Verification of Magnetic Dipole Moment of ION1 Torque Coils Backed with Aluminum

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Introduction

A previous experiment has verified that the ION1 torque coil magnetic moment is within 4.9% of the theoretical value. However, as the ION2 torque coils will be mounted to a panel containing Aluminum, the effects of this need to be explored. The setup in this experiment is almost identical to the one presented in Andy Pukniel's "Verification of the Magnetic Dipole Moment of ION's Torque Coils" from the Spring of 2008 semester [1]. Please refer to that document for a detailed explanation of the theory involved in this experiment. The specific additions to the experimental setup, the necessary equations, and the final results comparing the theoretical to the experimental values for the magnetic moment are discussed.

Background

The following is taken from Reference 1. The experiment, outlined in greater detail in a later section, consists of a torque coil that is freely suspended from a fishing line and is immersed in an externally-generated magnetic field. The coil produces a magnetic dipole that is always perpendicular to both the string and the coil face, and whose direction is determined according to the right-hand rule. When the coil is immersed in an external magnetic field, **B,** the torque generated by the coil tends to align the magnetic dipole moment with the field. This can be seen directly from Equation 1. Note that there is no torque generated if the magnetic dipole moment is already aligned with the magnetic field as in the right-most diagram.

$$\mathbf{T}_B = \mathbf{m} \times \mathbf{B} \qquad (1)$$

The Earth's magnetic field in almost all geodetic locations has all three of its components (in the ECF frame) be nonzero. As a result, if the coil's magnetic dipole moment is not aligned with the Earth's field, the torque exerted on the coil will have all three components be nonzero as well. However, for the coils considered in this experiment, the torque generated by the magnetic moment is insufficient to overcome the torque applied by the gravitational force. This forces the magnetic torque to manifest itself only as the rotation around the string (yaw motion) and allows us to ignore the two 'out-of plane' components of the external magnetic field. In other words, only the 'horizontal' (in the plane of rotation) component can be considered. Since the cross product in Equation 1 vanishes when **m** and **B** align, it is a natural choice to define this as the equilibrium position. Any perturbation from the equilibrium state will induce a restoring torque and form a plane oscillator. The period of oscillation will depend on the strength of the magnetic dipole moment and the strength of the external magnetic field. Therefore, by varying the external field and measuring the frequency, it is possible to calculate the magnetic moment of the coils. The derivation and details follow in the next section [1].

There is concern that the presence of sheet Aluminum will have an effect on this process. As Aluminum is a metal, a current will be induced in it, which may in turn affect the magnetic field strength. Furthermore, Aluminum is paramagnetic material. This means that it will exhibit weak magnetic behavior in the presence of a magnetic field [2]. In addition, some Aluminum alloys contain small amounts of ferromagnetic elements which will also create a magnetic field [3]. The result of these effects needs to be experimentally determined.

Theory

A detailed derivation of the equations used in this experiment is given in Reference 1. The frequency of the oscillating system is as follows.

$$f = \frac{1}{2\pi}\sqrt{\frac{mB}{I}} \qquad (2)$$

The magnetic field present in the test section of the Helmholtz coils is a follows.

$$B = B_{Earth} + B_A \quad (3)$$

Where B is the total magnetic field strength, B_{Earth} is the horizontal magnitude of the Earth's magnetic field. The governing equation for this system is

$$f^2 = \frac{m\mu_0 N}{4\pi^2 IR}\left(\frac{4}{5}\right)^{\frac{3}{2}} i + \frac{mB_{earth}}{4\pi^2 I} \qquad (4)$$

where f is the frequency of the torque coil oscillation, m is the magnitude of the magnetic moment, μ_0 is the permeability constant of 1.26 x10^{-6} Tm/A, i is the current through the Helmholtz coils, N is the number of turns of the Helmholtz coils, R is the radius of the Helmholtz coils, I is the moment of inertia of the torque coils and B_{earth} is the magnetic field strength of the Earth in the horizontal direction. It can then be seen that the square of the frequency of oscillation of the torque coils is directly related to magnetic moment. This allows the moment to be derived from known constants and experimentally measured frequency of oscillation [1]. This is, by measuring the frequency of oscillation for various values of current in the Helmholtz coils, one my fit a curve to find a numerical value of the slope of that relationship, and solve for the magnetic dipole moment given in the above equation.

