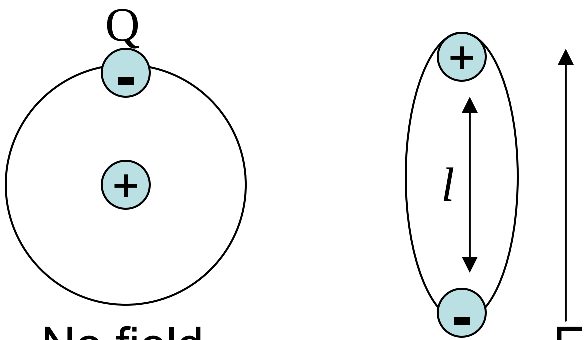
Electrical Properties of Matter

Fields and Waves
EE 6316
Spring 2008
1-30-2008

Topics:

- Review of Dipole, Polarization ,Susceptibility etc in isotropic medium
- Classical Harmonic Oscillator Model
  - Abraham Lorentz Equation
  - Damping
  - Dispersion plots
- Application of the model : some examples
- Plasma
- Dielectric behavior in anisotropic medium
  - Permittivity tensor
  - Example: Modulator

Review



$$p_i = Ql$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

where

$$\chi_e = \frac{1}{\varepsilon_0} \frac{\vec{P}}{\vec{E}} \qquad \vec{P} = \lim_{\Delta V \to 0} \sum_i \frac{p_i}{\Delta V}$$

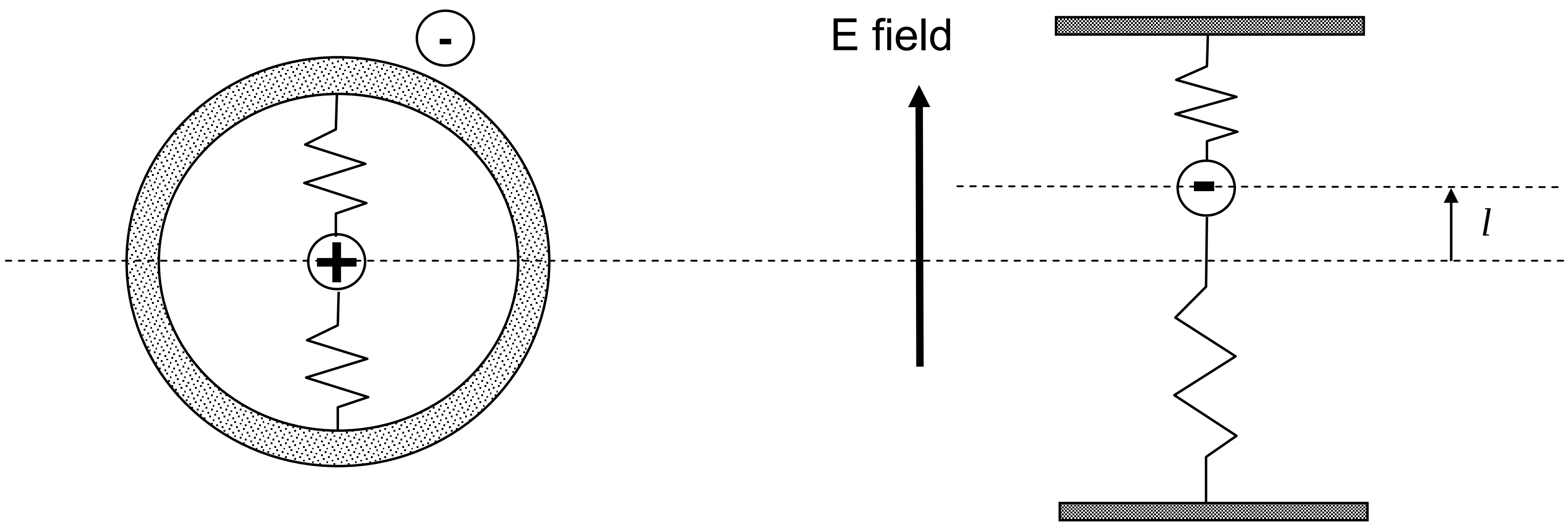
In terms of molecular polarizability,α

$$\vec{P} = N\alpha \vec{E}_{local}$$

α has contribution due to electronic, ionic and permanent dipole polarization

Classical Electron Oscillator (CEO)Model

- electron cloud is modeled as a spring mass system, with attractive electric force between nucleus and electron cloud as the spring providing the restoring force



