Optimal Sensor Coverage

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“Since the fabric of the world is the most perfect and was established by the wisest Creator, nothing happens in this world in which some reason of maximum or minimum would not come to light.”

- `Euler`

“The only difference between suicide and martyrdom is press coverage.”

- `Chuck Palahniuk`

“God used beautiful mathematics in creating the world.”

- `Paul Dirac`

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Chapter 1

Introduction

Most practical questions can be reduced to problems of largest and smallest magnitudes ... and it is only by solving these problems that we can satisfy the requirements of practice which always seeks the best, the most convenient.
P. L. Čebyšev

1.1 Coverage

Nowaday, sensors exist everywhere. They are used for monitoring battlefield, controlling traffic, watching environment, managing manufacture process, detecting disasters, examining human's bodies, and collecting data from hostile area, etc.. In many applications, the coverage is a fundamental requirement and hence becomes an important issue in study of sensor systems, especially wireless sensor networks. For example, when a request comes to ask for information on enemy's activities in a certain area of battlefield, a set of sensors are required to activate for covering (sensing) the target area.

Given a target area or a set of target points, the coverage is usually to aim at finding a set of sensors for covering given target area or all given target points. A set of sensors is called a *sensor cover* if it can realize the coverage, i.e., covers a given target area or all given target points. In the literature, a related problem is often called a coverage problem or a sensor cover problem, e.g., the minimum connected sensor cover problem and the maximum lifetime coverage problem.

The coverage of target area can often be converted into the coverage of a set of target points. For a sensor, its sensing area is a disk. Therefore, this problem can be formulated as follows: Given a set $\mathcal{D}$ of n disks with possibly different radius, and a target area Ω, find a set $\mathcal{P}$ of target points such that a subset $\mathcal{D}'$ of $\mathcal{D}$ covers target area Ω if and only if $\mathcal{D}'$ covers all target points in $\mathcal{P}$.

There are several ways in the literature for choosing the set of target points, $\mathcal{P}$.

For any area A, denote by ∂A the boundary of A. Note that circles ∂D for all $D \in \mathcal{D}$ divide the area Ω into many small areas. The first way is to choose an interior point (Fig. 1.1) from each small area. Let $a_1, ..., a_{m_a}$ be all chosen points. The following theorem indicates that they can be target points to replace the target area Ω.

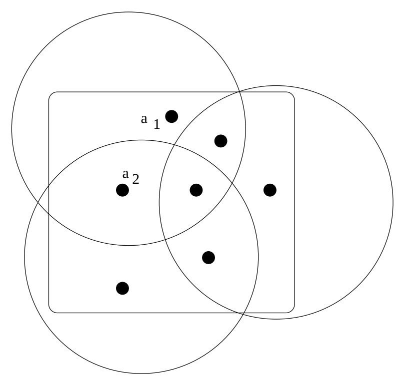

Figure 1.1: In each small area, choose an interior point as a target point.

Theorem 1.1.1 (Wu et al. [64]) *Suppose $\partial\Omega$ has $O(n^2)$ intersetion points with ∂D for all $D \in \mathcal{D}$. Then $m_a = O(n^2)$. Moreover, any disk subset $\mathcal{D}' \subseteq \mathcal{D}$ covers area Ω if and only if every a_i is an interior point of D for some $D \in \mathcal{D}'$.*

Proof. By Euler's formula,

$$(m_a + 1) + (v' + v'') = e + 2$$

where

$$\begin{aligned} v' &= |\{x \in \Omega \mid x \text{ is an intersection of } \partial D \text{ and } \partial D' \text{ for some } D, D' \in \mathcal{D}\}|, \\ v'' &= |\{x \mid x \text{ is an intersection of } \partial\Omega \text{ and } \partial D \text{ for some } D \in \mathcal{D}\}|, \end{aligned}$$

and e is the number of segments on the boundaries of small areas in Ω; they result from $\partial\Omega$ and ∂D for $D \in \mathcal{D}$, cutting by intersection points. Note that $v' + v'' = O(n^2)$. To show $m_a = O(n^2)$, it suffices to prove $e = O(n^2)$.

First, note that since $v'' = O(n^2)$, the number of segments on $\partial\Omega$ is $O(n^2)$. Now, we show that the number of segments on ∂D for $D \in \mathcal{D}$ is also $O(n^2)$. To do so, we first remove $\partial\Omega$ and consider only ∂D for $D \in \mathcal{D}$. We claim that the number of pieces resulting from cutting all ∂D for $D \in \mathcal{D}$ with intersection points is at most $2n(n-1)$. We show this claim by induction on n. For $n = 1$, it is trivially true. Next, consider n disks with assumption that the claim for $n-1$ disks is true. A new circle ∂D can intersect $n-1$ circles with at most $2(n-1)$ intersection points. Those intersection points would cut ∂D into at most $2(n-1)$ segments. Each of those intersection points may also break some segments, on other $n-1$ circles, into more segments. However, each intersection point can break at most one such segment into two. Therefore, at most $4(n-1)$ segments would be increased. The total number of segments on n circles is at most

$$2(n-1)(n-2) + 4(n-1) = 2n(n-1).$$

Although those segments can be further cut by $\partial\Omega$, the increasing number of segments is at most $O(n^2)$. Therefore, $e = O(n^2)$. This completes the proof of the first half of this theorem.

For the second half of this theorem, it suffices to note that no boundary cuts any small area. Thus, each a_i is covered by a disk if and only if the small area with representative a_i is covered by the disk. Thus, $\mathcal{D}'$ covers area Ω if and only if every small area is covered by a disk in $\mathcal{D}'$ if and only if all representatives $a_1, a_2, ..., a_{m_a}$ are covered by the disk subset $\mathcal{D}'$. Moreover, each a_i is covered by a disk if and only if it is covered by the interior of the disk. $\square$

The second way is to choose intersection points of circles ∂D for $D \in \mathcal{D}$. Let $b_1, ..., b_{m_b}$ be all such intersection points lying in the interior of Ω (Fig.1.2). Since two circles have at most two intersection points, we have $m_b = O(n^2)$. The following theorem indicates that those intersection points can be used for target points to replace the target area.

Theorem 1.1.2 (Wu et al. [64]) *Suppose $m_b \geq 1$ and for every disk D, every maximal connected piece of ∂D lying in the interior of Ω contains at least one b_i. Then a disk subset $\mathcal{D}'$ covers area Ω if and only if every b_i lies in the interior of the union $\cup_{D \in \mathcal{D}'} D$.*

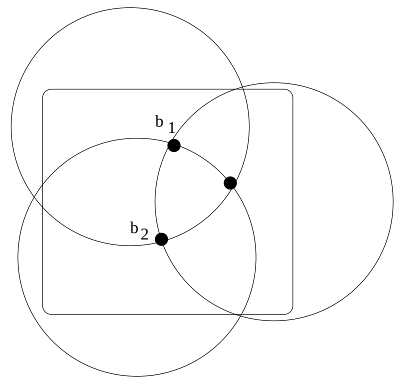

Figure 1.2: The second way is to choose all intersection points of disk boundaries lying in the interior of area.

Proof. Note that all $b_1, .., b_{m_b}$ lie in the interior of Ω. If $\mathcal{D}'$ covers Ω, then clearly, every b_i is in the interior of the union $\cup_{D \in \mathcal{D}'} D$. Thus, the necessity is true. Next, we show the sufficiency.

For contradiction, suppose the sufficiency is not true. Then there exists a point p in Ω, not covered by the union $\cup_{D \in \mathcal{D}'} D$. However, since $m_b \geq 1$ and b_1 is covered by the union $\cup_{D \in \mathcal{D}'} D$, there must exist a boundary piece of this union separating p and b_1, and lying in the interior of Ω. If this piece comes from boundaries of at least two disks (Fig.1.3(a)), then it must contains an intersection point b_i which is not an interior point of the union $\cup_{D \in \mathcal{D}'} D$, contradicting the theorem condition. If this piece of the boundary of $\cup_{D \in \mathcal{D}'} D$ comes from one disk (Fig.1.3(b)), then it must be a maximal connected piece lying in the interior of Ω and hence contains some point b_i which is not an interior point of the union $\cup_{D \in \mathcal{D}'} D$, again contradicting the theorem condition. $\square$

When a large amount of sensors deployed into an area Ω, the assumption in Theorem 1.1.2 holds easily. However, the assumption may not be held when the number of sensors is small. In such a case, we can consider to use the first method.

