

## On positive-influence target-domination

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**Abstract** Consider a graph  $G = (V, E)$  and a vertex subset  $A \subseteq V$ . A vertex  $v$  is positive-influence dominated by  $A$  if either  $v$  is in  $A$  or at least half the number of neighbors of  $v$  belong to  $A$ . For a target vertex subset  $S \subseteq V$ , a vertex subset  $A$  is a positive-influence target-dominating set for target set  $S$  if every vertex in  $S$  is positive-influence dominated by  $A$ . Given a graph  $G$  and a target vertex subset  $S$ , the positive-influence target-dominating set (PITD) problem is to find the minimum positive-influence dominating set for target set  $S$ . In this paper, we show two results: (1) The PITD problem has a polynomial-time  $(1 + \log\lceil\frac{3}{2}\Delta\rceil)$ -approximation in general graphs where  $\Delta$  is the maximum vertex-degree of the input graph. (2) For target set  $S$  with  $|S| = \Omega(|V|)$ , the PITD problem has a polynomial-time  $O(1)$ -approximation in power-law graphs.

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## 1 Introduction

Consider a graph  $G = (V, E)$  and a vertex subset  $A \subseteq V$ . A vertex  $v$  is *positive-influence dominated* by  $A$  if either  $v$  is in  $A$  or at least half the number of neighbors of  $v$  belong to  $A$ . The positive-influence dominating has been studied extensively in the literature [6, 12, 14] due to its applications in social networks.

A positive comment produces a positive-influence and a negative comment generates a negative-influence. Every person is influenced by his friends positively or negatively, and tends to adopt a behavior with more positive-influence. For example, more children begin smoking when parents are smoking [10]. This is the background of the concept of positive-influence dominating.

Suppose a company wants to advertise its product by sending out some free samples for generating positive-influence for dominating a group of target members in a social networks. For example, if the product is woman's clothing, then the target group should consist of only women. To dominate all target members by positive-influence, how can samples used be distributed to minimize the total number of free samples? This viral marketing problem can be formulated as the positive-influence target-dominating set (PITD) problem in the following way: given a graph  $G$  and a subset  $S$  of vertices, find the minimum vertex subset  $A$  such that every vertex in  $S$  is positive-influence dominated by  $A$ . Each vertex in  $S$  is called a *target* and  $S$  is called a target set. In this paper, we show two results:

1. The PITD problem has a polynomial-time  $[\log \Delta + O(1)]$ -approximation in general graph  $G$  where  $\Delta$  is the maximum vertex-degree of  $G$ .
2. For target set  $S$  with  $|S| = \Omega(|V|)$ , the PITD problem has a polynomial-time  $O(1)$ -approximation in power-law graphs.

## 2 Preliminaries

Consider a finite set  $X$  and a function  $f : 2^X \rightarrow \Re$ . Function  $f$  is called a *submodular* function if for any two subsets  $A$  and  $B$  of  $X$ ,

$$f(A) + f(B) \geq f(A \cup B) + f(A \cap B).$$

Function  $f$  is *monotone nondecreasing* if

$$A \subset B \Rightarrow f(A) \leq f(B).$$

A monotone nondecreasing submodular function  $f$  with  $f(\emptyset) = 0$  is called a *polymatroid* function. The followings are well-known properties of submodular and monotone nondecreasing functions and polymatroid functions, which can be found in [8, 11, 13].

**Lemma 1** *Function  $f$  is monotone nondecreasing submodular if and only if for any element  $x \in X$ , and two subsets  $A$  and  $B$  with  $A \subset B$ ,*

$$\Delta_x f(A) \geq \Delta_x f(B)$$

where  $\Delta_x f(A) = f(A \cup \{x\}) - f(A)$ .

**Lemma 2** *Suppose  $f$  is a monotone nondecreasing submodular function. Then for any constant  $c$ ,  $\min(c, f)$  is monotone nondecreasing submodular. Moreover, if  $f$  is a polymatroid function and  $c > 0$  is a constant, then  $\min(c, f)$  is also a polymatroid function.*

**Lemma 3** *If  $f$  and  $g$  are two polymatroid functions, then  $f + g$  is also polymatroid.*

Consider a polymatroid function  $f$  on  $2^X$  and a nonnegative cost function  $c : X \rightarrow \mathfrak{R}_+$ . Define  $c(A) = \sum_{v \in A} c(v)$ . The following is called the *submodular cover problem*:

$$\begin{aligned} & \min c(A) \\ & \text{subject to } A \in \Omega(f) \end{aligned}$$

where  $\Omega(f) = \{A \mid f(A) = f(X)\}$ . There is a greedy approximation algorithm with a Theorem on its performance for the submodular cover problem which can be found in [8, 13].