Mechanical Equivalent model of CEO

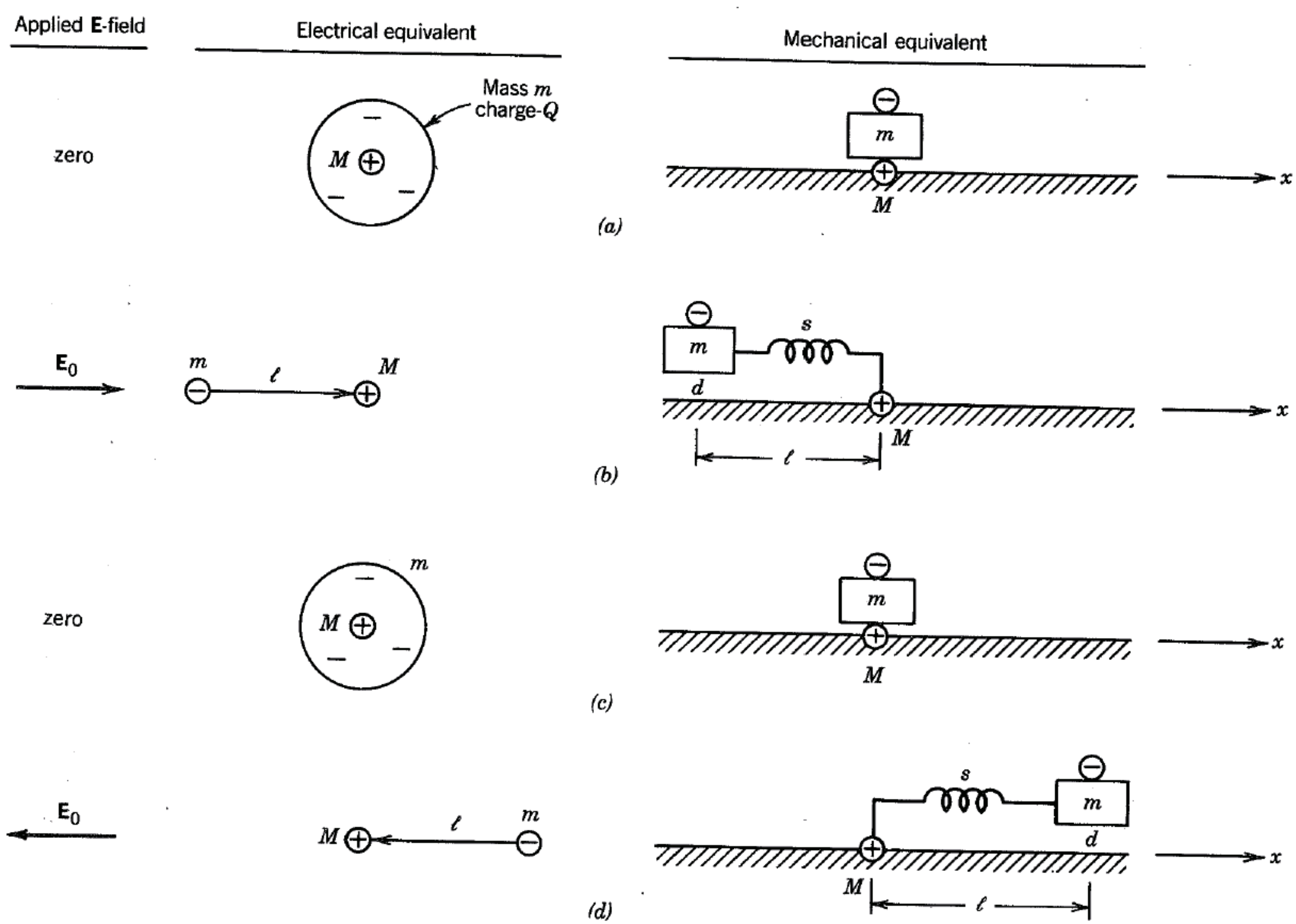


FIGURE 2-16 Electrical and mechanical equivalents of a typical atom in the absence of and under an applied electric field.