Calculation of the Theoretical Magnetic Dipole Moment

The following section is taken directly from Reference 1. The magnetic dipole moment induced by a current flowing through a rectangular loop is calculated according to the following equation:

$$\mathbf{m} = NAi\mathbf{a}_n \quad (5)$$

where N is the number of loops, A is the cross-sectional area, i is the current flowing through the wire, and $\mathbf{a}_n$ is the direction normal to the coil and is found according to the right-hand-rule. ION's magnetic torque coils are based on a typical electronics board design where 0.007in in diameter traces are etched onto a standard printed circuit board in a wind-down pattern. It is possible to place consecutive traces 0.007in apart on a single layer and to combine multiple layers to form a single torque coil. At present time, the design uses 30 loops on each of 4 layers for a combined total of 120 loops.

In order to accurately calculate the magnetic dipole moment, the varying sizes of consecutive loops must be taken into account. Using Equation 5 and given the dimensions of the most outer loop as a_{outer} and b_{outer} as well the thickness and separation of the traces, which in this case are equal, as w, it is possible to write the magnetic dipole moment as:

$$\mathbf{m} = 4 \cdot i \cdot \sum_{N=0}^{29} \left[\left(a_{outer} - 4 \cdot N \cdot w \right) \left(b_{outer} - 4 \cdot N \cdot w \right) \right] \mathbf{a}_n \quad (6)$$

Appendix A includes a short code that is used to compute the magnetic dipole moment for a manufactured coil with 120 turns, a_{outer} and b_{outer} as 58.42 mm and 43.18mm respectively [1].

Experimental Setup

There are several pieces of equipment involved in this experiment. The Helmholtz coils were constructed specifically for this series of experiments. Twenty turns of 18 gauge magnet wire were wrapped around each end of a 12.75 inch OD, 12 inch long, PVC pipe. Four rectangular holes were cut at ninety degrees from each other to create access to the test section. The torque coils used in the experiment were originally constructed for ION1. Figure 1 below shows their final configuration where they are powered by solar cell. This was done to reduce external forces on the torque coils due to wiring.

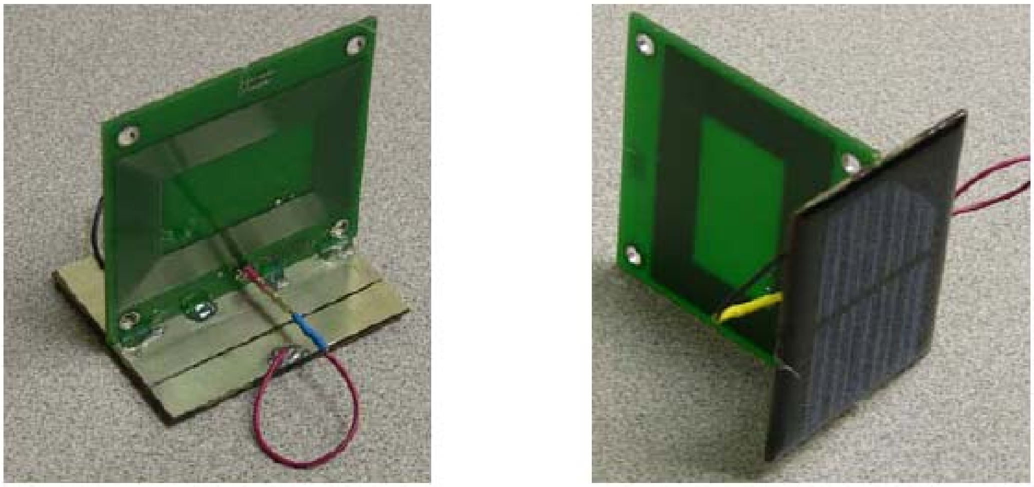

Figure 1: Torque Coils with Attached Solar Cell

For this experiment, the torque coils were also tested with backing of Aluminum 1100. This backing was 50.73 mm by 63.46 mm by 0.2 mm. Its mass was 1.8 grams. It was lightly taped to the face of the torque coil.

