EE 2310 Homework #4 Solutions – Complex Digital Circuits

1. Given the truth table to the right, express its Boolean expression in SOP form. Then construct the Karnaugh Map, simplify the Boolean expression by determining the prime implicants, and draw the new, simplified circuit (you do not have to draw the original circuit).

w	x	y	z	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

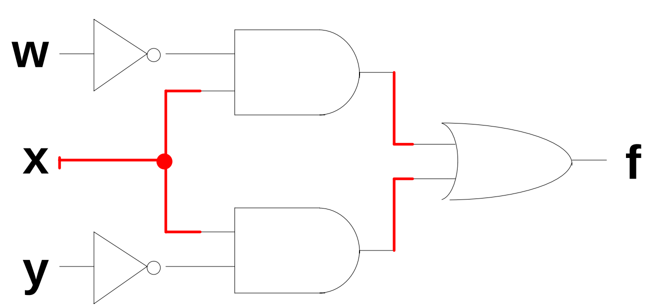
yz

wx	00	01	11	10
00				
01	1	1	1	1
11	1	1		
10				

Original Boolean expression:

$$f = \overline{w}x\overline{y}\overline{z} + \overline{w}x\overline{y}z + \overline{w}xyz + \overline{w}xy\overline{z} + wx\overline{y}\overline{z} + wx\overline{y}z.$$

Simplified expression: $f = \overline{w}x + x\overline{y}.$



Simplified circuit:

2. **We have not done many exercises involving POS Boolean Algebra expressions. This example shows that sometimes the POS version of Boolean expression is the much more convenient. For the truth table on the right, fill in the K-map and write both the <u>original</u> and <u>simplified POS expressions</u>. Then draw the simplified circuit (only).**

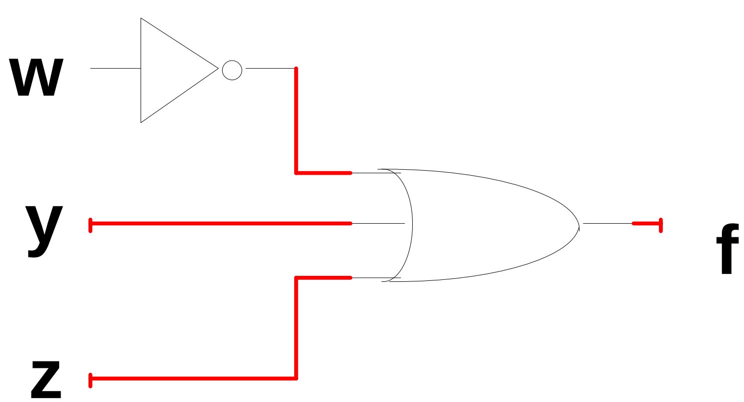
w	**x**	**y**	**z**	**f**
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

y+z

w+x	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	1	1	1
10	0	1	1	1

Original Boolean expression: $f = (\overline{w} + \overline{x} + y + z) \cdot (\overline{w} + x + y + z).$

Simplified expression: $f = \overline{w} + y + z\,.$



Simplified circuit:

3. **Another way to represent the Boolean expression for a set of logic elements is simply to designate the squares filled with ones on the standard Karnaugh map diagram. For instance (see lecture seven notes), one way of expressing a Boolean function is:**

$$f = \Sigma m\,(4,5,6,7,c,d,e,f)$$

Given the above expression, write the original and simplified SOP Boolean expressions, and draw only the simplified circuit.

wx \ yz	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

Simplified circuit:

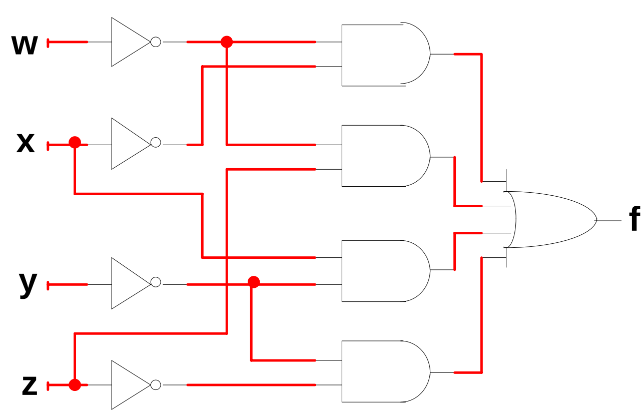
Original expression:

$$f = \bar{w}x\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}xyz + \bar{w}xy\bar{z} + wx\bar{y}\bar{z} + wx\bar{y}z + wxyz + wxy\bar{z}.$$

Simplified expression: $f = x.$

4. **Consider another logic expression: f = Σm (0, 1, 2, 3, 4, 5, 7, 8, 12, 13). Show the Karnaugh map for this Boolean expression, write the simplified the expression, and draw the simplified circuit.**

