Fairness and Failures of Coordinating Contracts: on the optimality of pooling

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Experimental studies of coordinating contracts find that these contracts often fail to coordinate channels. One important cause of this failure is that responders reject a significant proportion of offers. These rejections may be due to information asymmetry between proposers and responders about the responders’ preferences. Our study characterizes the supplier’s optimal contract in a setting with inequality aversion preferences. We prove that the optimal contract is (almost surely) pooling, calling for a “shut down” of the most inequity-averse types whenever the types’ density function is continuous, and derive the optimal rejection rate. In terms of managerial insights and testable predictions, we demonstrate that the equivalence of coordinating contracts can break down in this environment – the optimal contract can be implemented with a minimum-order-quantity contract but not with a two-part tariff.

Key words: behavioral operations; mechanism design; fairness; supply chain coordination

1. Introduction

In today’s world, “... supply chain competes with supply chain and the success of any one company will depend upon how well it manages its supply chain relationships.” (Christopher 2010). The main challenge facing supply chains is that decentralized decision-making by self-interested firms may result in poor overall performance. In an early paper, Spengler (1950) shows how double marginalization arising under wholesale pricing results in inefficient outcomes. To overcome the issue, several other types of contracts, so called coordinating, have been proposed: two-part tariff (TPT), quantity discount (QD), buyback (BB), revenue-sharing (RS), minimum-order quantity (MOQ), etc. (Jeuland and Shugan 1983, Moorthy 1987, Pasternack 1985, Cachon and Lariviere 2005, Cachon 2003). Such contracts are sufficiently flexible and, according to theory, can be designed to align incentives across the supply chain and ensure the optimal overall performance.
However, those coordinating contracts tested using controlled laboratory experiments with human subjects, did not perform as well as theory predicts. Lim and Ho (2007), Ho and Zhang (2008) and Katok and Wu (2009) test the performance of two-part-tariff (TPT), quantity discount (QD), buyback (BB), and revenue sharing (RS) contracts, and find that these theoretically coordinating contracts bring only marginal efficiency improvement over wholesale pricing. These studies identify several behavioral factors that affect the contract performance, but a common finding is that a significant proportion of contracts proposed by suppliers is rejected by retailers. For example, in Lim and Ho (2007) experiment retailers rejected 11% of the two-block contracts and 15% of the three-block contracts proposed by the suppliers. Even higher rejection rates, 26% for the TPT contracts and 18% for QD contracts, have been reported in the Ho and Zhang (2008) experiment. At the same time, efficiency of the accepted contracts in these studies was quite high, at around 94%. That is, contract rejections are a primary reason of the poor performance in those studies.

Experimental economics has a long and influential literature dealing with other-regarding preferences, going back to the first experiment on the \textit{Ultimatum Game} (Guth et al. 1982). In the ultimatum game, the proposer offers a division of a fixed amount of money between herself and the recipient. If the recipient agrees, both players make corresponding earnings, and otherwise both earn nothing. In contrast to the subgame perfect equilibrium solution that dictates that the proposer should end up with most of the pie, and the responder should not reject any positive offer, ultimatum game results feature high rejection rates for small offers, and generally equitable splits. Forsythe et al. (1994) suggest that in the ultimatum game rejections may occur due to the fact that the proposer does not know the recipient’s preferences for fairness. In other words, rejections happen due to incomplete information regarding fairness preferences (Bolton and Ockenfels 2000) when proposers offer a small share of the total pie to responders who happen to be strongly concerned with receiving their fare share. However, rejections may also happen due to “noisy decisions”, a form of bounded rationality captured by random choice models (Su 2008). The responder accepts or rejects an offer at random but better offers are more likely to be accepted. To separate the influence of different factors causing rejections Katok and Pavlov (2013) conduct an experiment in which they manipulate the responders’ degrees of bounded rationality, strength of preferences for fairness, and private information about fairness. They find that the single most influential cause of rejections, accountable for about 2/3 of the rejection rate, is information asymmetry regarding the strength of fairness preferences.

It is well understood that, in general, any relevant information privately held by some agent(s) in a supply chain tends to be detrimental to overall performance. Therefore, in supply chain literature, a number of studies derive optimal contracting mechanisms when some relevant information such as production cost or demand forecast is private (Ha 2001, Cachon and Lariviere 2001, Corbett et al.
and investigate the performance of supply chain contracts under information asymmetry (Kalkanci et al. 2011, 2014). However, the structure of the optimal contract when fairness concerns are private information has not been established yet. Even though behavioral operations literature is growing fast (Bendoly et al. 2006, 2010, Katok 2011) most behavioral models in this literature assume full information about behavioral factors (Ho and Zhang 2008, Cui et al. 2007). Katok et al. (2014) is the only study in the supply chain contracting literature we are aware of that explicitly analyzes the effect of fairness preferences that are private information. They analyze the wholesale price contract, and show that in equilibrium the rejection rate is zero (under most conditions). The intuition behind this result is that the equilibrium wholesale price is low enough so that even the most extremely inequity-averse type can make the profits equitable by choosing a sufficiently small (but positive) order quantity. While this intuition helps provide insight into other contracts, and the analysis may be extended to two-part tariff or some other contractual arrangements, the main open question remains: “What is the supplier’s optimal contract when preferences for fairness are retailer’s private information?”