Mathematical development of CEO Model

In the presence of an applied electric field: Abraham Lorentz Equation

$$m\frac{d^2 l}{dt^2} + d\frac{dl}{dt} + sl = Q\vec{E}(t) = Q\vec{E}_0 e^{j\omega t}$$

d: Damping coefficient

s: spring constant

$$\Rightarrow \frac{d^2 l}{dt^2} + 2\alpha\frac{dl}{dt} + \omega_0^2 l = \frac{Q}{m}\vec{E}_0 e^{j\omega t}$$

where

$$\alpha = \frac{d}{2m}$$

$$\omega_0 = \sqrt{\frac{s}{m}}$$:Resonant frequency

$$l_{ss} = l_0 e^{j\omega t} = \frac{\frac{Q}{m}\vec{E}_0 e^{j\omega t}}{\left(\omega_0^2 - \omega^2\right) + j\omega\left(\frac{d}{m}\right)}$$

Steady state solution

CEO Model :Damping Conditions

Condition	Classification of Solution
1) $\alpha > \omega_0$	overdamped
2) $\alpha = \omega_0$	Critically damped
3) $\alpha < \omega_0$	Underdamped

Frequency dependent dielectric response: Dispersion

Polarization: $$\vec{P} = \frac{N\left(\frac{Q_2}{m}\right)\vec{E}}{\left(\omega_0^2 - \omega^2\right) + j\omega\left(\frac{d}{m}\right)}$$

Permittivity: $$\varepsilon = \varepsilon_0 + \frac{\vec{P}}{\vec{E}} = \varepsilon_0 + \frac{N\left(\frac{Q_2}{m}\right)}{\left(\omega_0^2 - \omega^2\right) + j\omega\left(\frac{d}{m}\right)} = \mathrm{Re}(\varepsilon) - j\,\mathrm{Im}(\varepsilon)$$

Dispersion Relation of Permittivity

Similar relationship for relative permittivity hence refractive index can be derived. The real and imaginary parts of refractive index are related to each other through Kramers-Kronig relationship.