*Greedy Algorithm SC*

$A \leftarrow \emptyset$ ;

while  $f(A) < f(X)$  do

choose  $x = \operatorname{argmax}_{x \in X} \frac{\Delta_x f(A)}{c(x)}$

and set  $A \leftarrow A \cup \{x\}$ ;

output  $A$ .

**Theorem 4** *If  $f$  is a polymatroid integer function on  $2^X$ , then Greedy Algorithm SC produces an approximation solution for the submodular cover problem within a factor of  $1 + \ln \gamma$  from the optimal where  $\gamma = \max_{x \in X} f(\{x\})$ .*

### 3 In general graphs

Consider a graph  $G = (V, E)$  and a set of targets,  $S \subseteq V$ . For any  $v \in V$ , denote  $\deg_A(v) = |\{(u, v) \mid u \in A \text{ or } v \in A\}|$ . For any vertex subset  $A$ , define

$$f_S(A) = \sum_{v \in S} \min \left( \left\lceil \frac{1}{2} \cdot \deg(v) \right\rceil, \deg_A(v) \right).$$

Then we have the following.

**Lemma 5** For any given subset of vertices  $S$ ,  $f_S$  is a polymatroid function.

*Proof* We claim that  $g(A) = \text{deg}_A(v)$  is a polymatroid function for any  $v \in V$ . Note that  $g(\emptyset) = 0$ . By Lemma 1, it is sufficient to show that for any vertex  $x \in V$  and two vertex subsets  $A$  and  $B$  with  $A \subset B$ ,  $\Delta_x g(A) \geq \Delta_x g(B)$ . We divide the proof into three cases.

*Case 1.*  $v \in A$ . Then  $g(A) = g(A \cup \{x\}) = g(B) = g(B \cup \{x\}) = \text{deg}(v)$ . Hence,  $\Delta_x g(A) = \Delta_x g(B) = 0$ .

*Case 2.*  $v \in B \setminus A$ . Then  $g(B) = g(B \cup \{x\}) = \text{deg}(v)$ . Hence,  $\Delta_x g(A) \geq \Delta_x g(B) = 0$ .

*Case 3.*  $v \notin B$ . If  $v \neq x$ , then we have

$$\begin{aligned} \Delta_x g(A) &= g(A \cup \{x\}) - g(A) \\ &= |(A \cup \{x\}) \cap N(v)| - |A \cap N(v)| \\ &\geq |(B \cup \{x\}) \cap N(v)| - |B \cap N(v)| \\ &= \Delta_x g(B), \end{aligned}$$

where  $N(v)$  is the set of neighbors of  $v$ . If  $v = x$ , then we have

$$\begin{aligned} \Delta_x g(A) &= g(A \cup \{x\}) - g(A) \\ &= \text{deg}(v) - |A \cap N(v)| \\ &\geq \text{deg}(v) - |B \cap N(v)| \\ &= \Delta_x g(B). \end{aligned}$$

Now, by our claim and Lemmas 2 and 3,  $f_S$  is also a polymatroid function. □

**Lemma 6** A vertex subset  $A$  is a positive-influence dominating set for target  $S$  if and only if  $f_S(A) = f_S(V)$ .

*Proof* Note that  $v$  is positive-influence dominated by  $A$  if and only if  $\lceil \frac{1}{2} \cdot \text{deg}(v) \rceil \leq \text{deg}_A(v)$ . Now, the lemma follows immediately from this fact. □

**Theorem 7** Let a target set  $S$  be given. Then there exists a polynomial-time greedy algorithm which produces an approximation solution for the PITD problem, within a factor of  $1 + \ln \lceil \frac{3}{2} \Delta \rceil$  from the optimal where  $\Delta$  is the maximum vertex degree of graph  $G$ .

*Proof* By Lemma 6,  $\Omega(f_S)$  is the set of positive-influence dominating sets for target  $S$ . Thus, finding the minimum positive-influence dominating set for target  $S$  is the submodular cover problem with polymatroid function  $f_S$  and constant cost function  $c(u) = 1$  for every vertex  $u$ .