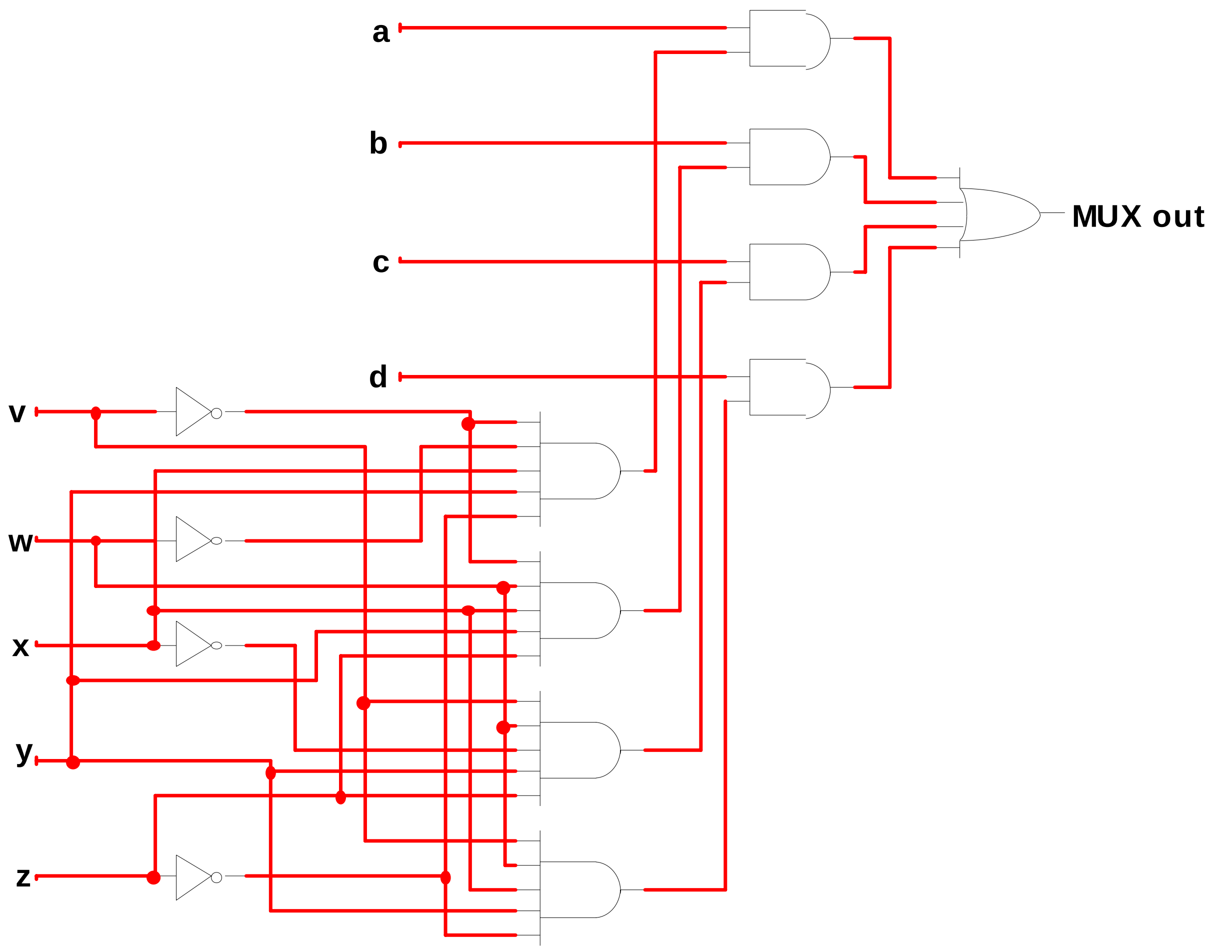
wx \ yz	00	01	11	10
00	1	1	1	1
01	1	1	1	
11	1	1		
10	1			



Simplified expression:

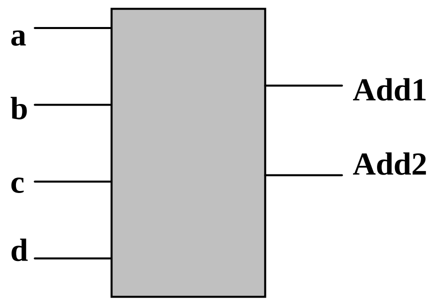
$$f = \bar{w}\bar{x} + \bar{w}z + x\bar{y} + \bar{y}\bar{z}$$

5. It is desired to multiplex four different input data lines onto one output. The signals are labeled a through d. Five address lines, (labeled "v" [MSB] through "z" [LSB]) control which line is to be multiplexed onto the output line. This five-input address can be regarded as a hexadecimal number which ranges from 0x00 to 0x1f. Input a is MUXed out on address 0x06, b on address 0x0f, c out on 0x1b, and d out on 0x1e. Using 2- and 5-input AND gates, a four-input OR gate, and inverters (NOTs), draw the MUX circuit.