Dispersion Plots

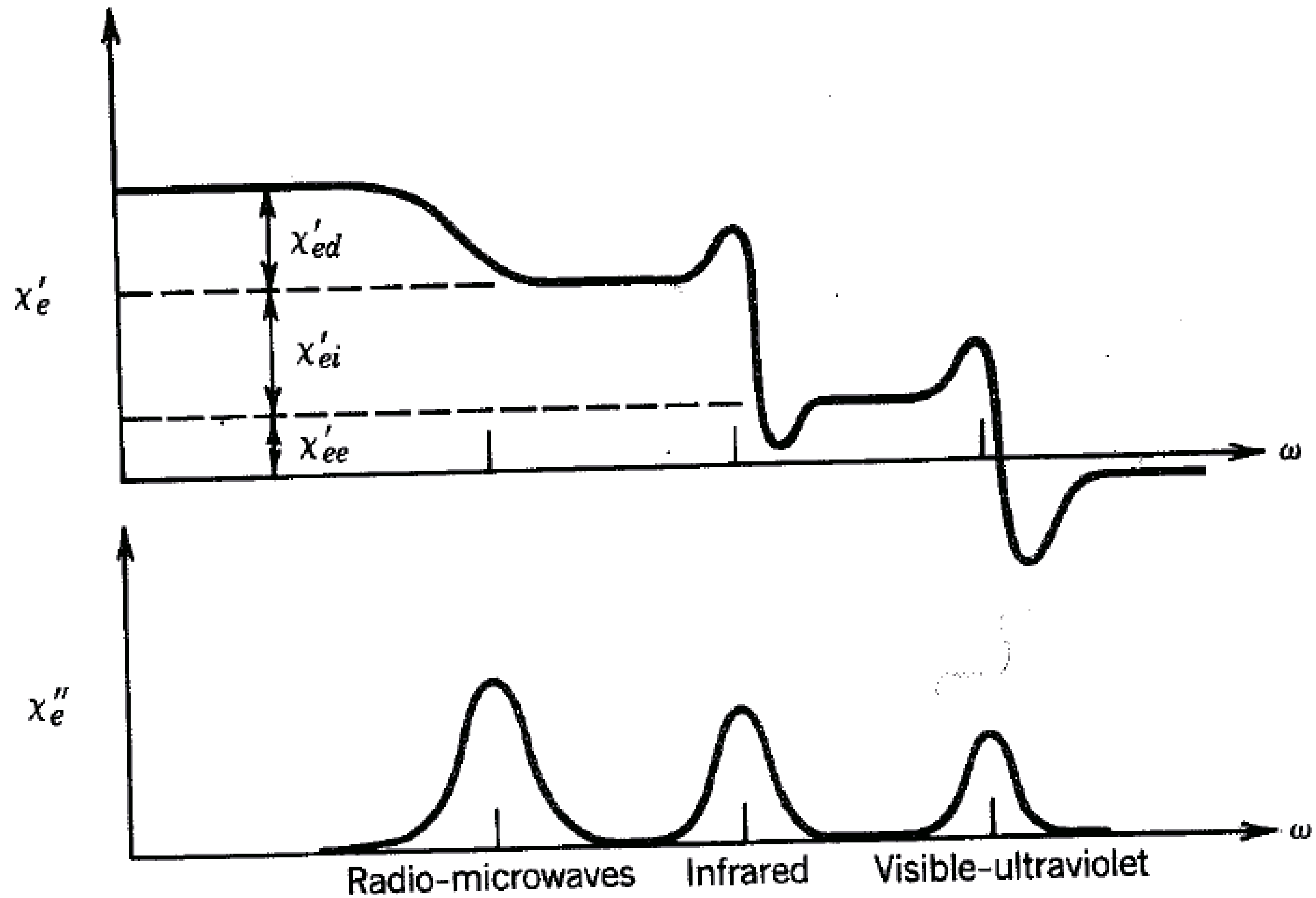


FIGURE 2-18 Electric susceptibility (real and imaginary) variations as a function of frequency for a typical dielectric.

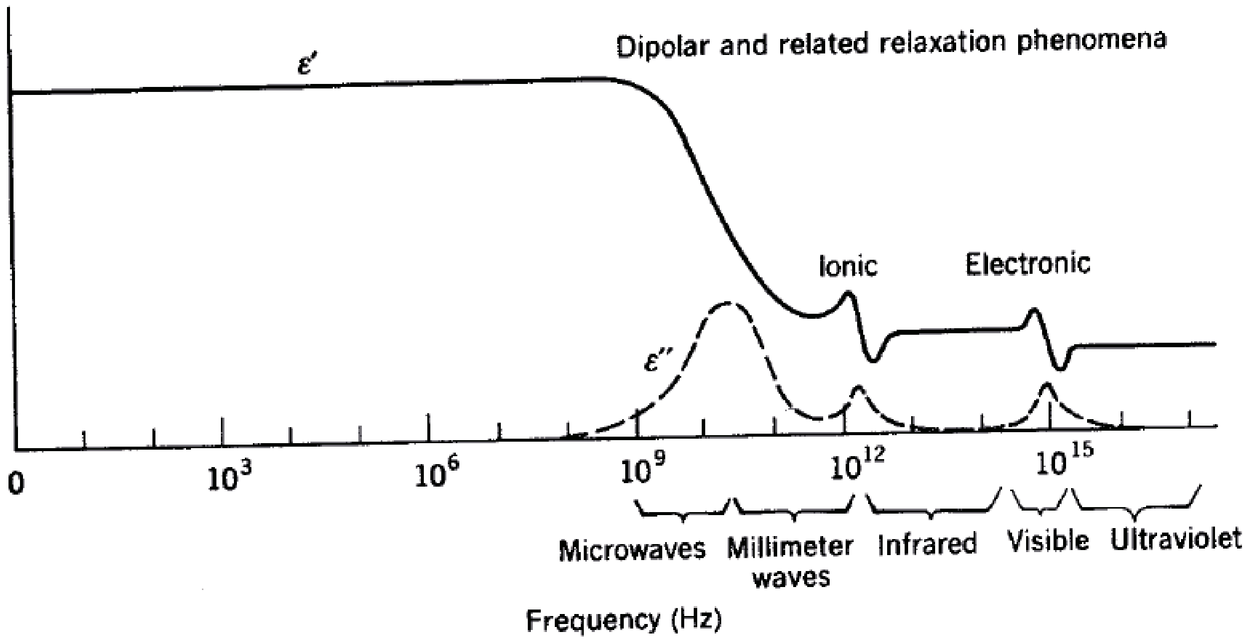


FIG. 13.2a Frequency response of permittivity and loss factor for a hypothetical dielectric showing various contributing phenomena.