By Lemmas 5, 6 and Theorem 4, Greedy Algorithm SC produces an approximation solution within a factor of  $1 + \ln \gamma$  from the optimal where  $\gamma = \max_{u \in V} f_S(\{u\})$ . Note that

$$f_S(\{u\}) = \sum_{v \in S} \min \left( \left\lceil \frac{1}{2} \cdot \text{deg}(v) \right\rceil, \text{deg}_{\{u\}}(v) \right).$$

In this sum, when  $v = u$ , the corresponding term is equal to  $\lceil \frac{1}{2} \cdot deg(u) \rceil$  since by our definition of  $deg_A(v)$ ,  $deg_{\{u\}}(u) = deg(u)$ . When  $v \neq u$  and  $(v, u) \in E$ , the corresponding term is equal to 1. When  $v \neq u$  and  $(v, u) \notin E$ , the corresponding term is equal to 0. Therefore,

$$\begin{aligned} \gamma &= \max_{u \in V} f_S(\{u\}) \\ &\leq \max_{u \in V} \left( deg(u) + \left\lceil \frac{1}{2} deg(u) \right\rceil \right) \\ &= \max_{u \in V} \left\lceil \frac{3}{2} deg(u) \right\rceil \\ &\leq \left\lceil \frac{3}{2} \Delta \right\rceil. \end{aligned}$$

□

### 4 In power law graphs

Next, we move our attention to power law graphs. A graph  $G$  is said to belong to class  $C(\alpha, \gamma)$  if  $G$  has no isolated vertex and, for  $k \geq 1$ , the number of vertices with degree  $k$  is  $\lfloor \frac{e^\alpha}{k^\gamma} \rfloor$ . Clearly, the maximum vertex degree  $\Delta = \lfloor e^{\alpha/\gamma} \rfloor$ . For the number  $n$  of vertices and the number  $m$  of edges, we have

$$n = \sum_{k=1}^{\Delta} \left\lfloor \frac{e^\alpha}{k^\gamma} \right\rfloor \leq \sum_{k=1}^{\Delta} \frac{e^\alpha}{k^\gamma} < 2n,$$

and

$$m = \frac{1}{2} \cdot \sum_{k=1}^{\Delta} k \left\lfloor \frac{e^\alpha}{k^\gamma} \right\rfloor \leq \frac{1}{2} \cdot \sum_{k=1}^{\Delta} \frac{e^\alpha}{k^{\gamma-1}} < 2m.$$

Usually, an online social network varies as time changes, which can be formulated by a family of power law graphs with a fixed  $\gamma$ . In this family,  $m$  and  $n$  are varied when  $\alpha$  is varied. Our study will emphase on a family of power law graphs with fixed  $\gamma > 2$  because many social networks are such a family of power law graphs [1–5,9].

Let us first show some properties for such a family of power law graphs  $C(\alpha, \gamma)$ .

**Lemma 8** *For any power law graph  $G \in C(\alpha, \gamma)$  with fixed  $\gamma > 2$ ,  $n \geq c_1 \cdot m$  where  $c_1$  is a constant depending on only  $\gamma$ .*

*Proof* It suffices to show that  $n/m$  is greater than a positive constant. Let  $\zeta(\gamma)$  be the Riemann Zeta function, i.e.,  $\zeta(\gamma) = \sum_{k=1}^{\infty} \frac{1}{k^\gamma}$ . Then

$$n/m \geq \frac{0.5 \sum_{k=1}^{\Delta} \frac{1}{k^\gamma}}{0.5 \sum_{k=1}^{\Delta} \frac{1}{k^{\gamma-1}}} \zeta(\gamma) > \zeta(\gamma - 1).$$

□

For any vertex subset  $A$ , denote  $deg(A) = \sum_{v \in A} deg(v)$ .

**Lemma 9** Suppose  $G$  is a graph without isolated vertex and  $S$  is a target set. Let  $A$  be a positive-influence target-dominating set in  $G$ . Then  $deg(A) \geq 0.5 \cdot |S|$ .