It is necessary for Theorem 1.1.2 to have the condition that every maximal connected piece of boundary of disk lying in the interior of Ω contains at least one b_i. A counterexample in Fig.1.4 shows that if this condition does not hold, then $\mathcal{D}$ may not cover whole area although all b_i are covered in the interior of the union of all disks. This condition also has some relationship

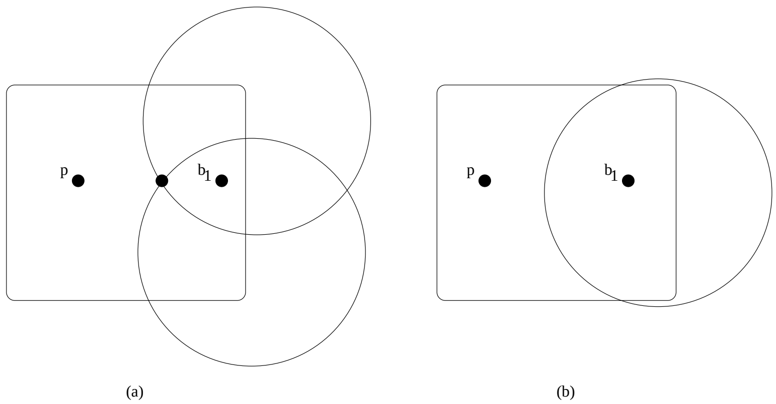

Figure 1.3: A piece of the boundary of the union of disks in $\mathcal{D}'$ separates p and b_1.

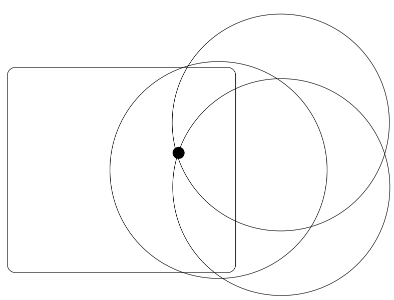

Figure 1.4: A counterexample.

with the condition in Theorem 1.1.1 that the number of intersection points of $\partial\Omega$ and boundaries of disks is $O(n^2)$. Indeed, when the number of intersection points of $\partial\Omega$ and boundaries of disks is more than $O(n^2)$. There may exist more than $O(n^2)$ maximal connected pieces of boundaries of disks, lying in the interior of Ω (Fig. 1.5), which implies the existence of at least one such maximal connected piece not containing any intersection point a_i because $m_a = O(n^2)$.

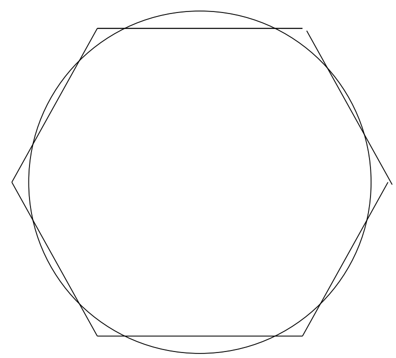

Figure 1.5: A circle may introduce $m \gg n^2$ maximal pieces in a regular polygon with m edges.

Hall [44] gave an interesting sufficient condition for convex area Ω and a set $\mathcal{D}$ of disks with the same radius to satisfy the condition in Theorem 1.1.2.

Theorem 1.1.3 (Hall [44]) *Consider a convex area Ω and a set $\mathcal{D}$ of disks with the same radius. Suppose convex area Ω contains one or more disk in $\mathcal{D}$. If for each $D \in \mathcal{D}$, either ∂D intersects at least one another $\partial D'$ lying the interior of Ω for some $D' \in \mathcal{D}$, or $D \cap \Omega = \emptyset$. Then convex area Ω and disk set $\mathcal{D}$ satisfy the condition that for every disk, every maximal connected piece of its boundary lying in the interior of Ω contains at least one b_i.*

Proof. First, we note that since Ω contains at least one disk in $\mathcal{D}$, each ∂D can have at most one maximal connected piece lying in the interior of Ω, which must contain an intersection point b_i since it intersects at least one disk lying in the interior of Ω. $\square$

When the number n of sensors is large, it may not be easy to compute points $a_1, a_2, ..., a_{m_a}$ because determination of all nonempty small areas may require to study 2^n systems of quadratic inequalities and for each system, determine whether there exists or not a solution of the system. However, computing $b_1, b_2, .., b_{m_b}$ may be relatively easy since we need only to solve $n(n-1)/2$ systems each of two quadratic equations. Therefore, in pratice, we often use $b_1, b_2, ..., b_{m_b}$ instead of $a_1, a_2, ..., a_{m_a}$. A valuable tip is how to deal with the case that there exists a maximal piece of boundary of sensing area of a sensor not containing any a_i. The following theorem suggests that we may arbitrarily choose an interior point on the maximal piece.

Theorem 1.1.4 *Consider an area Ω and a disk set $\mathcal{D}$. Suppose $c_1, c_2, ..., c_{m_c}$ are interior points of an area Ω, satisfying the following two conditions:*

(a) $\{b_1, b_2, ..., b_{m_b}\} \subseteq \{c_1, c_2, ..., c_{m_c}\}$.

(b) Every maximal piece of disk boundary lying in the interior of area Ω contains some point c_i.

Then, a disk subset $\mathcal{D}'$ covers Ω if and only if all $c_1, c_2, ..., c_{m_c}$ lie in the interior of the union $\cup_{D\in\mathcal{D}'} D$.

Proof. Similar to the proof of Theorem 1.1.2. □

Zhang and Hou [69] gave the following result.

Theorem 1.1.5 (Zhang and Hou [69]) *Consider a convex area Ω and a homogeneous disk set $\mathcal{D}$. Let $b_{m_b+1}, ..., b_{m'_b}$ be all intersection points of $\partial\Omega$ and circle ∂D for $D \in \mathcal{D}$. Assume $m_b \geq 1$. If every point of $b_1, b_2, ..., b_{m'_b}$ is covered by the interior of a disk in disk subset $\mathcal{D}'$, then Ω is covered by $\mathcal{D}'$.*

Proof. Suppose Ω contains a point p not covered by $\mathcal{D}'$. Then between p and b_1, there is a boundary of $\cup_{D\in\mathcal{D}'} D$. This boundary cannot contain anyone of $b_1, b_2, ..., b_{m'_b}$ since everyone of them is covered by the interior of a disk in $\mathcal{D}'$. Thus, this boundary must be the boundary of a disk D in $\mathcal{D}'$. However, this is impossible because D must contain b_1 and the circle $\partial D'$ passing through b_1 must intersect ∂D since D and D' have the same size of radius. □

In each of above methods, the number of selected target points is $O(n^2)$. Is it possible to select less number of target points to replace the target area? This is an interesting open problem.

Du and Wu [32] proposed an improvement for the first method as follows.

First, construct a directed graph on points $a_1, a_2, ..., a_{m_a}$ selected in the first method described as above. Two point a_i and a_j are said to be *adjacent* if two small areas represented by a_i and a_j have a boundary in common. This boundary must be a piece of ∂D for some disk $D \in \mathcal{D}$. If a_i lies outside of D and a_j lies inside of D, then we add an edge from a_i to a_j. The directed graph constructed in this way is denoted by $G(\Omega)$ (Fig.1.6).

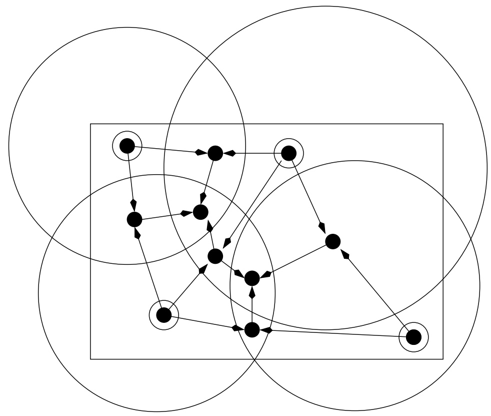

Figure 1.6: Graph $G(\Omega)$ with source nodes circled.

