Worksheet # 2A: DeMorgan's Law and Using K-Maps for Simplification

<u>DeMorgan's Law states the following two equivalences</u>:

1. $\overline{x + y} = \bar{x} \cdot \bar{y}$, **(OR version of DeMorgan's Law), and**
2. $\overline{x \cdot y} = \bar{x} + \bar{y}$ **(AND version of DeMorgan's Law).**

Let us consider these separately. We can also express $\overline{x \cdot y} = \bar{x} + \bar{y}$ **as** $\overline{\overline{x \cdot y}} = \overline{\bar{x} + \bar{y}}$ **(taking the complement of both sides, so that this does not change the relation). Thus, since we know that** $\bar{\bar{x}} = x$**, then we can say that the AND version of DeMorgan's law can also be expressed as:**

3. $\underline{x \cdot y = \overline{\bar{x} + \bar{y}}}$.

And likewise, the OR version, $\overline{x + y} = \bar{x} \cdot \bar{y}$ **can be expressed as:**

4. $\underline{x + y = \overline{\bar{x} \cdot \bar{y}}}$.

Incidentally, these two expressions are the proof that NAND and AND-NOT are <u>not the same functions</u>, and that NOR and OR-NOT are also not the same functions. Now, using the different forms of DeMorgan's Law above, find simpler versions of the following Boolean expressions:

- $f = x \cdot \left[\overline{\bar{y} + \bar{x}}\right]$

- $f = x + \overline{\left[\bar{y} \cdot \bar{x}\right]}$

- $f = \overline{x \cdot y} + \overline{w \cdot z}$

Simplification Using Karnaugh-Maps:

1. **Given the Boolean function $f = xy\bar{z} + x\bar{y}z + xyz$, plot the 1's on the K-Map below, find the simplified SOP function, and plot the simplified SOP circuit.**

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x \ yz	00	01	11	10
0				
1				

2. **Given the truth table, use the K-Map to develop a simplified POS expression and draw the simplified POS circuit.**

w \ y + z	00	01	11	10
00				
01				
11				
10				

w	x	y	z	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	0
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	0
1	1	0	1	
1	1	1	0	
1	1	1	1	

3. **A Boolean function is defined on a 4-D K-Map as $f = \sum_m 0,2,4,6$. Plot the 1's on the K-Map below, simplify in SOP form as much as possible, and draw the simplified SOP circuit.**

wx \ yz	00	01	11	10
00				
01				
11				
10				