*Proof* Note that at least  $|S|$  vertices are positive-influence dominated by  $A$  and each vertex has degree at least one. Thus,  $deg(S) \geq |S|$ . Since  $A$  is a positive-influence target-dominating set for target set  $S$ , each vertex  $v \in S$  has at least half the number of neighbors in  $A$ . Hence, we have  $deg(A) \geq 0.5deg(S) \geq 0.5|S|$ .  $\square$

**Lemma 10** For any constant  $c > 0$  and  $\gamma > 2$ , there exists a constant  $c_2 > 0$ , which depends on only  $c$  and  $\gamma$ , such that for any graph  $G$  in class  $C(\alpha, \gamma)$  with fixed  $\gamma > 2$  and any vertex subset  $A$ ,

$$deg(A) \geq cm \Rightarrow |A| \geq c_2n.$$

*Proof* Let  $k_0$  be the largest integer such that

$$|A| \leq \sum_{k=k_0}^{\Delta} \left\lfloor \frac{e^\alpha}{k^\gamma} \right\rfloor.$$

Then

$$cm \leq deg(A) \leq \sum_{k=k_0}^{\Delta} \frac{e^\alpha}{k^{\gamma-1}}. \tag{1}$$

Since  $\sum_{k=1}^{\Delta} \frac{1}{k^{\gamma-1}} \rightarrow \zeta(\gamma - 1)$  as  $\Delta \rightarrow \infty$ , there exists  $\Delta_0 > 0$  such that for  $\Delta > \Delta_0$ ,  $\sum_{k=1}^{\Delta} \frac{1}{k^{\gamma-1}} \geq 0.5\zeta(\gamma - 1)$ , where  $\Delta_0$  depends on only  $\gamma$ . Note that for  $1 \leq k \leq \Delta$ ,  $\lfloor \frac{e^\alpha}{k^\gamma} \rfloor \geq 1$ . Therefore, for  $1 \leq k \leq \Delta$ ,  $\lfloor \frac{e^\alpha}{k^\gamma} \rfloor > 0.5 \cdot \frac{e^\alpha}{k^\gamma}$ . Thus, for  $\Delta > \Delta_0$ ,

$$\begin{aligned} m &= \frac{1}{2} \sum_{k=1}^{\Delta} k \cdot \left\lfloor \frac{e^\alpha}{k^\gamma} \right\rfloor \\ &> \frac{1}{4} \sum_{k=1}^{\Delta} \frac{e^\alpha}{k^{\gamma-1}} \\ &\geq \frac{1}{8} \zeta(\gamma - 1)e^\alpha. \end{aligned}$$

Choose  $k_1 > 0$  such that

$$\sum_{k=k_1}^{\infty} \frac{1}{k^{\gamma-1}} < c\zeta(\gamma - 1)/8.$$

Then  $k_1$  depends on only  $c$  and  $\gamma$ . Moreover, for  $\Delta > \Delta_0$ ,

$$cm > \sum_{k=k_1}^{\infty} \frac{e^\alpha}{k^{\gamma-1}} > \sum_{k=k_1}^{\Delta} \frac{e^\alpha}{k^{\gamma-1}}. \tag{2}$$

Comparing (2) with (1), we obtain  $k_0 < k_1$ . It follows that

$$|A| > \sum_{k=k_1}^{\Delta} \left\lfloor \frac{e^\alpha}{k^\gamma} \right\rfloor \geq \frac{1}{2} \sum_{k=k_1}^{\Delta} \frac{e^\alpha}{k^\gamma} \geq \frac{n}{2} \cdot \frac{\sum_{k=k_1}^{\Delta} \frac{1}{k^\gamma}}{\sum_{k=1}^{\Delta} \frac{1}{k^\gamma}}.$$

Since

$$\frac{\sum_{k=k_1}^{\Delta} \frac{1}{k^\gamma}}{\sum_{k=1}^{\Delta} \frac{1}{k^\gamma}} \rightarrow \frac{\sum_{k=k_1}^{\infty} \frac{1}{k^\gamma}}{\zeta(\gamma)} \text{ as } \Delta \rightarrow \infty,$$

we can choose  $\Delta_1 \geq \Delta_0$  such that for  $\Delta > \Delta_1$ ,

$$\frac{\sum_{k=k_1}^{\Delta} \frac{1}{k^\gamma}}{\sum_{k=1}^{\Delta} \frac{1}{k^\gamma}} \geq \beta_1 = 0.5 \frac{\sum_{k=k_1}^{\infty} \frac{1}{k^{\gamma-1}}}{\zeta(\gamma)}.$$