Theorem 1.1.6 (Du [32]) *$G(\Omega)$ contains no cycle.*

Proof. Consider an edge (a_i, a_j). By construction of edge (a_i, a_j), we know that there exists a disk $D \in \mathcal{D}$ such that a_i is not in D and a_j is in D. Note that every edge crossing ∂D has a direction from outside to inside of D. Therefore, no path exists from a_j to a_i. This means that no cycle exists in $G(\Omega)$. □

A point a_i is called a *source node* if every edge incident to a_i is going out from a_i. Let A_s be the set of all source nodes in G (Fig. 1.6).

Corollary 1.1.7 *$A_s \neq \emptyset$.*

Proof. Clearly, $m_a \geq 1$. If a_1 is not a source node, then there exists an edge (a_{i_2}, a_1). If a_{i_2} is not a source node, then there exists an edge (a_{i_3}, a_{i_2}). This process ends at finding either a source node or a cycle, contradicting to Theorem 1.1.6. □

Theorem 1.1.8 (Du [32]) *A disk subset $\mathcal{D}'$ of $\mathcal{D}$ covers target area Ω if and only if $\mathcal{D}'$ covers all points in A_s.*

Proof. The "only if" part is trivial. We show the "if" part as follows.

For contradiction, suppose $\mathcal{D}'$ does not cover Ω. We take disks in $\mathcal{D} - \mathcal{D}'$ one by one and add to $\mathcal{D}'$ until $\mathcal{D}'$ becomes a maximal subset not covering Ω, that is, $\mathcal{D}'$ does not cover Ω and however, for any $D \in \mathcal{D} - \mathcal{D}'$, $\mathcal{D}' \cup \{D\}$ covers Ω. In this case, the uncovered area of Ω must contain a source node outside all disks in $\mathcal{D}'$, contradicting to the assumption that $\mathcal{D}'$ covers all points in S. □

How large is $|A_s|$? In many examples, $|A_s|$ is significantly smaller than m_a. However, Ding and Zang [30] indicated that $|A_s| = \Omega(n^2)$. As shown in Fig. 1.7, all n disks are divided into four groups and each contains $n/4$

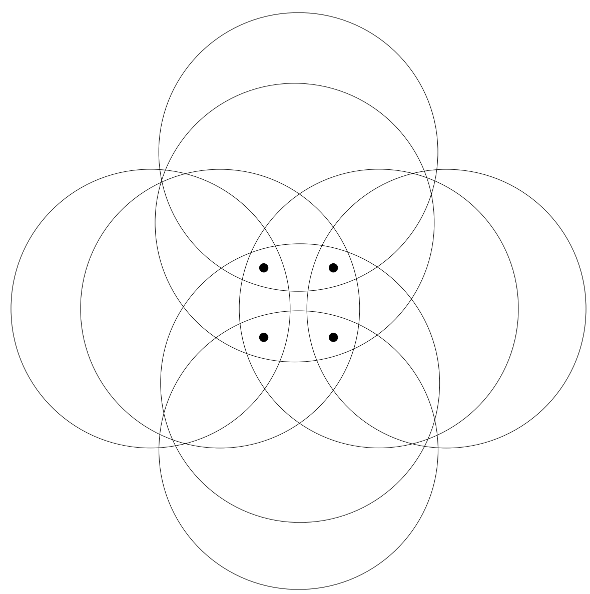

Figure 1.7: $|A_s| = \Omega(n^2)$.

disks. They form $n^2/16$ source nodes. Motivated from this fact, we have a conjecture as follows.

Conjecture 1.1.9 *For any area Ω, there exists a set of n disks, $\mathcal{D}$ such that if a finite set of points, A has the property that any disk subset $\mathcal{D}' \subseteq \mathcal{D}$ covers Ω if and only if $\mathcal{D}'$ covers A, then $|A| = \Omega(n^2)$.*

Sometime the boundary of $\partial\Omega$ may introduce too many source nodes (Fig.1.8). Du and Wu [32] also proposed a method to give an improvement in such a case as follows

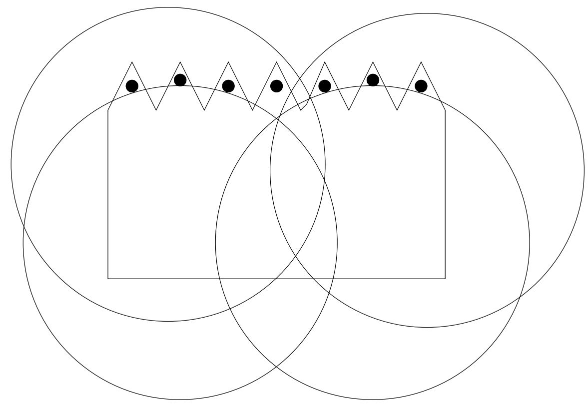

Figure 1.8: Nonsepable source nodes.

Two source nodes a_i and a_j in A_s is *separable* if there is a disk D in $\mathcal{D}$ such that a_i is not in D and a_j is in D, or vice versa. For any two source nodes in A_s, if they are not separable, then we delete one from A_s. Suppose A_s^* is obtained from A_s through this operation until no source node can be deleted any more, that is, every two source nodes in A_s^* are separable.

Theorem 1.1.10 (Du and Wu [32]) *A disk subset $\mathcal{D}'$ of $\mathcal{D}$ covers target area Ω if and only if $\mathcal{D}'$ covers all points in A_s^*.*

Proof. The proof is similar to the proof of Theorem 1.1.8. However, we should note that at end of the proof. The uncovered part of Ω may contain several source nodes which are not separable, one of which should belongs to A_s^*. $\square$

1.2 Connectivity

The connectivity is another important issue in the study of wireless sensor networks. In fact, after collect information, sensors need to deliver information to certain station, which requires the existence of a communication network connecting sensors. Actually, each sensor has a communication area, which is also a disk. The radius of communication area may be different from the radius of sensing area. A sensor can send information to another one if the latter lies inside the communication disk of the former. For a set of sensors, $\mathcal{S}$, the *communication network* for $\mathcal{S}$ is the directed graph with node set $\mathcal{S}$ and edge set

$$\{(s, s') \mid s' \text{ lies in the communication disk of } s\}.$$

Since a set of sensors owns a communication network, we may call a set of sensors as a sensor network or wireless sensor network to emphasize that the communication is wireless.

A sensor network is *homogeneous* if all sensors have the same communication radius R_c and sensing radius R_s. For a homogeneous sensor network, the communication network can be looked as an undirected graph. In general, we denote by $disk_r(o)$ a disk with center o and radius r, and by $circle_r(o)$ a circle with center o and radius r. In a homogeneous sensor network, for simplicity of notation, we may denote by $disk_c(o)$ (instead of $disk_{R_c}(o)$) the communication disk of sensor o, and by $circle_c(o)$ (instead of $circle_{R_c}(o)$) the boundary of the communication disk of sensor o. Similarly, denote by $disk_s(o)$ the sensing disk of sensor o and by $circle_s(o)$ the boundary of the sensing disk of sensor o.

Although in general the communication network of a sensor network is a directed graph, an undirected communication model is also considered in the literature [58]. In such a model, a communication link only in one direction would be ignored.

Surprisingly, the coverage and the connectivity sometimes have a close relationship. In the following, we would like to present two examples.