The experimental setup used here is based entirely off of the setup as documented in Reference 1. A description of the setup used will be included along with details of changes to setup. A figure of the setup is displayed below.

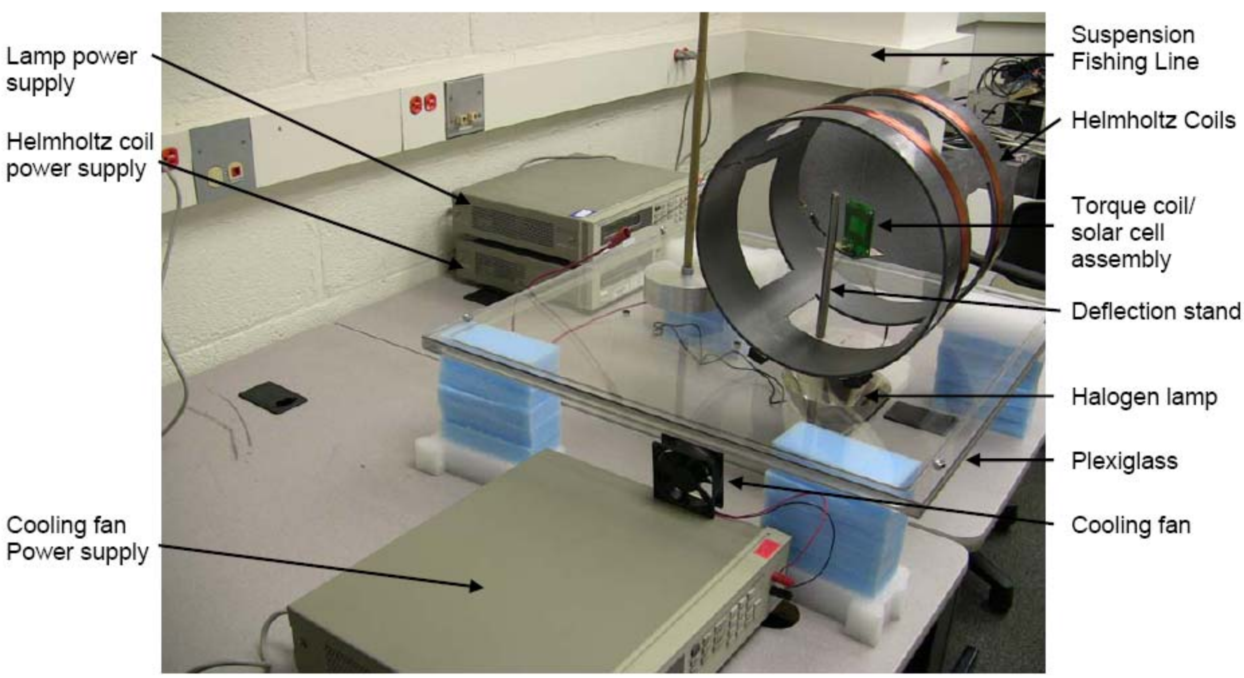

Figure 2: the Experimental Setup

As seen, the torque coil is suspended from a stand in the center of the Helmholtz coil. This is supported by a clear, Plexiglas panel which is elevated by foam blocks. This allows for the placement of the Halogen lamp which powers the solar cell and torque coil assembly. The decision was made to power the torque coils via solar cell because any wires fed to the torque coils caused large external torques on the coils preventing them from oscillating freely. As shown, a small cooling fan is included to dissipate the large amount of heat produced by the Halogen lamp. Two power supplies are included to power both the Helmholtz coils and the Halogen lamp. Table 1 below summarizes the settings for the power supplies.