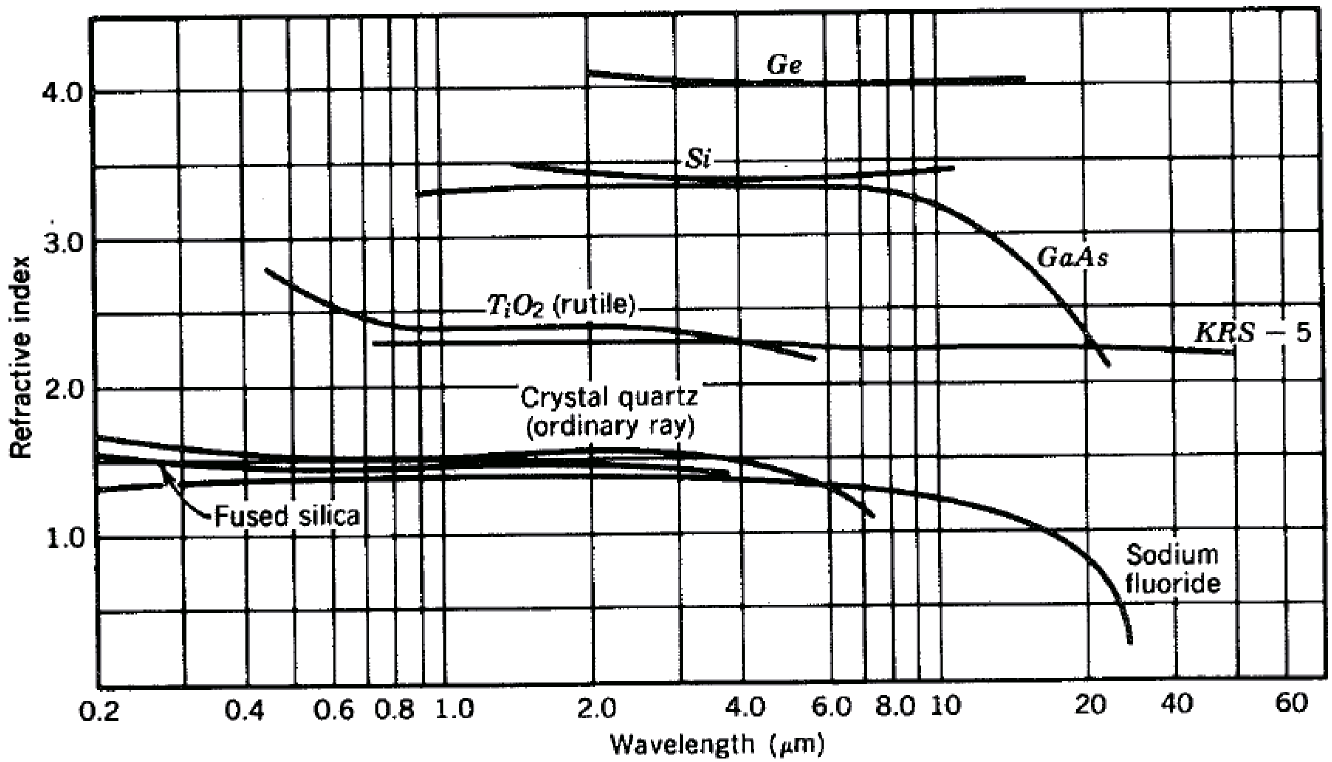


FIG. 13.2*b* Refractive index versus wavelength for several materials with useful values of optical and infrared transparency. Data from *American Institute of Physics Handbook.*[6]