The contribution of our study is that we (i) characterize the supplier’s optimal contract, (ii) use a different parameterization of the problem and the approach to proving the structure of the optimal contract than those of Baron and Myerson (1982) and Ha (2001), (iii) prove that the optimal contract can be implemented in practice by a simple minimum order quantity (MOQ) contract, (iv) show that information asymmetry regarding fairness destroys the equivalence between MOQ and TPT contracts, and (v) demonstrate, using results of past experimental studies, that one of the main testable predictions, regarding the optimal rejection rate, is in line with the empirical evidence.

2. The model
2.1. Standard contracting game

We consider a supply chain composed of a supplier and a retailer (further, we also refer to them as players). The supplier can manufacturer any $q \geq 0$ units of an infinitely divisible product which the retailer can buy from the supplier and sell on the market. We assume that the retailer’s revenue function $R(q)$ is concave, $R(0) = 0, R'(0) > 0$, while the supplier’s cost function $C(q) \geq 0, C(0) = 0$, is increasing convex such that the total profit of the supply channel, $\pi(q) = R(q) - C(q)$, is a concave function with a finite maximum. We denote the quantity maximizing the total channel profit $q^{FB}$ and call it the first-best solution. This setting encompasses standard cases such as downward sloping linear demand and the newsvendor problem, and excludes uninteresting trivial cases.

The timing of events is the following. First, the supplier offers a contract to the retailer. The form of the contract is $(q, t(q))$, where $q$ is the quantity sold to the retailer, and $t(q): t(0) = 0$ is
the payment to the supplier. The retailer orders the number of units they consider optimal, \( q^* \geq 0 \),
pays \( t^* = t(q^*) \) to the supplier, and takes this quantity to the market. The supplier earns the profit
\( \pi_S(q^*,t^*) = t(q^*) - C(q^*) \) and the retailer earns \( \pi_R(q^*,t^*) = R(q^*) - t(q^*) \). We use subscripts \( S \)
and \( R \) to distinguish variables that belong to the supplier and to the retailer. Superscripts are used
to distinguish variables that belong to different contracting games we analyze further. For brevity,
we sometimes omit function arguments whenever we think this will not cause confusion.

We assume that players are fair-minded. To account for their preferences, we adopt Cui et al.
(2007) specification which is a generalization of Fehr and Schmidt (1999) and a liner version of the
utility function used by Bolton and Ockenfels (2000). This utility function allows for a possibility
that parties consider payo\( s \) fair (or equitable) not only when they are equal but, more generally,
when \( \pi_R = \gamma \pi_S, \quad 0 \leq \gamma \leq 1 \). The retailer’s utility depends on the final profits, \( \pi_R \) and \( \pi_S \), as follows
(the supplier’s utility is defined analogously):

\[
U_R(\pi_R, \pi_S | \alpha_R, \beta_R) = \pi_R - \alpha_R (\gamma \pi_S - \pi_R)^+ - \beta_R (\pi_R - \gamma \pi_S)^+. \tag{1}
\]

The two terms subtracted from the retailer’s profit account for dis-utility from disadvantageous
(when \( \gamma \pi_S - \pi_R > 0 \)) and advantageous (when \( \pi_R - \gamma \pi_S > 0 \)) inequalities. To simplify the exposition
we use \( \gamma = 1 \) in the analysis below but the results immediately generalize for the case of an arbitrary
\( \gamma \). Factors \( \alpha_R \geq 0 \) and \( \beta_R \geq 0 \) are scaling coefficients capturing the strength of preferences for
fairness. Using mechanism design terminology, we call the pair \( \{\alpha_R, \beta_R\} \) a retailer type, which can
be common knowledge or private information of the retailer. Regarding \( \beta \), a substantial amount
of empirical evidence suggests that in the context we are analyzing, it is safe to consider \( \beta_R = 0 \).
De Bruyn and Bolton (2008) conduct a meta analysis using data sets from several bargaining
experiments and find that the model with \( \beta = 0 \) fits the data just as well as a model with \( \beta > 0 \).
Assuming \( \beta_R = 0 \) reduces the retailer type to \( \alpha_R \). There is also a technical reason to keep \( \beta_R = 0 \),
which is that \( \beta_R > 0 \) makes the problem of designing an optimal contract analytically intractable;
we explain this in more detail in Section 2.3. We do allow \( \beta_S > 0 \), however, because the supplier
does operate under the advantageous inequality, and the results of Cui et al. (2007) and Katok
et al. (2014) suggest that even if \( \beta_S \) is small it can still notably affect the channel performance.