Theorem 1.2.1 (Zhang and Hou [69]) *Consider a homogeneous sensor network $\mathcal{S}$ with $R_c \geq 2R_s$. If $\mathcal{S}$ is a sensor cover for a connected area Ω such that every sensor s in $\mathcal{S}$ has a sensing point in Ω, then $\mathcal{S}$ is a connected network.*

Proof. For any two sensors s and s', choose two points p and p' in Ω such that p is in the sensing area of s and p' is in the sensing area of s'. Since Ω is a connected area, we can draw a curve C connecting p and p', lying inside of Ω (Fig.1.9). Suppose C is covered by sensing areas of a sequence of sensors $s_1 = s, s_2, ..., s_k = s'$. Without loss of generality, we may assume that for any two distinct sensors s_i and s_j, $disk_s(s_i) \cap C \not\subseteq dis_s(s_j) \cap C$ and $disk_s(s_i) \cap C \not\supseteq disk_s(s_j) \cap C$. We may also assume that along curve C from s to s', a point would meet sensing areas of sensors in ordering $s_1, s_2, ..., s_k$. This means that $disk_s(s_i) \cap disk_s(s_{i+1}) \neq \emptyset$. Since $R_c \geq 2R_s$, we would have $s_i \in disk_c(s_{i+1})$ and $s_{i+1} \in disk_c(s_i)$, that is, edge (s_i, s_{i+1}) exists in communication network. Hence, s and s' is connected by path $(s = s_1, s_2, ..., s_k = s')$. Therefore, the communication network of $\mathcal{S}$ is connected. $\square$

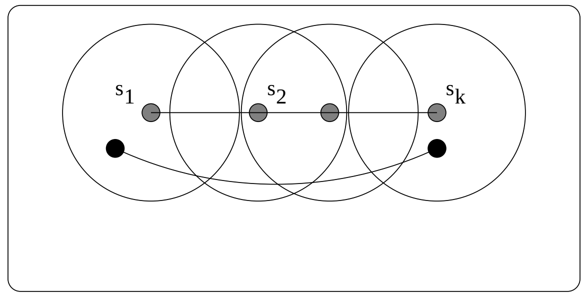

Figure 1.9: A curve C lies in area Ω.

A sensor cover $\mathcal{S}$ for a target area Ω is called a sensor k-cover if every point in Ω is covered by at least k sensors in $\mathcal{S}$. When an area has a sensor k-cover, we say that the area receives k-coverage. A graph $G = (V, E)$ is k-connected if removal $k-1$ nodes from G cannot destroy the connectivity, i.e., for any subset V' of at most $k-1$ nodes, $G[V - V']$, the subgraph induced by $V - V'$, is still connected. Theorem 1.2.1 can be generalized as follows.

Theorem 1.2.2 (Zhou, Das and Gupta [70]) *Consider a homogeneous sensor network $\mathcal{S}$ with $R_c \geq 2R_s$. If $\mathcal{S}$ is a sensor k-cover for a connected area Ω such that every sensor s in $\mathcal{S}$ has a sensing point in Ω, then $\mathcal{S}$ is a k-connected network.*

Proof. Since $\mathcal{S}$ is a sensor k-cover for Ω, every point in Ω can still be covered after delete at most $k-1$ sensors from $\mathcal{S}$. By Theorem 1.2.1, the sensor network is still connected after delete at most $k-1$ sensors. $\square$

In Theorems 1.2.1 and 1.2.2, the condition that every sensor s in $\mathcal{S}$ has a sensing point in Ω is quite natural since a sensor which does not cover any point in Ω can be deleted from a sensor cover or k-cover.

Theorem 1.2.3 (Xing et al. [65]) *Consider a homogeneous sensor network $\mathcal{S}$ with $R_c \geq 2R_s$. If $\mathcal{S}$ is a sensor k-cover for a convex area Ω, then for every two sensors u and v with their sensing areas lying in the interior of*

Ω, u and v are $2k$-connected in the communication network of $\mathcal{S}$, i.e., there are at least $2k$ node-disjoint paths between u and v.

Proof. Let (w, x) be the diameter of $circle_s(u)$ perpendicular to line (u, v) and (y, z) the diameter of $circle_s(v)$ perpendicular to line (u, v) (Fig. 1.10). Since $circle_s(u)$ and $circle_s(v)$ lie in the interior of Ω, the rectangle $wxyz$

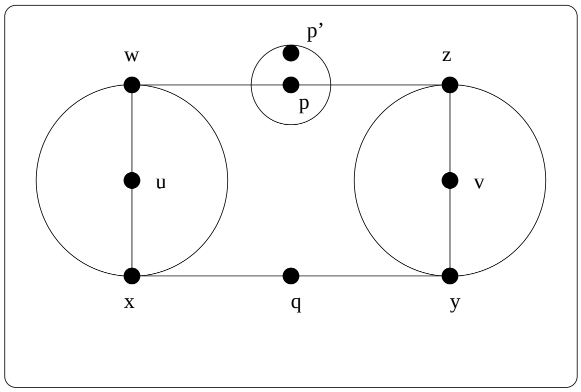

Figure 1.10: $circle_s(u)$ and $circle_s(v)$ lie in the interior of Ω.

lies in the interior of Ω. For contradiction, suppose u and v are not $2k$-connected. Then there exists a subset $\mathcal{M}$ of $2k - 1$ sensors in $\mathcal{S}$ such that removal $\mathcal{M}$ would break all connections between u and v. This implies that segment wz contains a point p not covered by the sensing disk of any sensor not in $\mathcal{M}$ and segment xy also contains a point q not covered by the sensing disk of any sensor not in $\mathcal{M}$. Note that the area A covered by sensing disks of sensors in $\mathcal{S} \setminus \mathcal{M}$ is a closed area and hence the area $\bar{A}$ not covered by any sensing disk of sensor in $\mathcal{S} \setminus \mathcal{M}$ is an open area. Moreover, p is an interior point of Ω. Therefore, p has an open neighborhood lying in the interior of Ω and also in $\bar{A}$. From this neighborhood, choose a point p' not in rectangle $wxyz$. Then p' and q have distance larger than $2R_s$ where R_s is the sensing radius of every sensor in $\mathcal{S}$. This means that each sensor in $\mathcal{M}$ can cover at most one of p' and q. Thus, one of p' and q cannot be sensed by k sensors in $\mathcal{M}$, a contradiction. $\square$

1.3 Energy Efficiency

The energy efficiency is an important issue in study of wireless sensor networks due to the following:

(a) Each senor is usually equipped with batteries and hence has a limited energy supply.

(b) In many applications, sensors are deployed into a hostile or dangerous region so that changing batteries is a mission impossible.

A lot of optimization problems stem from consideration of energy saving. When a target area is requested to be monitored for a long time, one may maximizes the lifetime of coverage by making a proper schedule of active and sleeping modes of sensors. However, when a duty can be finished within the lifetime of every sensor, the minimization on total energy consumption may be performed. If all active sensor have the same energy consumption rate, that is, consume the same amount of energy during the same time period, then the minimization of total energy is equivalent to the minimization of the number of active sensors.

The following are examples of optimization problems with objective functions coming from above consideration.

Problem 1.3.1 (Minimum Sensor Cover) *Given a set of sensors and a set of target points in the Euclidean plane, find a sensor cover with minimum cardinality.*

Problem 1.3.2 (Minimum Connected Sensor Cover) *Given a set of sensors and a set of target points in the Euclidean plane, find a connected sensor cover with minimum cardinality.*

Problem 1.3.3 (Maximum Lifetime Coverage) *Given a set of sensors with certain lifetime and a set of target points in the Euclidean plane, find a schedule for active/sleeping modes of sensors, under constraint that the total active time of each sensor does not exceed its lifetime, to maximize the lifetime of coverage where the coverage is alive at a moment if every target point is covered by at least one active sensor*

Problem 1.3.4 (Minimumm Weight Sensor Cover) *Given a set $\mathcal{A}$ of target points and a set $\mathcal{S}$ of sensors with positive weight $\mathcal{S} \to R^+$ in the Euclidean plane, find a sensor cover with minimum total weight.*

In above problems, we consider only target points because by Theorems 1.1.1 and 1.1.2, target area may be transformed into an equivalent set of target points in many cases. However, Theorems 1.2.1 and 1.2.2 indicate that sometime, such a transformation may loss the information on connectivity.

When this happens, that is, a result holds only for connected target area, we will mention it specially.

All optimization problems in above are classic in the literature and will be studied in this book.

1.4 Deployment

There are two different ways for deploying sensors. One is to deploy sensors randomly in a certain field. The other one is to put sensors in designated locations. These two ways may produce different problems, different algorithms, and different analysis methods. Therefore, some research results are related to certain deployment.