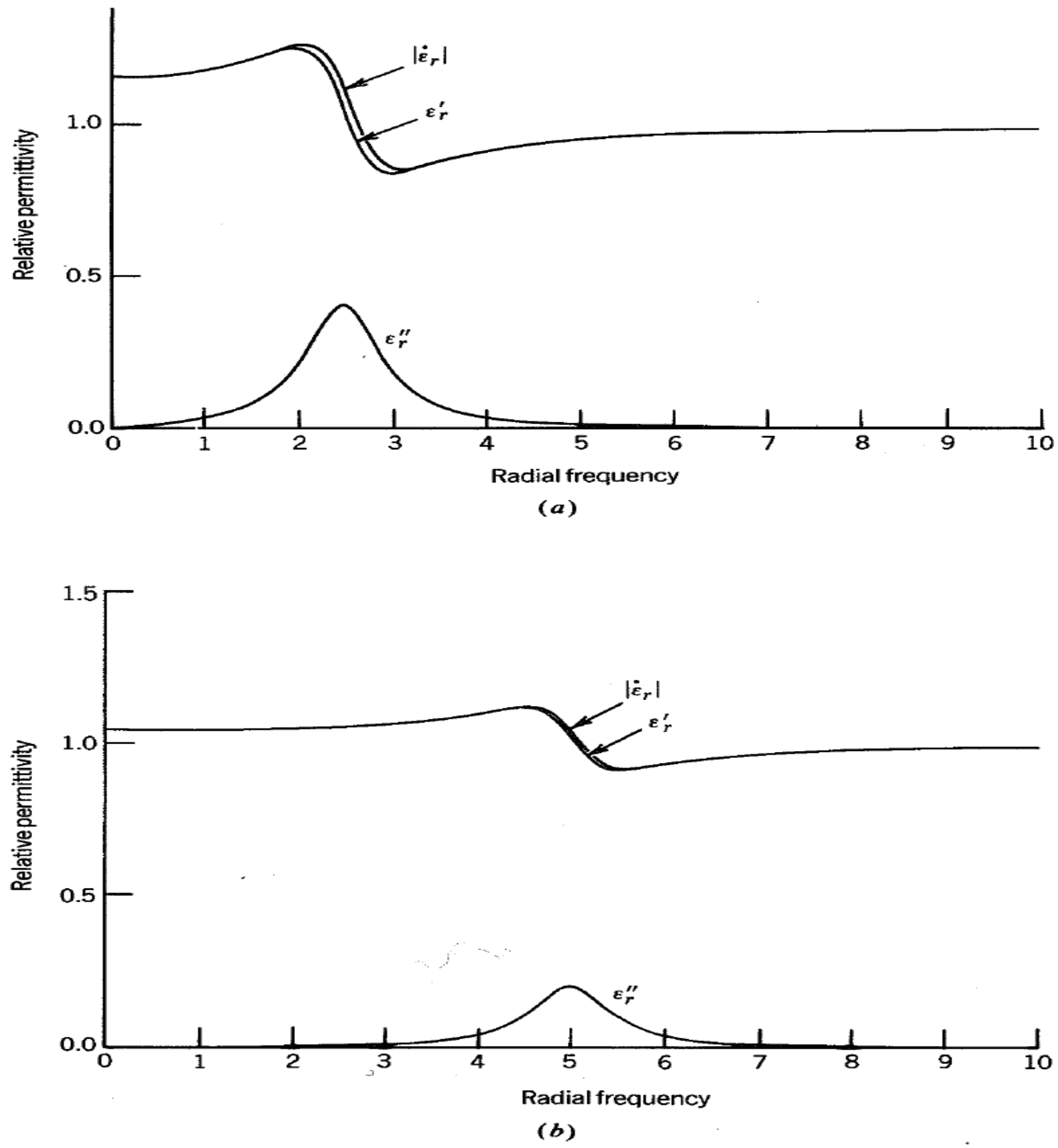


FIGURE 2-19 Typical frequency variations of real and imaginary parts of relative permittivity of dielectrics. (*a*) $N_e Q^2/\varepsilon_0 m = 1$, $d/m = 1$, $\alpha/\omega_0 = 1/5$, $\omega_0 = 2.5$. (*b*) $N_e Q^2/\varepsilon_0 m = 1$, $d/m = 1$, $\alpha/\omega_0 = 1/10$, $\omega_0 = 5$.