2.2. Generalized contracting game

To derive an optimal contract when the retailer’s preferences are private information we follow the
mechanism design approach (Myerson 1979). The key feature of our analysis, letting us streamline
the proofs and exposition, is that we formulate the problem as that of finding the location of the
best contract(s) in the contracting space, which is defined as a set of feasible contracting outcomes,
i.e. profits. To illustrate our approach and for further reference, we first analyze the case when
When the supplier knows $\alpha_R$ the contracting game introduced above is a dynamic game of complete and perfect information that can be solved using backwards induction. The supplier’s problem is therefore to find an optimal contract allowing for the retailer’s best-response:

$$\max_{t(q)} \pi_S (q^* (\alpha_R), t(q^* (\alpha_R)))$$

s.t.

$$q^* (\alpha_R) = \arg \max_q U_R ((q, t(q)) | \alpha_R)$$

$$U_R ((q^* (\alpha_R), t(q^* (\alpha_R))) | \alpha_R) \geq 0$$

Note that any $t(q)$ results in players’ profits $\tilde{\pi}_R (\alpha_R) = \pi_R (q^* (\alpha_R), t(q^* (\alpha_R)))$ and $\tilde{\pi}_S (\alpha_R) = \pi_S (q^* (\alpha_R), t(q^* (\alpha_R)))$. Therefore, instead of using $t(q)$ and having to solve for $q^* (\alpha_R)$ the supplier could offer a contract specifying the resulting profits $(\tilde{\pi}_R (\alpha_R), \tilde{\pi}_S (\alpha_R))$ directly. That is, the supplier’s problem can be re-stated in terms of profits, as follows:

$$\max \pi_S, \pi_R$$

s.t.

$$\pi_R - \alpha (\gamma \pi_S - \pi_R)^+ \geq 0$$  \hspace{1cm} (3)

$$\pi_R + \pi_S \leq 1$$  \hspace{1cm} (4)

$$\pi_R, \pi_S \geq 0$$  \hspace{1cm} (5)

In this formulation, (3) is the retailer participation constraint, meaning that the retailer does not accept contracts that result in negative utility. The inequality (4) states that contracts beyond the Pareto-frontier are infeasible (for clarity of exposition, we normalized the maximum total profit of the centralized channel to unity). Lastly, (5) are non-negativity constraints because, due to the definition (1), a negative profit implies a negative utility. They are redundant because of (3) but we included them because they help visualize the feasible space of the supplier’s problem as a triangle limited by the axes and the Pareto-frontier.

Figure (1) illustrates all the relevant parts of the problem and we will now use it to find the solution. Due to (2), the supplier needs to find a point within the feasible region as far to the right as possible, providing that it is acceptable for the retailer due to (3). Notice that any isouitility curve crosses the Pareto-frontier and that the lower the retailer’s utility the further to the right is the crossing point. Therefore, the optimal contract is at the intersection of the Pareto-frontier and the retailer’s isouitility curve corresponding to zero utility:

$$\begin{align*}
\pi_R - \alpha (\gamma \pi_S - \pi_R) &= 0 \\
\pi_R + \pi_S &= 1
\end{align*} \iff \begin{align*}
\pi^F_S &= \frac{\alpha+1}{(1+\gamma)\alpha+1} \\
\pi^F_R &= \frac{(1+\gamma)\alpha+1}{\alpha+1}
\end{align*}.$$  \hspace{1cm} (6)
2.3. Information asymmetry

In general, the number of retailer types is arbitrary and the type space can even be continuous. In our analysis, we adopt a discrete, $N$-type model, noting that from a practical standpoint any continuous distribution can be reasonably approximated by a discrete distribution via a sufficiently fine discretization. That is, we assume $N \geq 2$ retailer types, $\alpha_1 < \ldots < \alpha_N$, that are realized with probabilities $\rho_1, ..., \rho_N$ such that $\sum_{i=1}^{N} \rho_i = 1$. We additionally assume that $\rho_i$ themselves are drawn from a joint distribution without mass points.
The supplier’s problem of finding an optimal direct mechanism (a menu of contracts) is the following:

\[
\max_{\pi_S, \pi_R} EU_S = \sum_{i=1}^{N} \rho_i U_S (\pi_{R_i}, \pi_{S_i} | \alpha_S, \beta_S) \tag{7}
\]

subject to:

\[
U_R (\pi_{R_i}, \pi_{S_i} | \alpha_i) \geq 0 \tag{8}
\]

\[
\pi_{S_i} + \pi_{R_i} \leq 1 \tag{9}
\]

\[
U_R (\pi_{R_i}, \pi_{S_i} | \alpha_i) \geq U_R (\pi_{R_{i+1}}, \pi_{S_{i+1}} | \alpha_{i+1}) \tag{10}
\]

Constraints (8) and (9) are analogous to (3) and (4). Inequalities (10) are standard incentive-compatibility constraints. They ensure that a contract \((\pi_{S_i}, \pi_{R_i})\) will be accepted only by type \(\alpha_i\). We also assume that in the case of a tie between \((\pi_{S_i}, \pi_{R_i})\) and some other contract \(\alpha_i\) type chooses \((\pi_{S_i}, \pi_{R_i})\). Therefore, \(\rho_i\) in (7) correctly represents the true probability of \((\pi_{S_i}, \pi_{R_i})\) being selected. Note that this formulation allows all possible types of menus, from a fully separating menu in which there is a unique contract for each type, to “bunching”, when there are contracts to be chosen by several types, to a completely pooling contract, and, in addition, in each of those menus some types may be “shut down”, preferring to choose a null contract \((0,0)\).