For example, when sensors are randomly deployed into area Ω, the probability for three sensors to have their sensing area boundaries intersecting at a point is close to zero since the measure for such sensor positions is zero. We may take this advantage to give a simpler way to implement Theorem 1.1.2. In fact, Theorem 1.1.2 requires that each b_i is covered by the interior of the union of sensing disks of sensors in considered sensor subset, which means that such a covering for b_i may be realized by not only one sensing disks. The following result indicates that this situation can be changed under certain condition.

Theorem 1.4.1 (Wu *et al.***[64])** *Suppose $m_b \geq 1$ and for every sensor s, every maximal connected piece of its boundary of sensing area lying in the interior of Ω contains at least one b_i. If any three boundaries of sensing disks of sensors in $\mathcal{S}'$ do not intersect at a point, then Ω is covered by sensor subset $\mathcal{S}'$ if and only if every b_i for $1 \leq i \leq m_b$ is covered by the interior of sensing area of a sensor in $\mathcal{S}'$.*

Proof. Suppose b_i is the intersection point of boundaries of disks of sensors o and o'. Note that the union of sensing disks of o and o' cannot cover completely a neighbor area of b_i, i.e., b_i cannot be an interior point of the union. This means that b_i must be covered by the sensing area of the third sensor o''. Since the boundary of sensing disk of o'' cannot pass b_i, b_i must be covered by the interior of sensing disk of o''. Therefore, b_i is covered by the interior of sensing disks of sensors in $\mathcal{S}'$ if and only if b_i is covered by the interior of sensing disk of some sensor in $\mathcal{S}'$. Now, this theorem follows immediately from this fact and Theorem 1.1.2. □

Therefore, we have

Corollary 1.4.2 *Suppose* $m_b \geq 1$ *and for every sensor* s*, every maximal connected piece of its boundary of sensing area lying in the interior of* Ω *contains at least one* b_i*. Then for random deployment of sensors, with probability one, the target area* Ω *is covered by a subset of sensors,* $\mathcal{S}'$ *if and only if every* b_i *for* $1 \leq i \leq m_b$ *is covered by the interior of sensing area of a sensor in* $\mathcal{S}'$*.*

Let us look at another example. Suppose sensor locations can be designed at any points in the Euclidean plane to monitor a set of given target points. Then we may obtain a minimization problem different from previous ones on coverage as follows.

Problem 1.4.3 Minimum Disk Cover*: Given a set of target points in the Euclidean plane and a set of disks, find a way to place the minimum number of disks in order to cover all target points.*

Chapter 2

Sensor Cover

the only way Bex would miss this would be if she were unconscious. And tied up. And in a concrete bunker. In Siberia.
ALLY CARTER *Don't Judge a Girl by Her Cover*

2.1 Motivation and Overview

Consider an instance of the Minimum Sensor Cover problem (Problem 1.3.1), consisting of a set of target points, $\mathcal{A}$, and a set of sensors, $\mathcal{S}$, lying in the Euclidean plane. For each $s \in \mathcal{S}$, let $A(s)$ denote the subset of target points lying in the sensing area of sensor s. Then Minimum Sensor Cover can be seen as a special case of the following set cover problem.

Problem 2.1.1 (Minimum Set Cover) *Given a collection $\mathcal{C}$ of subsets of a finite set X with property that $\cup_{A \in \mathcal{C}} A = X$, find a subcollection $\mathcal{C}' \subseteq \mathcal{C}$ with minimum cardinality and with property that $\cup_{A \in \mathcal{C}'} A = X$.*

Any subcollection $\mathcal{C}'$ with property that $\cup_{A \in \mathcal{C}'} A = X$ is called a *set cover*. Therefore, we may simply say that the problem is to find a minimum set cover from a given set cover.

The minimum set cover problem is a well-known NP-hard combinatorial optimization problem. We may list some important results about its approximation solutions as follow.

Theorem 2.1.2 (Chvátal[29],Johnson[47],Lovász[52]) *The minimum set cover problem has a polynomial-time $H(n)$-approximation where $n = |X|$ and $H(n) = \sum_{i=1}^{n} \frac{1}{i} < 1 + \ln n$ is called a harmonic function.*

Theorem 2.1.3 (Feige [35]) *For $\rho < 1$, there is no polynomial-time $\rho \ln n)$-approximation for the minimum set cover problem unless $NP \subseteq DTIME(n^{O(\log\log n)})$.*

Theorem 2.1.4 (Raz and Safra [57]) *There is a costant $c > 0$ such that the existence of polynomial-time $(c \ln n)$-approximation for the minimum set cover problem implies $NP = P$.*

Theorem 2.1.5 (Trevisan [62]) *There exists positive constants B_0 and c such that for every $B \geq B_0$, it is NP-hard to approximate the minimum set cover problem with input that every subset in $\mathcal{C}$ has size at most B, in polynomial-time and within a factor $\ln B - c \ln\ln B$ from optimal.*

Clearly, the approximation algorithm for the minimum set cover also works for the minimum sensor cover problem. Therefore, the minimum sensor cover has a polynomial-time $(1 + \ln n)$-approximation. By Theorems 2.1.3, 2.1.4 and 2.1.5, the polynomial-time $(1 + \ln n)$-approximation is most likely the best possible for the minimum set cover problem. However, the minimum sensor cover problem has a geometric background and is also called a gometric set cover problem. Hence, it has better approximations. Actually, in study of the minimum set cover, if X is a geometric universe and each subset in $\mathcal{C}$ is a geometric area such as a square or a disk, then the problem is called the minimum *geometric set cover* problem. More procisely, the minimum sensor cover problem is a minimum geometric set cover problem with disks.

For unit disks (or say for homoginious wireless sensor networks), it has been known for many year that there are several polynomial-time $O(1)$-approximations for the minimum sensor cover problem [11, 14, 55, 21]. A big progree was made by Mustafa and Ray [54]. They showed that the minimum sensor cover problem has PTAS (Polynomial-Time Approximation Scheme). More importantly, they showed a planar expansion theorem which has many applications. However, the PTAS often has a high running time and hence has a limitation in practice. Therefore, it is stiil worth studing $O(1)$-appproximations with fast running time. The following problem is still open.

Open Problem 2.1.6 *Is there an approximation algorithm with performance ratio at most two and running time $O(n^2)$?*

To this end, we would like to include some existing techniques for design $O(1)$-approximations, especially a technique using an important combinatorial tool, ε-net.

There are some variations on the sensor cover problem. For examples, the sensor location is variable or the radius of sensing disk is variable or the objective function is different [**?**]. They are motived from different applications.

2.2 Planar Expansion Theorem

In this section, we show the following theorem.

Theorem 2.2.1 (Planar Expansion Theorem,Mustafa and Ray [54]) *There exist two universal positive constants $K \geq 3$ and c such that for any $k \geq K$, the following holds: Let $G = (R, B, E)$ be a bipartite graph on red and blue vertex sets R and B and $|R| \geq 2$. If for every $B' \subseteq B$ with $|B'| \leq k$, $|N_G(B')| \geq |B'|$, then*

$$|B| \leq (1 + c/\sqrt{k})|R|, \tag{2.1}$$

where $N_G(B') = \{u \in R \mid (u, v) \in E \text{ for some } v \in B\}$.

To prove Planar Expansion Theorem, we need the following results.

Theorem 2.2.2 (Chrobak and Eppstein [28]) *Every undirected graph can be oriented such that every node has an in-degree at most three.*

Proof. We show, by induction, a result a little stronger than the theorem as follows.

Every undirected graph can be oriented such that every node has an in-degree at most three and each exterior node has an in-degree at most two.

The theorem holds trivially For graph with at most three nodes. Consider a graph G with $n \geq 4$ nodes. Then G must contain an exterior node v with at most two adjacent exterior nodes. Let G' be the graph obtained from G by deleting v. By induction hypothesis, G' has the required orientation. Now, for each edge (v, x), if x is an exterior node of G, then assign direction from x to v to this edge and if x is not an exterior node of G, then assign direction from v to x to the edge. Note that the existence of edge (v, x) implies that x is an exterior node of G'. Thus, we obtain a required orientation for G. □

Theorem 2.2.3 (Koutis and Miller [50]) *Any planar graph with n nodes can be partitioned into groups of size at most k such that the number of edges between different groups is at most $\gamma n/\sqrt{k}$ where γ is a fixed constant.*

Now, we start the proof of Panar Expansion Theorem.