Application of CEO Model: Example

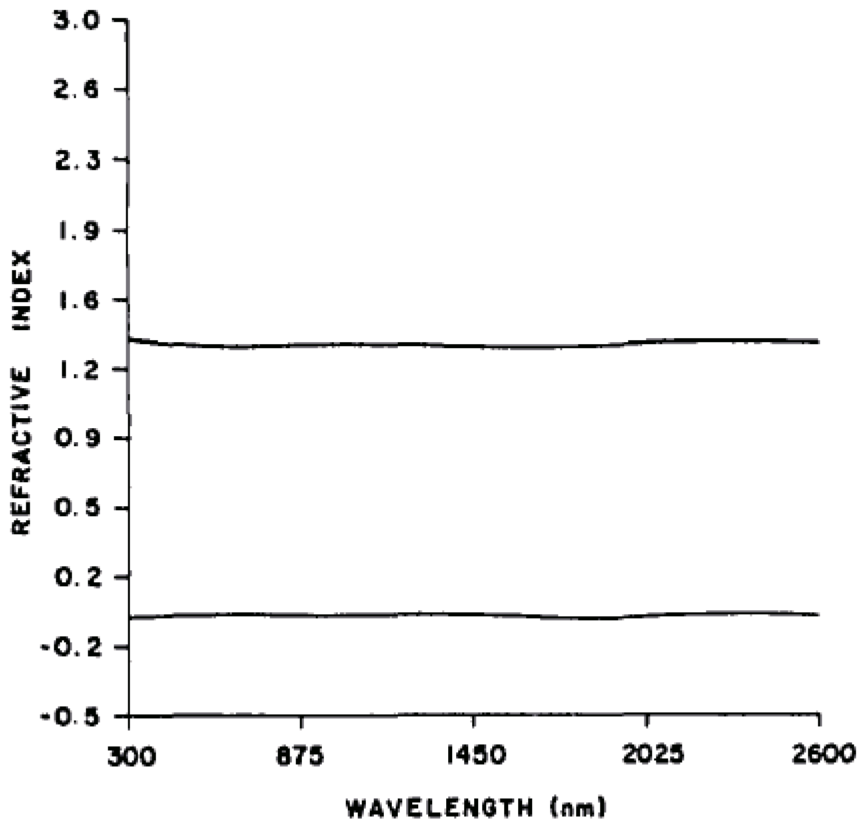


Fig. 5. Real and imaginary parts of the refractive index for MgF_2 calculated from the best-fitted parameters from two contributing oscillators.

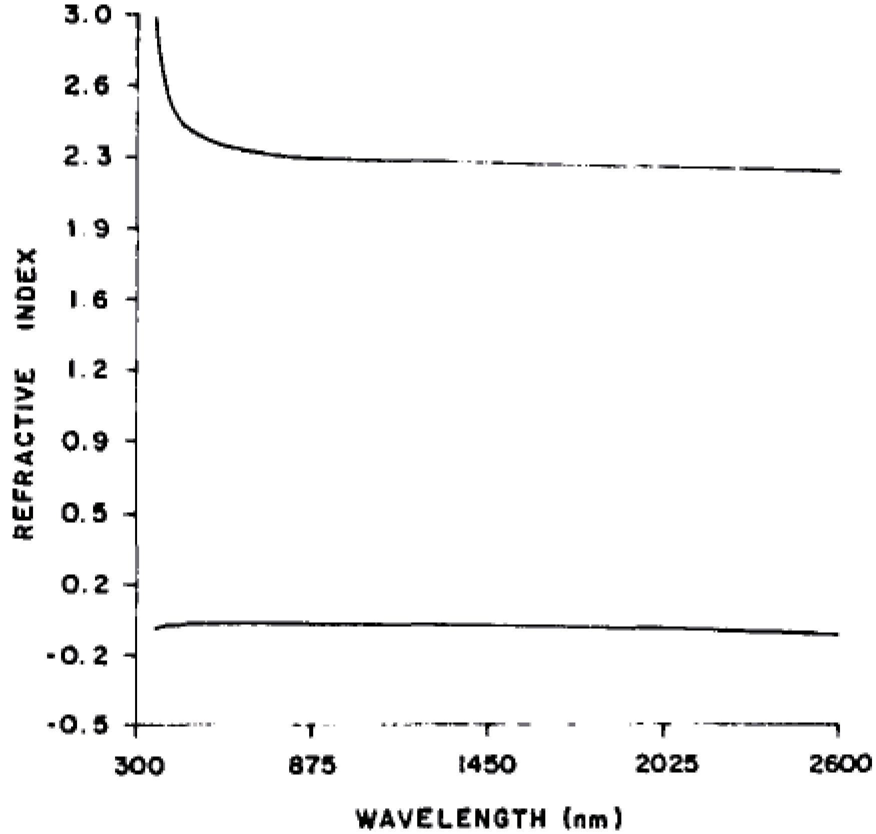


Fig. 6. Complex refractive index for ZnS. The dispersion is higher for the UV region. Two oscillators contribute to this calculation.