It is worth noting that the only IC constraints in this formulation are upward-binding constraints (10) whereas, generally, incentive compatibility requires that \(U_R (\pi_{R_i}, \pi_{S_i} | \alpha_i) \geq U_R (\pi_{R_k}, \pi_{S_k} | \alpha_k)\), \(\forall j \neq k\). However, because we assumed \(\beta_R = 0\) the retailer’s utility (1) satisfies the single-crossing condition and (10) are sufficient (Maskin and Riley 1984). If the single-crossing property does not hold, which is the case with (1) when \(\beta_R > 0\), any possible combination of IC constraints may turn out binding in the optimal menu and, therefore, all possible combinations of the constraints must be considered. This makes a general \(N\)-type discrete case of adverse selection problem analytically intractable (Lafoi and Martimort 2002, p. 93).

**Lemma 1.** Contracts on the optimal menu never result in disadvantageous inequity for the supplier, i.e. \(\pi_R \leq \gamma \pi_{S_i}, \forall i\).

This lemma allows us to streamline some of the further analysis but, most importantly, it shows that the assumption \(\beta_R = 0\) involved no loss of generality. By ignoring constraints that involve \(\beta_R\) we analyzed a relaxed problem and showed that its solution satisfies the unrelaxed one with all the possible constraints allowing for \(\beta_R > 0\).

**Proposition 1.** The optimal menu is (almost surely) unique and contains a single contract located on the Pareto frontier.
Perhaps the most important practical implication of this result is that the optimal menu turns out to be very simple. On the theory side, it is worth noting that our non-standard parameterization allowed a proof revealing the key driver of this result - the linearity of the problem (7) - (10). Had we chosen a standard \((q,t)\) parameterization this intuition might not be as clear.

**Proposition 2.** The optimal cutoff type, \(\hat{\alpha}\), is the largest solution to the following equation:

\[
\frac{f(\hat{\alpha})}{F(\hat{\alpha})} = \frac{(1 - 2\beta_S)\gamma}{(1 + (1 + \gamma)\hat{\alpha})(1 + \hat{\alpha} - \beta_S - \hat{\alpha}\beta_S(1 - \gamma))}.
\]

(11)

If the solution does not exist then all types participate.

The characterization of the cut-off type allows us not only to compute its exact value but, more importantly, to better understanding drivers the rejection rate. It is immediate to see that Eq. (11) has a solution for any distribution with a continuous density function because for any support \([\alpha_L, \alpha_M]\): \(\alpha_L \geq 0, \alpha_M \leq \infty\), the reverse hazard rate showing on the LHS decreases from infinity to zero: \(\lim_{\epsilon \to 0} \frac{f(\alpha_L + \epsilon)}{F(\alpha_L + \epsilon)} = \infty\) and \(\frac{f(\alpha_M)}{F(\alpha_M)} = 0\). Interestingly, in the case of distributions with a discontinuous density, the optimal rejection rate can be zero even if the upper bound of the support is arbitrarily large. For example, consider a uniform distribution \(U(0, \alpha_M)\). It is discontinuous at the ends of the support. The LHS of (11) is then \(\frac{1}{\hat{\alpha}}\), which is always larger than the RHS although \(\alpha_M\), the most inequity-averse type, can be arbitrarily large. Further, we refer to the right hand side of (11) as the cutoff function (COF). One can show that it is monotone decreasing both in \(\hat{\alpha}\) and \(\beta_S\). Therefore, if \(\hat{\alpha}\) belongs to an interval in which \(f(\alpha)/F(\alpha)\) is decreasing (e.g., the right tail of a distribution of the exponential family) then an increase in \(\beta_S\) leads to a lower rejection rate. Figure 2 presents more insights into comparative statics based on empirical evidence.

### 3. Implementation and testable predictions

The optimal contract characterized by Propositions 1 and 2 is a “point” contract. Expressing it in quantity-payment terms results in a single point \((q,t)\) whereas the supply chain contracting research has been focusing on contracts used in practice such as MOQ, QD and TPT but in these contracts the retailer has a continuum of quantities to choose from. Therefore, a question of immediate interest is which of these contracts, if any, can implement the optimal contract.

**Proposition 3.** (i) The optimal contract can be implemented with MOQ. (ii) In general, the optimal contract cannot be implemented with TPT.

This proposition has two important implications. The first, of course, is that in practice an optimal contract can be implemented by a simple means; a regular MOQ contract suffices. The second is that incomplete information about preferences for fairness breaks the equivalence between two coordinating contracts, MOQ and TPT.
The optimal cutoff type is determined by the intersection of the $f(\alpha)/F(\alpha)$ graph with a relevant curve from a family of COF parametrized by the supplier’s $\beta$. Lower curves from the family correspond to higher values of $\beta$.

One testable prediction follows immediately from Proposition 3: MOQ should result in a higher expected profit for the supplier than TPT. By extension, considering that MOQ is an extreme case of a family of quantity-discount contracts, one can expect MOQ to be at least as good for the supplier as any QD.