Table 1: Power Settings for Equipment

Item	Voltage [V]	Current [A]
Halogen lamp	12	4.5
Helmholtz coils	Variable	2-5

In order to ensure the torque coil is displaced by a constant, small angle, marks were made on the Plexiglas panel that corresponds to the center line of the Helmholtz coils and 10^o from center. The deflection stand is then placed on the 10^o displacement mark so that the torque coil may rest against this stand and be displaced by that angle. This allows for repeatable displacement throughout the experiment as well as preservation of the small angle approximations used in the derivation of the governing equations.

The experiment is performed by measuring the period of the oscillating torque coils at various current levels for the Helmholtz coils. One starts by setting the displacement stand on the 10^o mark and resting the torque coil against it. The torque coil may oscillate slightly, it is best to wait for that motion to stop before gently removing the displacement stand. The coil will oscillate for several cycles before external torques, such as from tension in the suspension line, will cause the coil to stop. These oscillations are timed and recorded. It is best to perform several trials to ensure accurate results. This process is repeated at several intervals from 2 to 5 amps through the Helmholtz coils. The torque coils have a more pronounced oscillation on the upper end of that scale, so it is better to sample a greater number of points towards 5 amps.

Experimental Results

The experiment was performed by measuring the period of oscillation for varying currents through the Helmholtz coils from 2.0 to 5.0 Amps. The lower range of this current was determined to produce a magnetic field strength roughly ten times that of the local horizontal strength of the Earth's magnetic field. This will cause any effects the Earth's field or any other residual fields may have to be negligible. The upper range is limited by the power supplies used.

The average experimental data for the oscillating torque coil without the Aluminum backing is given below in Table 2.

Table 2: Results for Torque Coils Without Aluminum

Current [A]	Avg. Period [s]	Avg. f^2 [1/s^2]
2.5	11.59	0.0074
3.0	10.97	0.0083
3.5	10.38	0.0093
4.0	9.73	0.0106
4.5	9.26	0.0117
5.0	9.15	0.0119

Next, the averaged experimental data for the torque coils with the Aluminum backing is given below in Table 3.

Table 3: Results for Torque Coils With Aluminum

Current [A]	Avg. Period [s]	Avg. f^2 [1/s^2]
2.0	13.86	0.0052
3.0	12.06	0.0069
4.0	10.35	0.0093
4.5	9.61	0.0108
5.0	9.48	0.0111

Now, these data were used in MATLAB to create a linear curve fit. Figure 3 below displays the curve fit.