Proof of Planar Expansion Theorem. First, one may assume $|B| > |R|$ since, otherwise, one has $|B| \leq |R|$ and hence (2.1) holds. One may also assume that G is a maximal planar bipartite graph since, otherwise, one can add edges which do not effect the condition and the conclusion in Planar Expansion Theorem. As a maximal planar graph, G must be connected and has no leaf.

Let B' be the set of all blue nodes with degree at least three. Let $G' = (R, B', E')$ be the graph obtained from G by deleting all blue nodes with degree two and edges incident to them. Then every face of G' is a quatrangle with two red nodes and two blue nodes appearing alternatively.

In each face of G', connecting the two red nodes by an edge lying inside of the face would result a triangulation T. Let $G_r = (R, E_r)$ be the subgraph of T induced by all red nodes. Then each face of G_r contains exactly one blue node. For each blue node $b \in B'$, let $f(b)$ denote the face of G_r where b lies. Then a red node is adjacent to b if and only it is on the boundary of $f(b)$.

Next, for each $b' \in B \setminus B$, one would also like to assign b' to a face of G_r and this assigment is requited to satisfy the following conditions:

(a) All red neighbors of b' is in $f(b')$.

(b) Each face of G_r receives at most seven assigments (including the one in B').

Note that each in G, $b' \in B \setminus B'$ is adjacent to two red nodes r_1 and r_2 which are two endpoints of an edge between two faces of G_r. b' can be assigned to either one to satisfy condition (a). Moreover, for each pair of such red nodes r_1 and r_2, there are at most two blue nodes b'_1 and b'_2 adjacent to both r_1 and r_2 since, otherwise, the existence of the third one b'_3 would imply $|N_G(\{b'_1, b'_2, b'_3\})| = |\{r_1, r_2\}| < |\{b'_1, b'_2, b'_3\}|$, contradicting assumption of the theorem. Next, we determine where b'_1 and b'_2 are assigned.

Define graph $G_b = (B', E_b)$ by defining $(b_1, b_2) \in E_b$ if and only if $f(b_1)$ and $f(b_2)$ are adjacent in G_r. Clearly, G_b is the planar dual graph of G_r. By Theorem 2.2.2, G_b can be oriented such that each node has in-degree at most three. Now, when $b' \in B \setminus B'$ is required to be assigned $f(b_1)$ or $f(b_2)$, choose $f(b_1)$ if edge (b_1, b_2) is oriented from b_2 to b_1, and choose $f(b_2)$, otherwise. This assignment also meets the condition (b).

By Theorem 2.2.3, graph G_b can be partitioned into several parts $B'_1, B'_2, ..., B'_h$, each of size at most $k/7$ such that the total number of edges between different parts is at most $(\gamma/\sqrt{k/7})|B'|$. Let B_i be the set of blue nodes assigned to some face $f(b)$ for $b \in B'_i$. Then by condition (b), $|B_i| \leq 7|B'_i| \leq k$. Thus, $|B_i| \leq |N_G(B_i)|$.

All nodes in $N_G(B_i)$ lies in area $Q_i = \cup_{b \in B'_i} f(b)$. They can be divided into two parts R_i and R'_i where R_i is the set of red nodes lying in the interior of area Q_i and R'_i is the set of red nodes lying on the boundary of Q_i. Clearly

$$\sum_{i=1}^{h} |R_i| \leq |R|.$$

Next, we want to establish an upper bound for $\sum_{i=1}^{h} |R'_i|$. Note that each $f(b)$ is a simply connected area with a cycle as its boundary. The boundary of Q_i is obtained from boundaries of $f(b)$'s for $b \in B'_i$ through an operation $\oplus$. For any two cycles C and C', $C \oplus C' = (C \cup C') \setminus (C \cap C')$ is an edge-disjoint union of cycles. Therefore, the boundary of Q_i is an edge-disjoint union of cycles. Let E_i be the number of edges on the boundary of Q_i. Then

$$|R'_i| \leq |E_i|.$$

By the duality relationship of G_r and G_b, it is easy to see that $|E_i$ is also the number of edges between part B'_i and other parts. Therefore,

$$\sum_{i=1}^{h} |R'_i| \leq 2 \cdot (\gamma/\sqrt{k/7})|B'| \leq (2\gamma\sqrt{7}/\sqrt{k})|B|.$$

It follows that

$$|B| = \sum_{i=1}^{h} |B_i| \leq \sum_{i=1}^{h} |N_G(B_i)| \leq \sum_{i=1}^{h} (|R_i| + |R'_i|) \leq |R| + \frac{2\gamma\sqrt{7}}{\sqrt{k}} \cdot |B|.$$

Hence,

$$|B| \leq \frac{1}{1 - \frac{2\gamma\sqrt{7}}{\sqrt{k}}} \cdot |R| \leq (1 + \frac{4\gamma\sqrt{7}}{\sqrt{k}})|R|,$$

when $\frac{2\gamma\sqrt{7}}{\sqrt{k}} \leq 1/2$, i.e., $k \geq 102\gamma^2$. This means that Planar Expansion Theorem holds for choose $K \geq \max(2, 102\gamma^2)$ and $c = 4\gamma\sqrt{7}$. $\square$

2.3 PTAS

In this section, we show that Minimum Sonser Cover has a PTAS.

Theorem 2.3.1 (Mustafa and Ray [54]) *Minimum Sonser Cover has a PTAS.*

To do so, we first show a corollary of Plana Expansion Theorem. Consider an instance of Minimum Sensor Cover, consisting of a set of target points, $\mathcal{A}$, and a set of sensors, $\mathcal{S}$, lying in the Euclidean plane. $\mathcal{S}$ is a sensor cover for $\mathcal{A}$. A sensor cover $\mathcal{S}' \subseteq \mathcal{S}$ for $\mathcal{A}$ is said to be k-tight if for any subset $\mathcal{U} \subseteq \mathcal{S}'$ with $|\mathcal{U}| \leq k$, there is no subset $\mathcal{V} \subset \mathcal{S}$ with $|\mathcal{V}| < |\mathcal{U}|$ such that $(\mathcal{S}' \setminus \mathcal{U}) \cup \mathcal{V}$ is still a sensor cover.

Corollary 2.3.2 *Let K and c be two universal constants in Planar Expansion Theorem. Suppose $k \geq K$. If $\mathcal{S}'$ is a k-tight sensor cover, then*

$$|\mathcal{S}'| \leq (1 + c/\sqrt{k}) \cdot opt,$$

where opt is the size of the minimum sensor cover.

Proof. If there is a sensor s covering all target points, then for $k \geq 2$, $\{s\}$ is a unique k-tight sensor cover and clearly the inequality holds. Next, assume that no sensor covers all target points so that $opt \geq 2$. Let $\mathcal{S}^*$ be a minimum sensor cover. Denote $X = \mathcal{S}' \setminus \mathcal{S}^*$, $Y = \mathcal{S}' \cap \mathcal{S}^*$ and $Z = \mathcal{S}^* \setminus \mathcal{S}'$. Let A be the set of target points not covered by any sensor in Y. Construct a bipartite graph $G = (X, Z, E)$ by connecting $x \in X$ and $z \in Z$ with an edge (x, z) if and only if there exists a target point $a \in A$ covered by both sensors x and z. Then for every subset $U \subseteq X$ with $|U| \leq k$, we must have $|N_G(U)| \geq |U|$ since, otherwise, one has $|N_G(U)| < |U|$ and $(Y \cup (X \setminus U) \cup N_G(U)$ is still a sensor cover, which contradicts assumption that $\mathcal{S}'$ is a k-tight sensor cover.