Another testable prediction is about the optimal rejection rate stated in Proposition 2. However, a single experiment with an MOQ contract may not be sufficient. The issue is that testing requires knowing: (i) the rejection rate under an MOQ contract, and (ii) the empirical distribution of the fairness parameter. However, obtaining both from the same MOQ data set is problematic because a single experiment with an MOQ contract may not be sufficient. The issue is that testing requires only its range. Therefore, we use the results of two independent experiments. Katok et al. (2014) provide an empirical distribution of the retailer’s fairness parameter $\alpha$ (Figure 3 on page 296) and estimate that $\gamma = 0.83$ (Tables 2 and 3 on page 295). Figure 2 presents the pdf and cdf of the empirical distribution, their ratio $\frac{f(\alpha)}{F(\alpha)}$ and a family of cutoff functions RHS of (11) parametrized by $\beta_S$. The graph of $\frac{f(\alpha)}{F(\alpha)}$ crosses the topmost COF curve ($\beta_S = 0$) around $\hat{\alpha} = 0.39$. Using the cdf graph one finds that $F(0.39) = 0.78$, i.e. the optimal rejection rate is 22%, which is close to 19.52% reported in Katok and Pavlov (2013). To estimate the effect of $\beta_S$ (comparative statics) on $\hat{\alpha}$, notice that the graph of $\frac{f(\alpha)}{F(\alpha)}$ crosses the lowest COF ($\beta_S = 0.3$) around $\hat{\alpha} = 0.43$, resulting in a slightly lower rejection rate around 20%. That is, the effect of supplier’s altruism on the optimal contract is insignificant unless, of course, $\beta_S \geq \frac{1}{1+\gamma} \approx 0.55$ because in that case,
the supplier prefers to offer a fair contract with $\pi_R = \gamma \pi_S$ (this follows immediately from Eq. (1)), acceptable to every retailer type.

4. Discussion and summary

We analyze the contracting problem in a setting in which the retailer and the supplier have preferences for fairness, and the retailer’s preferences are private information. In this setting, we characterize the optimal contract. We find that the contract is pooling with a possible shutdown of the most inequity-averse types. We derive our results in two steps. First, Proposition 1 establishes the structure of the optimal contract. The novelty of our approach is that it makes use of the fact that the profit of the informed party does not depend on their type (in contrast to settings analyzed, for example, in Baron and Myerson (1982) when cost is private information). This allows us to parametrize the problem in a novel way, casting it in terms of profits (final outcomes) rather than in terms of contract parameters. The most important implication is that this parameterization naturally leads to a proof revealing the key factor driving the result: the linearity of the problem in decision variables. Second, Proposition 2 characterizes the optimal cutoff type and its main implication is that the optimal rejection rate is positive for any type distribution with a continuous density. A somewhat counter-intuitive aspect of this result is that the types that the optimal contract “shuts down” may be arbitrarily close to zero. Overall, our characterization of the optimal contract suggests that incomplete information regarding fairness makes supply chain coordination generally impossible.

The rest of our results may also provide insights helpful both for practitioners and academics. First, Proposition 3 Part (i) shows that the optimal contract can be easily implemented in practice by a regular MOQ contract. Based on that, we use Eq. (11) to predict the rejection rate under MOQ based on the distribution of types obtained by Katok et al. (2014) from a wholesale pricing experiment and find that it is very close to the 20% observed in an independent experiment of Katok and Pavlov (2013) testing the performance of MOQ contracts. This implies that due to information asymmetry regarding fairness preferences, the supply chain efficiency is capped at around 80%. Therefore, it does not seem surprising anymore that at least in the experiments using downward sloping linear demand, the coordinating contracts provide only a marginal efficiency improvement over the theoretical 75% efficiency of wholesale pricing (Katok et al. 2014, show that the wholesale price contract performance is not significantly affected by information asymmetry regarding fairness concerns). Thus, our result helps close a puzzling gap between theory and practice:

“Can we explain why these contracts have not completely eliminated the Pareto inferior wholesale-price contract?” (Cachon 2003, p. 330)

Another issue our work illuminates is a choice between equivalent, according to standard theory, coordinating contracts:
“While it is possible to identify some differences among the contracts (e.g., different administrative costs, different risk exposures, etc.) none of them is sufficiently compelling to explain why one form should be adopted over another.” (Cachon 2003, p.257)

In fact, it appears that two-part-tariff contracts are not very common in B2B environments, while various versions of all-unit quantity discount contracts are much more prevalent (Munson and Rosenblatt 1998, Kolay et al. 2004). This is consistent with our observation that the optimal contract under incomplete information can be implemented as a MOQ contract, but not as a TPT contract. Proposition 3 Part (ii) shows that equivalence between two otherwise coordinating contracts, MOQ and TPT, breaks down under information asymmetry regarding fairness preferences. Our study directly extends a stream of research on the role of behavioral factors on the supply chain performance (Cui et al. 2007, Ho et al. 2014, Niederhoff and Kouvelis 2016, Lim and Ho 2007, Ho and Zhang 2008, Katok and Wu 2009), and also complements studies on the role of other-regarding preferences in other operations management contexts (Avci et al. 2014, Roels and Su 2013).