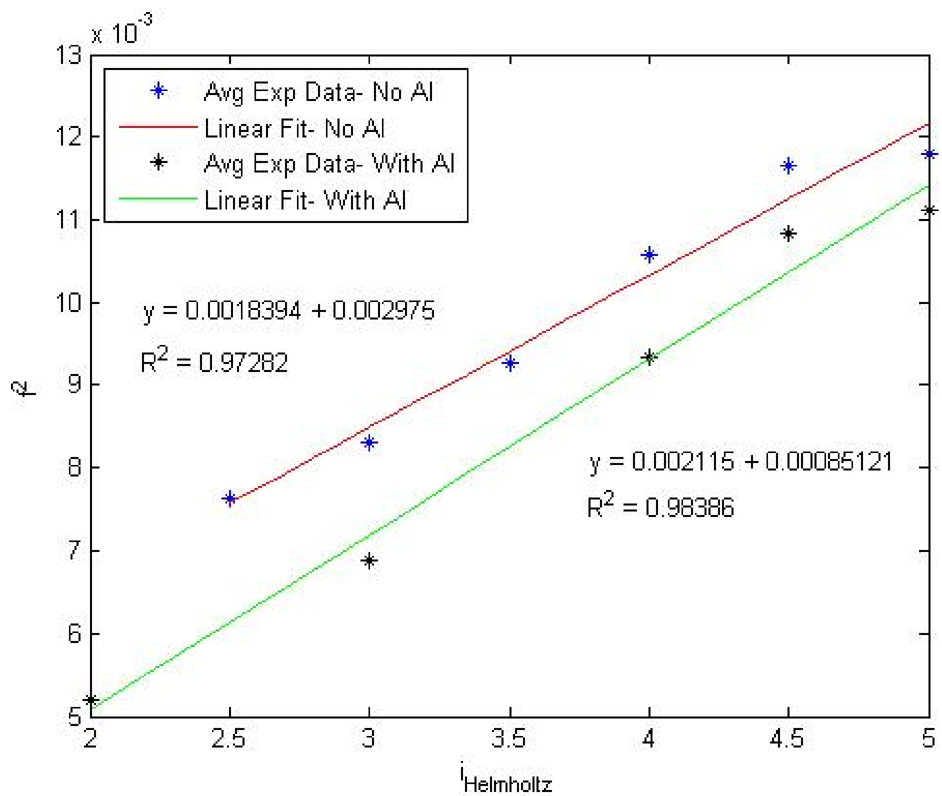

Figure 3: Plot of Data and Linear Fit

The current generate by the solar cell was also measured using a multi-meter connected in series with the torque coil. It measured 44.7 mA. Using these results, the experimental dipole moment was compared to the theoretical dipole moment. The table below shows those results.

Table 4: Theoretical and Experimental Dipole Moment Strenghts

	Theoretical Dipole Moment [A*m^2]	Experimental Dipole Moment [A*m^2]	Percent Error
No Aluminum	0.00869	0.00818	-5.87%
With Aluminum	0.00869	0.00951	9.53%

Appendix A and Appendix B contain the Matlab code used to calculate the experimental and theoretical dipole moment values, respectively.

Conclusions

It is clear that by adding a backing of Aluminum, the magnetic dipole strength is increased by a non-negligible amount. These results also verify the accuracy of the theoretical dipole moment for the torque coils without Aluminum.

There have been several attempts made to try to quantify the impact that the Aluminum has on the dipole moment. Aluminum is a paramagnetic element, meaning that in the presence of a magnetic field it will add energy to that field [2]. However, after examining the relative permeability of Aluminum to free space, it was determined to be 1.0000222 [4]. This means that the paramagnetic effects are quite small and not responsible for an almost 10% increase in magnetic dipole moment.

It is probable that the Aluminum is responding to the electric field produced by the moving current in the torque coils and the Helmholtz coils. However, as the Aluminum is oscillating in the electric field of the Helmholtz coils which is dependant of the current through the Helmholtz coils. This effect has proven difficult to quantify.

Ultimately, it is known that the Aluminum 1100 specimen marginally increases the magnetic dipole moment of the torque coils. This means that they would need to be calibrated on ION2 to ensure the appropriate moment is produced for a given amount of power applied. It would be recommended to take a direct measurement of magnetic field strength to accurately calibrate the torque coils.

Furthermore, the results have shown that by adjusting the material in the vicinity of the torque coils, the dipole moment may be increased for a fixed amount of power. Should ION2 require greater torque, it would be possible to accomplish this by placing an amount of ferrous material near the torque coils.

References

[1] Pukniel, Andy. “Verification of the Magnetic Dipole Moment of ION’s Torque Coils.” Spring, 2008.

[2] “Paramagnetism.” http://en.wikipedia.org/wiki/Paramagnetism (Cited April 1, 2008).

[3] “Chemical Composition Limits- Aluminum Alloys.” http://www.luskmetals.com/chemalum.html (Cited April 1, 2008).

[4] “Magnetic Permeability.” http://en.wikipedia.org/wiki/Magnetic_permeability (Cited April 28, 2008).