Can we apply Planar Expansion Theorem to get the following?

$$|X| \leq (1 + c/\sqrt{k}) \cdot |Z|.$$

If yes, then this inequality implies

$$|\mathcal{S}'| \leq (1 + c/\sqrt{k}) \cdot \mathcal{S}^*.$$

However, we cannot because G may not be planar and hence Planar Expansion Theorem can not apply to G. To use Planar Expansion Theorem, we need to find a planar subgraph H of G such that for each target point $a \in A$, H contains an edge whose two endpoints (two sensors) both cover a. With this property, for any $U \subseteq X$, $Y \cup ((X \setminus U) \cup N_H(U))$ is also a sensor cover. Hence, for $U \subseteq X$ with $|U| \leq k$, $|N_H(U)| < |U|$. Then, by Planar Expansion Theorem,

$$|X| \leq (1 + c/\sqrt{k}) \cdot |Z|.$$

Next, we show the existence of subgraph H. The subgraph H is constructed as follows.

Draw a variation of Voronoi diagram for $X \cup Z$ by defining

$$Voro(o) = \{u \mid \|u, o\| - R_s(o) = \min_{v \in X \cup z}(\|u, v\| - R_s(v))\},$$

where $R_s(o)$ is the sensing radius of sensor $o \in X \cup Z$. It is not hard to verify that $Voro(o)$ is a closed convex region. We may still call $Voro(o)$ the Voronoi cell of o. Connect $x \in X$ and $z \in Z$ with an edge (x, z) if and only if Voronoi cell of x and Voronoi cell of z are adjacent.

Clearly, H is a planar subgraph of G. The remaining is to show that H satisfies the required condition that for any target point $a \in A$, H contains an edge with two endpoints both covering a.

For a target point $a \in A$, a must be covered by a sensor x in X and a sensor z in Z. In the Euclidean plane where they lie, let $[x, a]$ and $[a, z]$ be two straight line segments connecting x and a, and a and z, respectively. Note that for any sensor $w \in X \cup Z$, if its Voronoi cell $Voro(w)$ intersects $[x, a]$ or $[a, z]$, then w covers a. In fact, suppose without loss of generality that $Voro(w) \cap [x, a] \neq \emptyset$. Let u be a point in $Voro(w) \cap [x, a]$. Then

$$\begin{aligned}
\|a, w\| - R_s(w) &\leq \|a, u\| + \|u, w\| - R_s(w) \\
&\leq \|a, u\| + \|u, x\| - R_s(x) \\
&= \|a, x\| - R_s(x).
\end{aligned}$$

Since x covers a, one has $\|a, x\| - R_s(x) \leq 0$. Hence $\|a, w\| - R_s(w) \leq 0$. This means that w covers a. covers a. Now, going from x to z along $[x, a]$ and $[a, z]$, one would find a sequence of sensors in $X \cup Z$, $(x = s_1, s_2, ..., s_k = z)$, such that

(a) every s_i for $i = 1, 2, ..., k$ covers a, and

(b) $Voro(s_i)$ and $Voro(s_{i+1})$ are adjacent for $i = 1, 2, k - 1$.

In this sequence, there exists i such that $s_i \in X$ and $s_{i+1} \in Z$. Thus, (s_i, s_{i+1}) is an edge in H, meeting the requirement for target point a. □

The following algorithm with local search finds a k-tight sensor cover.

input a set of target points, $\mathcal{A}$, and a set of sensors, $\mathcal{S}$.
Choose $k \geq K$ such that $c/\sqrt{k} \leq \varepsilon$.
$\mathcal{S}' \leftarrow \mathcal{S}$; **while** $\mathcal{S}'$ is not k-tight **do**
find a subset $\mathcal{U} \subseteq \mathcal{S}'$ with $|\mathcal{U}| \leq k$
and $\mathcal{V} \subseteq \mathcal{S}$ with $|\mathcal{V}| < |\mathcal{U}|$
such that $(\mathcal{S}' \setminus \mathcal{U}) \cup \mathcal{V}$ is a sensor cover,
and set $\mathcal{S}' \leftarrow (\mathcal{S}' \setminus \mathcal{U}) \cup \mathcal{V}$; **output** $\mathcal{S}'$.

Let $n = |\mathcal{S}|$. Since the "while" loop runs at most n time and each iteration takes at most $n^k \cdot n^{k-1}$ time, the running time of this algorithm is n^{2k}.

By Corollary 2.3.2, this algorithm gives an approximation with performance ratio $1 + c/\sqrt{k}$ for $k \geq K$ and hence gives a PTAS for Minimum Sensor Cover.

2.4 $O(1)$-Approximation

2.5 ε-Net

Chapter 3

Connected Sensor Cover

I realize love isn't about sex. It's about connection.
CHUCK PALAHNIUK

3.1 Motivation and Overview

The minimum connected sensor cover problem (Problem 1.3.2) was first proposed by Gupta, Das and Gu [42]. They presented a greedy algorithm with performance ratio $O(r \ln n)$ where n is the number of sensors and r is the *link radius* of the sensor network, i.e., for any two sensors s and s' with a sensing point in common, there exists a path between s and s' with hop distance at most r in communication network. Zhang and Hou [69] studied the minimum connected sensor cover problem in homogeneous wireless sensor networks with property that $R_c \geq 2R_s$. They showed that in this case, the coverage of a connected target area implies the connectivity. This result was generalized by Zhou, Das and Gupta [70] to the m-connectivity that if every point in a connected target area is covered by at least m sensors, then those sensors induce an m-connected sensor network. Xing *et al.* [65] presented a coverage configuration protocol which can give different degree of coverage requested by applications. Bai *et al.* [2] studied a sensor deployment problem regarding the coverage and connectivity. Alam and Haas [1] studied this problem in three-dimensional sensor networks.

Funke *et al.* [36] improved approximation algorithms for the minimum connected sensor cover problem by allowing sensos to vary their sensing radius. With variable sensing radius and commication radius, Zhou, Das, and

Gupta [71] designed a polynomial-time approximation with performance ratio $O(\log n)$. Chosh and Das [40] designed a greedy approximation using maximal independent set and Voronoi diagram. They determined the size of connected sensor cover produced by their algorithm. However, no comparison with optimal solution, that is, no analysis on approximation performance ratio is given.

For homogeneous wireless sensor networks, Wu *et al.*[64] improved the ratio $O(r \ln n)$ of Gupta, Das and Gu [42] by presenting two approximation algorithms with performance ratios $O(r)$ and $O(\log^3 n)$, respectively. They also made following conjecture.

Conjecture 3.1.1 (Wu *et al.*[64]) *There exists a polynomial-time constant-approximation for the minimum connectes sensor cover problem.*

3.2 NP-hardness

In this section, we show the NP-hardness of the minimum connected sensor cover problem.

Theorem 3.2.1 *The minimum connected sensor cover problem is NP-hard.*

Proof. The following NP-complete problem can be found in [**?**, **?**].

Problem 3.2.2 (Planar-4-CVC) *Given a planar graph $G = (V, E)$ with node degree at most 4 and a positive integer $k \leq |V|$, determine whether there is a* connected vertex cover *of size k, i.e., a subset $V' \subseteq V$ with $|V'| = k$ such that for each edge $\{u, v\} \in E$ at least one of u and v belongs to V' and the subgraph induced by V' is connected.*

We are going to construct a polynomial-time many-one reduction from this problem to the following decision version of the minimum connected sensor cover problem.

Problem 3.2.3 (Connected Sensor Cover) *Given a set of target points, $\mathcal{A}$, a set of homogeneous sensors, $\mathcal{S}$ on the Euclidean plane, and a integer $k > 0$, is there a sensor cover of size at most k?*

Consider an instance of the planar-4-CVC problem, consisting of a graph $G = (V, E)$ with vertex degree at most 4 and a positive integer k. First, we construct a new graph as follows. Note that we can embed G into the plane

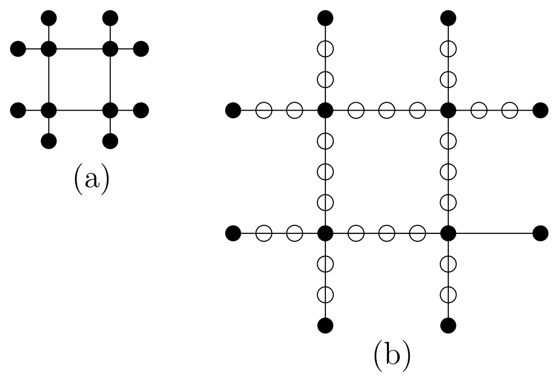

Figure 3.1: (a) A planar graph G. (b) The constructed new graph G'. The dark circled points are vertices of G and the light circled points are new vertices.

so that all edges consist of horizontal and vertical segments of lengths being an integer at least 4, so that every two edges meet at an angle of 90^o or 180^o. Add new vertices on the interior of each edge in G to divide the edge into a path of many edges, each of length exactly one. Denote by W the set of all such new vertices. (See Fig. 3.1. New vertices are light circled points.) At each vertex of G', put a sensor with sensing radius one and communication radius one.