Lastly, our study establishes a closer link between supply chain contracting and bargaining. When testing contract performance in the laboratory, if the bargaining is implemented as a “take-it-or-leave-it” offer by the supplier to the retailer (Lim and Ho 2007, Ho and Zhang 2008, Katok and Pavlov 2013) the supply chain contracting game closely resembles the Ultimatum Game. When the bargaining protocol is different from “take-it-or-leave-it” coordinating contracts may perform better. This may be due to a decrease in information asymmetry that may result from observing offers and counteroffers, or to the effect of the “face-to-face” protocol on outcomes reported from early bargaining experiments (Roth 1995). It is also possible that observing offers and counteroffers may shift the perception of what is fair ($\gamma$). Haruvy et al. (2015) further explore this explanation by analyzing a model of structured bargaining, together with a set of laboratory experiments, that demonstrate that a less restrictive bargaining protocol improves coordinating contract performance significantly relative to the “take-it-or-leave-it” protocol.

References


Appendix

A. Proofs

[Proof of Lemma 1] By contradiction.

Suppose there exists type $\alpha_k$ such that in an optimal menu satisfying constraints (8), (9) and (10), its contract results in a disadvantageous inequality for the supplier, i.e. $\pi_{R_k} > \gamma \pi_{S_k}$. The supplier’s utility from this contract is $U_S(\pi_{R_k}, \pi_{S_k}) = \pi_{S_k} - \alpha_S(\pi_{R_k} - \gamma \pi_{S_k})$. Increasing the supplier’s profit in this contract to $\pi'_S : \pi_{R_k} = \gamma \pi'_{S_k}$ increases the supplier’s utility by turning the second term to zero. Since the retailer’s utility for $\pi_{R_k} > \gamma \pi_{S_k}$ is simply $\pi_{R_k}$ then changing the supplier’s profit from $\pi_{S_k}$ to $\pi'_S$ does not affect the retailer’s utility and the new menu satisfies all the constraints. Hence, the original menu was not optimal, contrary to the assumption.

[Proof of Proposition 1] By contradiction. By Lemma 1, the supplier never experiences the disadvantageous inequality under the optimal menu and $\alpha_S$ are irrelevant. To shorten the formulas, first consider the following problem that assumes also $\beta_S = 0$:

$$\max_{\pi_{S_i} \leq \pi_{R_i}} E\pi_S = \sum_{i=1}^{N} \rho_i \pi_{S_i}$$ (EC.1)

s.t. $\forall i = 1, \ldots, N$

1. Suppose there exists an optimal menu with three or more different contracts and there are at least three types that accept three adjacent contracts designed for lowest types (in the order of decreasing profits): $(\pi_{S_{1,k}}, \pi_{R_{1,k}}), (\pi_{S_{k+1,m}}, \pi_{R_{k+1,m}}), (\pi_{S_{m+1,n}}, \pi_{R_{m+1,n}})$, where $k \geq 1$, $m \geq k + 1$, $n \geq m + 1$. This is the most general case of semi-pooling (“bunching”) allowing for a possibility that $k$ lowest types prefer the first contract, the next $m - (k + 1)$ types prefer the second and the next $n - (m + 1)$ types prefer the third contract. We omit the detailed proof of the case with only two contracts on the menu because it is can be obtained with a minor modification of the argument that follows (the required modification explained below).

2. By definition, since the menu is optimal the objective function cannot be strictly improved by modifying any part of the menu. That is, the following program, capturing the contribution of the first two contracts to the supplier’s expected profit, must have the same contracts as the solution (possibly non-unique):

$$\max_{\pi_{S_{1,k}}, \pi_{R_{1,k}}, \pi_{S_{k+1,m}}, \pi_{R_{k+1,m}}} \pi_{S_{1,k}} \sum_{i=1}^{k} \rho_i + \pi_{S_{k+1,m}} \sum_{i=k+1}^{m} \rho_i$$ (EC.5)

s.t.

$\pi_{S_{1,k}} + \pi_{R_{1,k}} = 1$ (EC.6)

$\pi_{R_{1,k}} = \alpha_k \left( \pi_{S_{1,k}} - \pi_{R_{1,k}} \right) = \pi_{R_{k+1,m}} - \alpha_k \left( \pi_{S_{k+1,m}} - \pi_{R_{k+1,m}} \right)$ (EC.7)
The resulting equations in the matrix form as

\[
\begin{align*}
\pi_{Rk+1,m} - \alpha_m (\pi_{Sk+1,m} - \pi_{Rk+1,m}) &= \pi_{Rm+1,n} - \alpha_m (\pi_{Sm+1,n} - \pi_{Rm+1,n}) \\
\pi_{S1,k} &\geq \pi_{Sk+1,m} \\
\pi_{Sk+1,m} &\geq \pi_{Sm+1,n}
\end{align*}
\]  

(EC.8) (EC.9) (EC.10)

