Sol 3.1 :- Follow Balanis and get to $= n s$ 3.8 and 3.11

Replace the operators $\frac{\partial}{\partial t}$ and $\frac{\partial^2}{\partial t^2}$ by $j\omega$ and $-\omega^2$ in these to get the result. This is done because the time harmonic fields are of the form $e^{j\omega t}$.

Sol 3.2 :- $$\frac{\partial^2 f}{\partial x^2} = -\beta_x^2 f \quad (1)$$

plugging in $f_1(x) = A_1 e^{-j\beta_x x} + B_1 e^{+j\beta_x x}$ in above:

$$\frac{\partial^2}{\partial x^2}\left\{A_1 e^{-j\beta_x x} + B_1 e^{+j\beta_x x}\right\} = -\beta_x^2 \left[A_1 e^{-j\beta_x x} + B_1 e^{+j\beta_x x}\right]$$

$$\Rightarrow -\beta_x^2 \left\{A_1 e^{-j\beta_x x} + B_1 e^{+j\beta_x x}\right\} = -\beta_x^2 \left[A_1 e^{-j\beta_x x} + B_1 e^{+j\beta_x x}\right]$$

Hence $f_1(x)$ is a solution of (1).

Lets try

$f_2(x) = C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)$ in 1

$$\Rightarrow \frac{\partial^2}{\partial x^2}\{C_1 \cos \beta_x x + D_1 \sin \beta_x x\} = -\beta_x^2\{C_1 \cos \beta_x x + D_1 \sin \beta_x x\}$$

$$\Rightarrow -\beta_x^2\{C_1 \cos \beta_x x + D_1 \sin \beta_x x\} = -\beta_x^2\{C_1 \cos \beta_x x + D_1 \sin \beta_x x\}$$

Hence $f_2(x)$ is also a solution.

Sol 3.3 :- The second complex exponential $= S\varepsilon = B_3 e^{j\beta_z z}$

all variations are time harmonic.

$$\Rightarrow (S\varepsilon)_{x,t} = \mathrm{Re}\{B_3 e^{j(\beta_z z + \omega t)}\}$$

$$= B_3 \cos(\omega t + \beta_z z)$$

Now $\omega t + \beta_z z_p = \text{Constant}$

$$\Rightarrow \frac{d}{dt}[\omega t + \beta_z z_p] = 0$$

$$\Rightarrow \omega + \beta_z \frac{dz_p}{dt} = 0 \Rightarrow \boxed{V_p = -\frac{\omega}{\beta_z}}$$

Sol 3.4 :-
$$\nabla^2 E_x - \gamma^2 E_x = 0 \qquad E_x = E_x(x,y,z)$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

Using seperation of variables ; $E_x = XYZ$

$$\Rightarrow YZ\frac{\partial^2 X}{\partial x^2} + XY\frac{\partial^2 Z}{\partial z^2} + XZ\frac{\partial^2 Y}{\partial y^2} - \gamma^2 E_x = 0$$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \gamma^2 - \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} - \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}$$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \gamma_x^2$$

as the right hand side of above equation dosent depend on X.

$$\Rightarrow \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = \gamma^2 - \gamma_x^2 - \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \gamma_y^2$$

$$\Rightarrow \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \gamma^2 - \gamma_x^2 - \gamma_y^2 = \gamma_z^2$$

Hence $\gamma_x^2 + \gamma_y^2 + \gamma_z^2 = \gamma^2$

Now $\frac{1}{X}\frac{d^2X}{dx^2} = \gamma_x^2$

$$\Rightarrow \frac{d^2X}{dx^2} - \gamma_x^2 X = 0$$

Has solutions of the form :-

$$X = A_1 e^{-\gamma_x x} + B_1 e^{\gamma_x x}$$

or $X = C_1 \cosh(\gamma_x x) + D_1 \sinh(\gamma_x x)$

Y, Z have similar solutions as well.