Appendix A: Matlab code used to calculate theoretical dipole moment

```
function [m_theor] = m_theor_calc(current)
% Dimensions of the coil
a1=.04318; %meters
b1=.05842; %meters
% Trace thickness
x=.000711; %meters
A_tot=0;
for i=1:30
A(i)=(a1-x*(i-1))*(b1-x*(i-1));
A_tot=A_tot+A(i);
end
%total magnetic dipole moment
m_theor=4*current*A_tot; % 4layers for each coil
```

Appendix B: Matlab Code used to calculate experimental dipole moment

```
%John Warner March 6th, 2008
%This program was originally written by Andy Pukniel and was slightly
%modified to use the data collect in the Aluminum torque coil test.
clear all; close all; clc;
format long
% List of currents sampled NO ALUMINUM DATA
I.current=[2.5,3.0,3.5,4.0,4.5,5.0];
% Measured period at each frequency
I1.time_raw=[11.5, 11.2, 11.0 ,11.9 ,10.7 ,11.43 ,10.95 ,12.2, 11.8 ,11.0,
...
    13.0, 11.81, 11.57, 12.06, 11.67];
I2.time_raw=[9.76, 11.12, 10.63, 10.36, 11.3, 11.3, 11.3, 10.62, 11.88, ...
    10.64, 11.28, 10.7, 11.2, 11.5];
I3.time_raw=[10.74, 10.26, 10.10, 10.42, 10.4, 10.0, 11.0, 10.5, 10.21, ...
    10.3, 10.7, 10.5, 10.1, 10.62, 10.23, 10.10, 10.29];
I4.time_raw=[9.2, 9.3, 9.8, 9.9, 8.88, 9.70, 9.53, 9.54, 10.1, 10.2, ...
    9.6, 10.5, 10.36, 9.77, 10.10, 10.07, 9.27, 9.50, 9.40, 9.57, ...
    9.52, 9.99, 9.50, 9.91, 9.98];
I5.time_raw=[9.13, 9.41, 9.18, 9.24, 9.04, 8.67, 8.97, 9.81, 9.70, ...
    9.14, 9.42, 9.22, 9.99, 8.94, 9.2, 9.7, 9.6, 8.9, 9.4, 9.16, ...
    9.39, 9.40, 8.43, 9.17, 9.22, 9.08, 9.42, 9.41];
I6.time_raw=[8.99, 8.87, 9.56, 8.66, 9.31, 8.6, 8.6, 9.5, 8.9, ...
    9.3, 9.7, 9.3, 9.47, 9.48, 9.02, 9.2];

% Average the measurments at each current
I1.time=mean(I1.time_raw);
I2.time=mean(I2.time_raw);
I3.time=mean(I3.time_raw);
I4.time=mean(I4.time_raw);
I5.time=mean(I5.time_raw);
I6.time=mean(I6.time_raw);
%Construct vector of times
I.time=[I1.time,I2.time,I3.time,I4.time,I5.time,I6.time];
%convert period to frequency
for i=1:length(I.time)
```

```
I.f(i)=1/I.time(i);
end
%Apply a linear fit to the averaged data. (fit_result=p1*x+p2)
[fit_result,gof]=fit(I.current',I.f.^2','poly1');
%fit statistics
r_square=gof.rsquare;
rmse=gof.rmse;
%visual verification
plot(I.current,I.f.^2,'b*');
hold on
x=[min(I.current):.1:max(I.current)];
y=fit_result.p1*x+fit_result.p2;
plot(x,y,'r-');
xlabel('i_H_e_l_m_h_o_l_t_z');ylabel('f^2');

text(mean(I.current)+.2,mean(I.f.^2),['y = ',num2str(fit_result.p1),' +
',num2str(fit_result.p2)])
text(mean(I.current)+.2,mean(I.f.^2)-.0005,['R^2 = ',num2str(r_square)]);
%computing the (experimentally obtained) magnetic dipole moment
%recall that f^2=m*mu*N*i/(4*pi^2*I*R)*(4/5)^(3/2)+m*Bearth/(4*pi^2*I) where
%I is the moment of inertia of the tested torque coil and i, R, and N are
%current, radius, and number of turns (in each coil) in the Helmholtz coils
respectively.