The constraint (EC.6) is necessary because otherwise the problem would be unbounded and it is binding because otherwise \(\pi_{S1,k}\) could be made arbitrarily large by adjusting \(\pi_{R1,k}\) so as to satisfy (EC.7), which is the only other constraint that might prevent \(\pi_{S1,k}\) from an arbitrarily large increase. Constraints (EC.7) and (EC.8) are the only two relevant incentive-compatibility constraints. That is, (EC.7) implies that types smaller than \(\alpha_k\) strictly prefer \((\pi_{S1,k}, \pi_{R1,k})\) to any other contract and, similarly, (EC.8) implies that types choosing \((\pi_{Sk+1,m}, \pi_{Rk+1,m})\) prefer it to other contracts. We included constraints (EC.9) and (EC.10) because, by assumption, the three contracts are different, i.e. \(\pi_{S1,k} > \pi_{S1,k} > \pi_{Sm+1,n}\) but if we find that at least one of these constraints is binding we will come to a contradiction.

3. For brevity, denote \(\rho_{1,k} = \sum_{i=1}^k \rho_i\) and \(\rho_{k+1,m} = \sum_{i=k+1}^m \rho_i\). The Lagrangian of the above problem is then

\[
L = \pi_{S1,k}\rho_{1,k} + \pi_{Sk+1,m}\rho_{k+1,m} + \\
\nu_{1,k} (\pi_{S1,k} + \pi_{R1,k} - 1) \\
\lambda_{1,k} (\pi_{R1,k} - \alpha_k (\pi_{S1,k} - \pi_{R1,k}) - \pi_{Rk+1,m} + \alpha_k (\pi_{Sk+1,m} - \pi_{Rk+1,m})) + \\
\lambda_{k+1,m} (\pi_{Rk+1,m} - \alpha_m (\pi_{Sk+1,m} - \pi_{Rk+1,m}) - \pi_{Rm+1,n} + \alpha_m (\pi_{Sm+1,n} - \pi_{Rm+1,n})) + \\
\mu_{1,k} (\pi_{S1,k} - \pi_{Sk+1,m}) + \\
\mu_{k+1,m} (\pi_{Sk+1,m} - \pi_{Sm+1,n})
\]

(EC.11)

where \(\nu_{1,k}, \lambda_{1,k}, \lambda_{k+1,m}, \mu_{1,k}\) and \(\mu_{k+1,m}\) are five Lagrange multipliers associated with constraints (EC.6)–(EC.10), accordingly.

4. The first order conditions result in the following system of linear equations with respect to Lagrange multipliers:

\[
\begin{align*}
\frac{\partial L}{\partial \pi_{S1,k}} &= 0 \iff \rho_{1,k} + \nu_{1,k} - \lambda_{1,k}\alpha_k + \mu_{1,k} = 0 \\
\frac{\partial L}{\partial \pi_{Sk+1,m}} &= 0 \iff \rho_{k+1,m} + \lambda_{1,k}\alpha_k - \lambda_{k+1,m}\alpha_m + \mu_{k+1,m}\pi_{Sk+1,m} = 0 \\
\frac{\partial L}{\partial \pi_{R1,k}} &= 0 \iff \nu_{1,k} + (1 + \alpha_k) \lambda_{1,k} = 0 \\
\frac{\partial L}{\partial \pi_{Rk+1,m}} &= 0 \iff \lambda_{k+1,m} (1 + \alpha_m) - \lambda_{1,k}(1 + \alpha_k) = 0
\end{align*}
\]

(EC.12) (EC.13) (EC.14) (EC.15)

5. By assumption, \(\mu_{1,k} = \mu_{k+1,m} = 0\) and so the terms containing them cancel out. However, re-writing the resulting equations in the matrix form as

\[
\begin{bmatrix}
1 & -\alpha_K & 0 \\
0 & \alpha_k & -\alpha_m \\
1 & 1 + \alpha_k & 0 \\
0 & -1 - \alpha_k & 1 + \alpha_m
\end{bmatrix}
\begin{bmatrix}
\nu_{1,k} \\
\lambda_{1,k} \\
\lambda_{k+1,m}
\end{bmatrix} = 
\begin{bmatrix}
-\rho_{1,k} \\
-\rho_{k+1,m} \\
0 \\
0
\end{bmatrix}
\]

(EC.16)
makes it obvious that in this system of four equations with three unknowns the equations are linearly independent, except possibly in a special case. Therefore, generally, the system has no solution. Using two equations to eliminate two out of three variables and requiring that the remaining two equations must have the same solution gives the special condition when this is possible:

\[ \rho_{k+1, m} = \frac{\alpha_m - \alpha_k}{(\alpha_m + 1) (2\alpha_k + 1)} \rho_{1,k}. \]  

(EC.17)

However, since by our assumption \( \rho's \) are drawn from a distribution without mass points, the probability that this condition is satisfied is zero. Hence, we conclude that at least one of the constraints (EC.9) and (EC.10) must be binding.

6. Note that this argument is exhaustive and could be formally presented as induction. The preceding analysis assumes that all types currently bundled at \( (\pi_{S_k+1, m}, \pi_{R_{k+1, m}}) \) can be moved only altogether, i.e. we do not consider splitting them into smaller bundles. However, since the number of types is arbitrary the argument covers the cases of those “would be” smaller bundles showing that they would be merged with other bundles.