Here,  $\Delta_1$  depends on  $k_1$  and  $\gamma$ , and hence depends on only  $c$  and  $\gamma$ . Therefore, for  $\Delta > \Delta_1$ ,

$$|A| > n(\beta_1/2).$$

For  $\Delta \leq \Delta_1$ , we note  $\Delta = e^{\lfloor \alpha/\gamma \rfloor}$  and hence  $e^\alpha \leq (\Delta_1 + 1)^\gamma$ . Therefore,

$$n \leq \sum_{k=1}^{\Delta} \frac{e^\alpha}{k^\gamma} \leq \beta_2 = \sum_{k=1}^{\Delta_1} \frac{(\Delta_1 + 1)^\gamma}{k^\gamma}.$$

Since  $|A| \geq 1$ , we have that for  $\Delta \leq \Delta_1$ ,

$$|A| \geq n \cdot \frac{1}{\beta_2}.$$

Set  $c_2 = \min(\beta_1/2, 1/\beta_2)$ . Then

$$|A| \geq c_2 n.$$

Since  $\beta_1$  and  $\beta_2$  depend on only  $c$  and  $\gamma$ ,  $c_2$  depends on only  $c$  and  $\gamma$ . □

**Theorem 11** *For any family of power-law graphs  $G = (V, E)$  in  $C(\alpha, \gamma)$  with fixed  $\gamma > 2$  and target set  $S$  with  $|S| \geq \mu n$  for a constant  $\mu > 0$ , there exists a polynomial-time approximation algorithm for PITD problem, with constant performance ratio depending on only  $\mu$  and  $\gamma$ .*

*Proof* Suppose  $A$  is a positive-influence target-dominating set for target set  $S$ . By Lemma 9,  $\deg(A) \geq 0.5|S|$ . Since  $|S| \geq \mu n$  and  $n \geq c_1 m$  by Lemma 8, we have  $\deg(A) \geq 0.5|S| \geq 0.5\mu n \geq 0.5\mu c_1 \cdot m$ . By Lemma 10,  $|A| \geq c_2 n$  where  $c_2$  depends on  $0.5\mu c_1$  and  $\gamma$ , i.e., depends on  $\mu$  and  $\gamma$ . Since  $|A| \leq n$ , the ratio of sizes of any two

positive-influence target-dominating sets is upper bounded by  $1/c_2$ . This means that any positive-influence target-dominating set for target  $S$  is a  $(1/c_2)$ -approximation solution.  $\square$

Given a graph  $G = (V, E)$  and a fraction  $r$  with  $0 < r \leq 1$ , the positive-influence partial-dominating set (PIPD) problem is to find the minimum vertex subset  $A$  such that at least a portion  $r$  of vertices are positive-influence dominated by  $A$ . With above approach, we can also show the following.

**Theorem 12** *For any family of power-law graphs  $G$  in class  $C(\alpha, \gamma)$  with fixed  $\gamma > 2$ , the PIPD problem has a polynomial-time approximation with a constant performance ratio depending on only  $\gamma$  and  $r$ .*

*Proof* Suppose that at least  $rn$  vertices are positive-influence dominated by  $A$  and each vertex has degree at least one. Let  $B$  denote the set of vertices positive-influence dominated by  $A$ . Then  $|B| \geq rn$  and  $\deg(B) \geq rn$ . Since each vertex  $v \in B$  has at least a half number of neighbors in  $A$ , we have  $\deg(A) \geq 0.5rn$ . By Lemma 8,  $\deg(A) \geq 0.5rn \geq 0.5rc_1 \cdot m$ . By Lemma 10,  $|A| \geq c_2n$ , where  $c_2$  depends on only  $r$  and  $\gamma$ . This means that every feasible solution for the PIPD problem is a  $(1/c_2)$ -approximation.  $\square$

## 5 Discussion

Note that the positive-influence dominating set (PIDS) problem in [6, 7] is exactly the PITD problem in case  $S = V$  and the PIPD problem in case  $r = 1$ . Therefore, all lower bound results in [6, 7] can be extended to the PITD problem and the PIPD problem. Thus, we have the following.

**Theorem 13** (a) *In general graphs, the PITD (and the PIPD) problem has no polynomial-time  $(0.5 - \varepsilon)$   $\ln n$ -approximation for any  $\varepsilon > 0$  unless  $NP \subseteq DTIME(n^{O(\log \log n)})$ .*

(b) *In power-law graphs, both the PITD and the PIPD problems are NP-hard.*

This lower-bound indicates that the result in Theorem 7 is almost the best possible. However, it is an open problem whether there exists a PTAS for the PITD (or the PIPD) problem.

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