Ref: *APPLIED OPTICS* / Vol. 27, No. 12 /pp.2549-2553/1998

Further Application of CEO Model

- This model can be extended to model optical processes in semiconductors
  - Spontaneous and Stimulated emission
  - Rabi Oscillation
  - Collision Broadening
  - Radiative lifetimes etc

Plasma

- Plasma is a sea of free electrons in a background of positive ions of same density.
- Due to existence of free electrons, they are very conductive.
- Plasma response to electrical field is very strong too

Dynamics of Plasma

- Motion of free electrons is governed by collision frequency *f*

$$m\frac{d\vec{v}}{dt} = -e\vec{E} - m\vec{v}f$$

For sinusoidal variation:

$$\Rightarrow \vec{v} = -\frac{e\vec{E}}{m(f + j\omega)}$$

Convection Current:

$$\vec{J} = -ne\vec{v} = \frac{ne^2\vec{E}}{m(f + j\omega)}$$

Inserting in Curl Eqn:

$$\nabla X\vec{H} = j\omega\varepsilon_0\vec{E} + \vec{J} = j\omega\varepsilon_0\vec{E} + \frac{ne^2\vec{E}}{m(f + j\omega)}$$

$$= j\omega\left[\left(\varepsilon_0 - \frac{ne^2}{m(f^2 + \omega^2)}\right) - j\frac{ne^2 f}{m\omega(f^2 + \omega^2)}\right]\vec{E}$$

Real Part

Imaginary part=0 as f $\rightarrow$ 0

Plasma Frequency

$$\varepsilon = \varepsilon_0 - \frac{ne^2}{m\omega^2} = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$where$$

Plasma frequency $$\omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m}}$$

Permittivity is negative for frequencies below plasma frequency. Physically this means wave is reflected off of plasma and attenuated inside.

Anisotropic Media: Dielectric tensors

$$\left[\vec{D}\right] = \left[\varepsilon\right]\left[\vec{E}\right]$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$D_x = \varepsilon_{xx} E_x + \varepsilon_{xy} E_y + \varepsilon_{xz} E_z$$

$$D_y = \varepsilon_{yx} E_x + \varepsilon_{yy} E_y + \varepsilon_{yz} E_z$$

$$D_z = \varepsilon_{zx} E_x + \varepsilon_{zy} E_y + \varepsilon_{zz} E_z$$

Electro-Optic Effect: Modulator

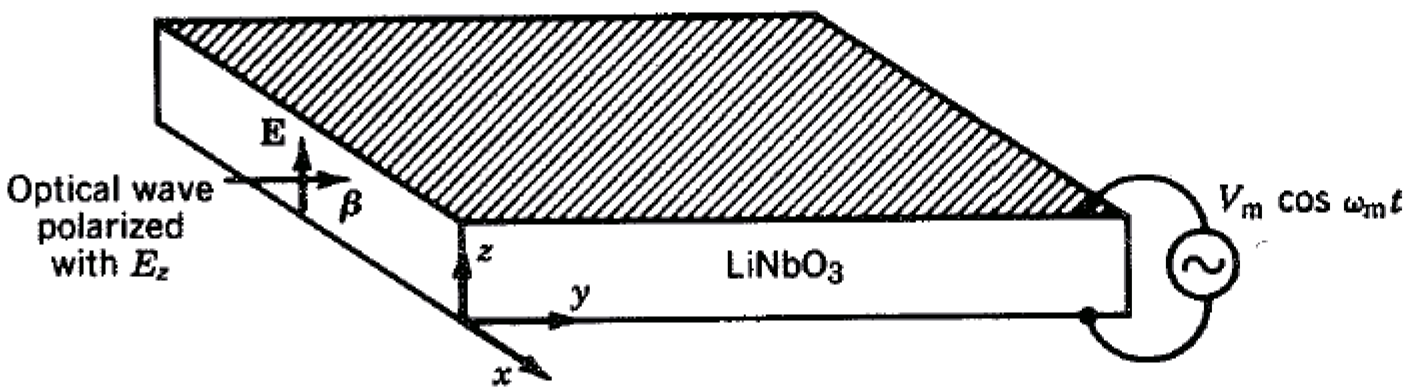


FIG. 13.11a Electro-optic phase modulation in $LiNbO_3$ crystal.

Modulation Index: $$\Delta\Phi_m = \frac{-\omega l r_{33} n_e^3 E_m}{2c}$$

Suggested Reading

- Chapter 2 :Advanced Engineering Electromagnetics *by* Constantine A. Balanis
- Chapter 13: Fields and Waves in Communication Electronics *by* Simon Ramo