7. A minor modification that we mentioned earlier would be required for the case of two contracts is a replacement of (EC.8) with a participation constraint for the highest type in that bundle. In effect, one needs to replace of the right-hand-side of (EC.8) with zero and then the same “merging” result obtains.

Finally, since the result obtains due to linearity of the Lagrangian in profits, the result holds not only for the profit-maximizing supplier but for a fair-minded supplier with \( \beta > 0 \) as well because the disutility term allowing for fairness is also linear in profits and adding it to the Lagrangian would not change the final result.

7. Proof of Proposition 2] Considering the results of Proposition 1, the supplier needs to find the optimal cut-off type by solving the following problem

\[
\max_{\hat{\alpha}, \pi_S(\hat{\alpha}), \pi_R(\hat{\alpha})} U_S(\pi_S(\hat{\alpha}), \pi_R(\hat{\alpha}) | \hat{\beta}_S) F(\hat{\alpha})
\]

s.t.

\[
\pi_S(\hat{\alpha}) + \pi_R(\hat{\alpha}) = 1,
\]

(EC.19)

\[
\pi_R(\hat{\alpha}) - \hat{\alpha} (\gamma \pi_S(\hat{\alpha}) - \pi_R(\hat{\alpha})) = 0.
\]

(EC.20)

The equality constraints (EC.19) and (EC.20) result in \( \pi_S(\hat{\alpha}) = \frac{\hat{\alpha} + 1}{(1 + \gamma) \hat{\alpha} + 1} \) and \( \pi_R(\hat{\alpha}) = \frac{\hat{\alpha} \gamma}{(1 + \gamma) \hat{\alpha} + 1} \). Substituting this expression into (1) and the result into (EC.18) gives the following single-variable unconstrained maximization problem:

\[
\max_{\hat{\alpha}} \left( \frac{1 + \hat{\alpha} - \beta_S - \hat{\alpha} \beta_S (1 - \gamma)}{1 + \hat{\alpha} (1 + \gamma)} \right) F(\hat{\alpha}).
\]

(EC.21)

Differentiating the objective function and rearranging the terms in the resulting FOC completes the proof:

\[
\frac{f(\hat{\alpha})}{F(\hat{\alpha})} = \frac{(2\beta_S - \beta_S - \gamma \beta_S (1 - \gamma))}{(1 + (1 + \gamma) \hat{\alpha} (1 + \alpha - \beta_S - \hat{\alpha} \beta_S (1 - \gamma))}. \]

(EC.22)

7. Proof of Proposition 3[i] By construction. Consider an MOQ contract \((q_{\min}, w)\) such that \( q_{\min} = q^{FB} \) and \( w \) is such that \( U_R(\pi_R(q^{FB}, w), \pi_S(q^{FB}, w) | \hat{\alpha}) = 0 \). Then all types \( \alpha \leq \hat{\alpha} \) accept because, due to (1),

\[
U_R(\pi_R(q^{FB}, w), \pi_S(q^{FB}, w) | \alpha) - U_R(\pi_R(q^{FB}, w), \pi_S(q^{FB}, w) | \hat{\alpha}) = \left( \hat{\alpha} - \alpha \right) \left( \gamma \pi_S(q^{FB}, w) - \pi_R(q^{FB}, w) \right) > 0.
\]

(EC.23)

(EC.24)
These types order $q = q_{\text{min}}$ because ordering $q > q_{\text{min}}$ implies a higher profit for the supplier, a lower total profit of the supply chain, and, therefore, a lower utility for the retailer than ordering $q = q_{\text{min}}$. Using the same steps one can show that types $\alpha > \hat{\alpha}$ reject the contract. Hence, this contract implements the optimal contract.

(ii) By contradiction. Suppose there exists a TPT that implements an optimal contract. By Proposition 1 it is pooling. To prove this is not the case, in general, it is sufficient to provide one counter-example. Consider a supply chain in which the retailer is facing a linear downward sloping demand $q = 1 - p$, where $q$ is the quantity sold and $p$ is the market clearing price. Let $c$ be the supplier’s marginal cost. The retailer’s and the supplier’s profits, given a contract $(L, w)$, are $\pi_R = pq - wq - L = (1 - q - w)q - L$ and $\pi_S = (w - c)q$, accordingly.

By Lemma 1, all participating retailer types experience a disadvantageous inequality. Thus, the utility of participating type $\alpha$ is $U_R(q|L, w, \alpha) = \pi_R - \alpha (\gamma \pi_S - \pi_R) = (1 + \alpha) ((1 - q - w)q - L) - \alpha \gamma ((w - c)q)$.

Solving the first-order condition $\frac{dU_R}{dq} = 0$ for $q(\alpha)$ gives $q(\alpha) = \frac{(\gamma p - w)\alpha + (1 - w)}{2(1 + \alpha)}$, and it is straightforward to verify that $q(\alpha) \neq \text{const}, \forall w$. This implies that at least some types will be ordering different quantities and, therefore, this TPT contract is not pooling.