Now, for each sensor w at a location in W, put a target w' as shown in Fig. 3.2 such that w' can be covered by only sensor w. Then every connected

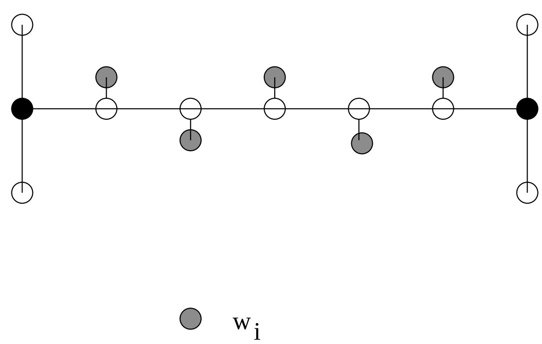

Figure 3.2: Add w_i''s.

sensor cover must contain W. Now, it is easy to see the following facts:

(1) W is a sensor cover.

(2) C is a connected vertex cover of G if and only if $C \cup W$ is a connected sensor cover.

Therefore, G has a connected vertex-cover of size at most k if and only if there exists a connected sensor cover of size at most $|W| + k$. □

3.3 $O(r)$-approximation

In this section, we present a polynomial-time $O(r)$-approximation for the connected sensor cover problem given by Wu *et al.* [64]. This approximation is constructed by using an approximation algorithm for the network Steiner tree problem as follows.

Problem 3.3.1 (Network Steiner Tree) *Given a graph $G = (V, E)$ with edge weight $c : E \to \Re$, and a node subset $P \subseteq V$, find a minimum total edge-weight tree interconnecting all nodes in P.*

The network Steiner tree is a classic combinatorial optimization problem. Currently, the best known polynomial-time approximation algorithm has the performance ratio 1.39 [12] which will be introduced in next section. The $O(r)$-approximation designed by Wu *et al.* [64] for the minimum connected sensor cover problem is as follows.

Step 1. Let $\mathcal{A}$ be the target set and $\mathcal{S}$ the sensor set in an input of the minimum connected sensor cover problem. Compute a $(1+\varepsilon)$-approximation $\mathcal{S}'$ for the minimum sensor cover problem on input target set $\mathcal{A}$ and input sensor set $\mathcal{S}$.

Step 2. Consider the communication network G on $\mathcal{S}$. Assign each edge with weight one. Compute a 1.39-approximation T for the network Steiner tree problem on $\mathcal{S}'$ as terminal set. Output the vertex set $\mathcal{S}_T$ of T.

Theorem 3.3.2 (Wu *et al.* **[64])** *$\mathcal{S}_T$ is a polynomial-time $O(r)$-approximation for the minimum connected sensor cover problem where r is the link radius of the sensor network, that is, for any two sensors s and s' with sensing areas intersecting, there exists a path between s and s' with hop distance at most r.*

Proof. Note that the cost of T is $|\mathcal{S}_T| - 1$. Suppose Opt_{csc} is an optimal solution for MIN-CSENC. Let $\mathcal{S}' \subseteq \mathcal{S}$ be a $(1+\varepsilon)$-approximation for the minimum sensor cover problem on input target set $\mathcal{A}$ and sensor set $\mathcal{S}$. For each sensor $s \in \mathcal{S}'$, find a sensor $s' \in Opt_{csc}$ such that $disk_{R_s}(s) \cap disk_{R_s}(s') \neq \emptyset$ and then connect s and s' by a shortest path. Then we obtain a tree with total cost at most

$$|Opt_{csc}| - 1 + |\mathcal{S}'|r \leq |\mathcal{S}'|(r+1) - 1 \leq (4+\varepsilon)|Opt_{csc}|(r+1) - 1.$$

Therefore,

$$|\mathcal{S}_T| - 1 \leq 1.39 \cdot ((1+\varepsilon)|Opt_{csc}|(r+1) - 1).$$

Hence

$$|\mathcal{S}_T| \leq 1.39(r+1)(1+\varepsilon) \cdot |Opt_{csc}|.$$

□

For homogeneous wireless networks with property that $R_c \geq 2R_s$, $r = 1$. Therefore, the minimum connected sensor cover problem has a polynomial-time constant-approximation. From the proof of Theorem 3.3.2, this constant could be $1.39(2 + 2\varepsilon)$.

Corollary 3.3.3 *When $R_c \geq 2R_s$, the minimum connected sensor cover problem has a polynomial-time $(2.78 + \varepsilon)$-approximation for any $\varepsilon > 0$.*

3.4 Network Steiner Tree

One of classical Steiner tree problem is the network Steiner tree problem. There are many publications in the literature on network Steiner trees [**?**]. In this section, we would like to introduce only the 1.39-approximation algorithm designed by Byrka *et al.* [12]. This is currently the best-known approximation for the network Steiner tree problem.

3.5 $O(\log^2 n \log m)$-Approximation

Suppose that in the minimum connected sensor cover problem, input consists of a set of n sensors, $\mathcal{S}$ and a set of m target points, $\mathcal{A}$ on the Euclidean plane. Then it can be easily reduced to the group Steiner tree problem defined as follows.

Problem 3.5.1 (Group Steiner Tree) *Consider a graph $G = (V, E)$ with positive edge length $w : E \to R_+$. Given m subsets of vertices, $g_1, ..., g_m$, find the shortest tree on a vertex subset which hits every g_i, $i = 1, ..., m$, where by hit, we mean that the intersection of two sets is not empty.*

To see the reduction, set $V = \mathcal{S}$, $w(s, s') = 1$ for every edge (s, s') in the communication network G of all sensors, and for each target point a_i, set a group g_i consisting of all sensors each covering a_i. Since every edge has length one, the total edge-length of a tree on a vertex subset $\mathcal{S}'$ is equal to $|\mathcal{S}'| - 1$, the objective function value of the minimum connected sensor cover problem minus one. Note that for positive numbers $a > b > 1$, we have inequality $a/b \leq (a-1)/(b-1)$. It follows that the following lemma holds.

Lemma 3.5.2 *If the group Steiner tree problem has a polynomial-time ρ-approxoimation, then so does the minimum connected sensor cover problem.*

Garg, Konjevod and Ravi [37] showed that for any $\varepsilon > 0$, there exists a polynomial-time algorithm which can produce an approximation solution for the group Steiner tree problem such that with probability $1 - \varepsilon$, the solution is $O(\log^2 n \log\log n \log m)$-approximation where n is the number of vertices and m is the number of groups in input. One of computation step in this algorithm has been improved by Fakcharoenphol *et al.* [34] from running time $O(\log n \log\log n)$ to $O(\log n)$. Therefore, we now have the following.

Lemma 3.5.3 *For any $0 < \varepsilon < 1$, there exists a polynomial-time approximation algorithm for the group Steiner tree problem, which with probability $1 - \varepsilon$, produces a $O(\log^2 n \log m)$-approximation.*

Therefore, we have

Theorem 3.5.4 *For any $0 < \varepsilon < 1$, there exists a polynomial-time approximation for the minimum connected sensor cover problem such that with probability $1 - varepsilon$, this algorithm produces a $O(\log^2 n \log m)$-aproximation solution.*

3.6 Metric Approximation

In this section, we introduce a tool to design rondom approximation algorithm for combinatorial optimization problems, such as the group Steiner tree.

3.7 Group Steiner Tree

3.8 $O(R_s/R_c)^2$)-Approximation

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