6. It is desired to design a simple encoder, which we did NOT discuss in class, that will output a 2-bit address of 00-11 when one of four input lines is raised high (1). An encoder is the reverse of a decoder, and outputs a unique n-bit address for each of 2^n individual inputs. Thus in the problem here, there will be four (4) inputs and a 2-bit output address. Assume that you have only 2-input AND and OR gates and inverters. Note that you must have a truth table for each of the two output address bits. The basic truth table for each one is made as usual. You may assume that only one of the encoder inputs goes high at one time (i.e., any time one line is high, the other three are low). This assumption will simplify the truth table dramatically.

The input lines are labeled a-d, the output lines Add1 (LSB) and Add2 (MSB). When a is high, the output address is 00, when b is high, the address is 01, when c is high, the address is 10, and when d is high, the address is 11. Fill in the truth table below for your address outputs. A block diagram of the device is shown below.

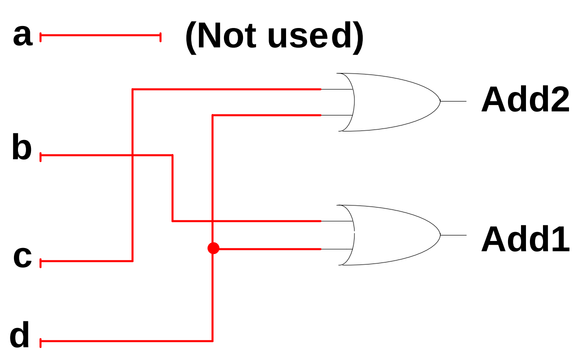


Encoder

Truth Table for Encoder

a	b	c	d	Add2	Add1
1				0	0
	1			0	1
		1		1	0
			1	1	1

Boolean Expressions: Add1 = b+d; Add2 = c+d



Note that there are NO inverters needed.

7. **Consider the problem of building a 1-bit <u>subtract only circuit</u>. This circuit will ONLY be used for subtraction, and may be used in an n-bit subtractor by tying n of these 1-bit subtractors together. Your problem is to design the 1-bit subtractor. This will be a full subtractor, which means that the inputs are x (the bit to be subtracted from), y (the bit that is subtracted from x), and borrow in (bi), the number that results when the adjacent column to the right has a y number bigger than x (or y and x are equal, but there is a borrow from the next-right column). The outputs are D (the difference) and the borrow out (bo). The two truth tables for D and bo are shown to the right, already filled out. MAKE SURE YOU UNDERSTAND THE TRUTH TABLE! Then fill in the two Karnaugh maps below, simplify as much as possible, write out the SOP Boolean expressions for D and bo, and lay out the two SOP digital circuits.**

bi	x	y	D	bo
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

Difference (D)
(No simplification possible)

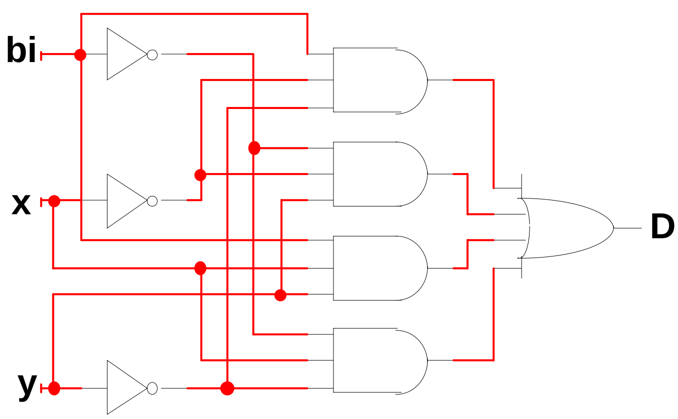
bi \ xy	00	01	11	10
0		1		1
1	1		1	

Borrow Out (bo)

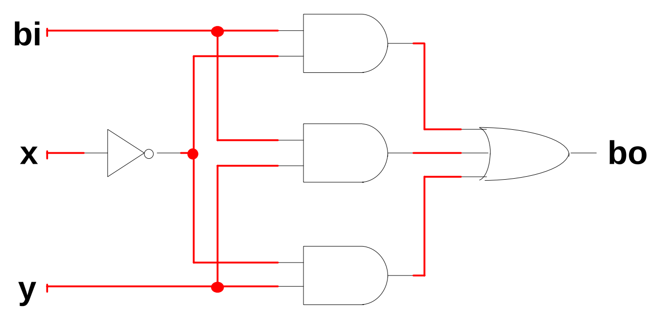
bi \ xy	00	01	11	10
0		1		
1	1	1	1	

$$D = b_i\bar{x}\bar{y} + \bar{b}_i\bar{x}y + b_ixy + \bar{b}_ix\bar{y}$$

$$b_O = b_i\bar{x} + b_iy + \bar{x}y$$



Difference



Borrow Out

8. **Consider the K-mapped SOP Boolean function $f = \sum_m 2,6,e$. The input variables operate in a restricted space. Input combinations are limited such that x can never be 0 when w is 1. Map both the 1's in the function and the special "don't care" conditions listed, develop the simplified SOP expression, and draw ONLY the simplified circuit.**

wx \ yz	00	01	11	10
00				1
01				1
11				1
10	X	X	X	X

$$f = y\bar{z}$$