%mu is the permeability constant.
mu=1.2566371e-6; % [m*T/A] = [N/A^2]
N=20;
R=0.32385/2; %[m] 12.75 in diameter PVC pipe;
Iyy=(11.923599)/(1000^2); %kg*m^2 Second term is the aluminum plate
i_torque_coil=0.0447; % measured torque coil current in A
m_exp=fit_result.p1*4*pi^2*Iyy*R/(mu*N)*(4/5)^(-3/2)
m_theoretical=m_theor_calc(i_torque_coil) %Am^2
percent_diff=abs(m_theoretical-m_exp)/m_theoretical*100
%
%  Below is the data WITH ALUMINUM
%
% Currents sampled
I.current=[2.0,3.0,4.0,4.5,5.0];
% Data sampled at each current
I1.time_raw=[13.6, 13.84, 14.40, 13.3, 13.2, 14.5, 13.9, 14.1, 13.65, ...
    14.54, 13.41];
I2.time_raw=[12.26, 12.44, 11.85, 11.57, 12.1, 12.1, 12.1, 12.5, ...
    11.2, 12.26, 12.4, 11.4, 12.2, 12.54, 11.92];
I3.time_raw=[10.4, 10.3, 10.9, 10.7, 10.07, 10.99, 11.07, ...
    10.43, 10.88, 9.81, 10.42, 10.03, 9.7, 10.1, 10.0, 10.3, ...
    10.59, 10.64, 9.98, 10.28, 10.50, 10.2, 10.3, 9.9, 10.3];
I4.time_raw=[9.1, 9.5, 9.59, 9.63, 9.7, 9.22, 9.99, 9.75, 9.98, 9.60, 9.62];
I5.time_raw=[9.66, 9.46, 9.88, 9.60, 9.21, 9.4, 9.6, 9.7, 9.3, 9.36, ...
    9.40, 9.41, 9.26, 9.5, 9.7, 9.4, 9.4, 9.4];

% Average of samples
I1.time=mean(I1.time_raw);
I2.time=mean(I2.time_raw);
I3.time=mean(I3.time_raw);
I4.time=mean(I4.time_raw);
I5.time=mean(I5.time_raw);
```

```
% Vector of average periods
I.time=[I1.time,I2.time,I3.time,I4.time,I5.time];
%convert period to frequency
I.f=zeros(1,length(I.time));
for i=1:length(I.time)
I.f(i)=1/I.time(i);
end

%Apply a linear fit to the averaged data. (fit_result=p1*x+p2)
[fit_result,gof]=fit(I.current',I.f.^2','poly1');
%fit statistics
r_square=gof.rsquare;
rmse=gof.rmse;
%visual verification
plot(I.current,I.f.^2,'k*');
hold on
x=[min(I.current):.1:max(I.current)];
y=fit_result.p1*x+fit_result.p2;
plot(x,y,'g-');
xlabel('i_H_e_l_m_h_o_l_t_z');ylabel('f^2');
legend('Avg Exp Data- No Al','Linear Fit- No Al','Avg Exp Data- With Al',
'Linear Fit- With Al',2);
text(mean(I.current)+.2,mean(I.f.^2),['y = ',num2str(fit_result.p1),' +
',num2str(fit_result.p2)])
text(mean(I.current)+.2,mean(I.f.^2)-.0005,['R^2 = ',num2str(r_square)]);
%computing the (experimentally obtained) magnetic dipole moment
%recall that f^2=m*mu*N*i/(4*pi^2*I*R)*(4/5)^(3/2)+m*Bearth/(4*pi^2*I) where
%I is the moment of inertia of the tested torque coil and i, R, and N are
%current, radius, and number of turns (in each coil) in the Helmholtz coils
respectively.
%mu is the permeability constant.
mu=1.2566371e-6; % [m*T/A] = [N/A^2]
N=20;
R=0.32385/2; %[m] 12.75 in diameter PVC pipe;
Iyy=(11.923599+ 0.7301932)/(1000^2); %kg*m^2 Second term is the aluminum
plate
i_torque_coil=0.0447; % measured torque coil current in A
m_exp=fit_result.p1*4*pi^2*Iyy*R/(mu*N)*(4/5)^(-3/2)
m_theoretical=m_theor_calc(i_torque_coil) %Am^2
percent_diff=abs(m_theoretical-m_exp)/m_theoretical*100
```