilon_0}}{\sqrt{\epsilon_0} + \sqrt{\epsilon}}$$

$$= \frac{2 \sqrt{\epsilon_0}}{\sqrt{\epsilon_0} + \sqrt{\epsilon}}$$