Push, Pull, or Both?
A Behavioral Study of Inventory Risk on Channel Efficiency

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In this paper we experimentally investigate how the allocation of inventory risk in a two-stage supply chain affects channel efficiency. We first evaluate two common wholesale price contracts that differ in which party incurs the risk associated with unsold inventory; a push contract in which the retailer incurs the risk, and a pull contract in which the supplier incurs the risk. Our experimental results show that a pull contract achieves higher channel efficiency than that of a push contract, and that behavior systematically deviates from the standard theory. Therefore, we extend the existing theory to incorporate a number of behavioral regularities and structurally estimate its parameters. The estimates suggest that a combination of errors with loss aversion organizes our data remarkably well. Following this we explore a third contract, the advanced purchase discount (APD) contract, which combines certain features of push and pull by allowing both parties to share the inventory risk. We apply our behavioral model to the APD contract in a separate experiment as an out-of-sample test and find that it accurately predicts channel efficiency and qualitatively matches decisions. Lastly, from a managerial perspective, we observe that the APD contract weakly Pareto dominates the push contract; retailers are better off and suppliers are no worse off under the APD contract.
1. Introduction

Location and ownership of inventory is one of the key drivers of supply chain performance. Even in a simple supply channel—single retailer, single supplier, and full information—researchers and companies find that common wholesale price contracts with different inventory allocations affect channel efficiency (e.g. Lariviere and Porteus 2001, Cachon 2003, Kaya and Özer 2012). Channel design decisions are made at the highest managerial levels, as they involve a number of difficult tradeoffs and have a direct effect on a firm’s survival. For instance, Randall, Netessine and Rudi (2002) provide examples of companies in which the difference between success and bankruptcy could be attributed to different inventory allocation strategies. Furthermore, Randall, Netessine and Rudi (2006) identify, and empirically investigate, how factors such as revenue, product variety, and profit margins, impact a firm’s decision to own the inventory.

Traditional channels use a “push” structure in which the retailer makes stocking decisions, owns the inventory, and thus incurs the holding cost as well as the cost of any unsold product. However, Internet-enabled technologies now permit other supply chains arrangements for allocating inventory ownership and risk (the cost of unsold inventory), which may affect channel profitability (see Netessine and Rudi 2006 and Cachon 2004).

Alternatively, under a “pull” inventory system, the supplier makes the stocking (production) decision, and therefore incurs most of the holding costs and inventory risk. The retailer provides a storefront (real or virtual), and products flow from the supplier to the end customer with minimal exposure of the retailer to inventory risk. One extreme implementation of the pull inventory system is a drop-shipping arrangement—the retailer is never exposed to the inventory at all—the suppliers ship to customers directly. Such arrangements are quite prevalent in e-commerce; Randall et al. 2006 report that between 23% and 33% of Internet retailers use drop-shipping exclusively, and the U.S. Census estimates that e-commerce sales by retailers totaled $194 billion in 2011, up 16.4% compared to 2010 (U.S. Census Bureau 2013). Additionally, supply chains selling specialty products utilize pull structures (Klein 2009). The popularity of these contracts has even created opportunities for companies to specialize in providing drop-shipping services for businesses (Davis 2013 provides an example of CommerceHub). Less extreme pull arrangements exist as well, including just-in-time (JIT) delivery—the supplier delivers in small batches, thus becoming effectively responsible for holding cost and inventory risk, and vendor-managed inventory (VMI), in which the supplier makes stocking decisions, but the retailer holds the physical inventory.
Another inventory structure, the “advance purchase discount” (APD) contract, combines the aspects of the push and the pull systems so that both parties share the inventory risk (Cachon 2004). Cachon (2004) provides the example of O’Neill Inc., a manufacturer of water-sports apparel, which successfully uses the APD contract. In another study, Tang and Girotra (2010) evaluate how an APD structure impacts Costume Gallery, a privately-owned wholesaler of dance costumes, and estimate that the company could increase its net profits by 17% if it adopted an APD contract.

The question of how to structure the channel to best allocate inventory risk, and the effect of inventory risk on channel performance, has been extensively studied analytically (Netessine and Rudi 2006, Cachon 2004, Özer and Wei 2006, Özer, Unca and Wei 2007). In practice, however, top-level managers make these decisions, since they are strategic, involve difficult tradeoffs, and, therefore, are not automated. Consequently, it is critical to understand how differences in inventory risk location affect profits, when decisions are made by human decision-makers. To gain insights into the role human judgment plays in channel design decisions, we conduct a set of laboratory experiments to explore human behavior in push and pull settings. We find that the standard model is qualitatively consistent with some of the aspects of the data, but we also identify systematic deviations. Therefore, we extend the standard model to account for behavioral regularities we observe. We characterize and derive the equilibrium predictions for a number of behavioral models that have been identified in the recent literature (Su 2008, Ho and Zhang 2008, Ho et al. 2010, Cui et al 2007), structurally estimate their parameters, and find that a simple model of loss aversion with random errors fits the data remarkably well.

We then find that this model makes accurate out-of-sample predictions about the performance of the APD contract, which includes push and pull features. We consider this an important contribution to the literature because identifying systematic deviations from standard theory, and incorporating these behavioral regularities into analytical models, helps to understand their causes and provides insights that result in designing contracting mechanisms that are behaviorally robust. Using our model, managers can take into account behavioral regularities when they design channel structures.

Our experimental results highlight a number of managerial insights. First, we find that the pull contract, contrary to theory, achieves the same supply chain efficiency as the APD contract. Second, our results indicate that when supply chain partners have the ability to pick among the three alternatives, the push contract should rarely be used as the APD contract Pareto dominates the traditional push contract. This implies that regardless of who has the most bargaining power in the channel, the APD contract should be favored over the push contract.
In the next section we detail our experimental design and provide a brief overview of the standard theory regarding the push and pull contracts. In Section 3, we present our experimental results for the push and pull contracts, starting with summary statistics and then proceed to develop and structurally estimate our behavioral models. Using the results from Section 3, in Section 4, we present and test a number of hypotheses for the APD contract. Lastly, in Section 5, we summarize our main findings and discuss directions for future research.

2. Experimental Design and Standard Theory

2.1. Experimental Design

We evaluated three supply chain contracts, each in a separate between-subjects experimental treatment. In the push and pull contracts, one party offers the contract and the other party sets the stocking quantity (or rejects the contract). To be consistent with this structure, in our push treatment, the supplier offers the contract and the retailer decides on the order quantity. Conversely, in our pull treatment, the retailer offers the contract, and the supplier decides on the production quantity. Thus, our push and pull treatments differ in which party proposes the wholesale price and which party sets the stocking quantity.

The APD contract differs from the push and pull contracts in that it includes two wholesale prices (we call them the regular wholesale and discount wholesale prices), where both parties may share the inventory risk. The retailer incurs the inventory risk for a quantity ordered in advance of realized demand (called the prebook quantity), and the supplier incurs the inventory risk on the difference between its production amount and the retailer's prebook quantity. Specifically, in the APD treatment, the supplier begins the sequence by first proposing the two wholesale prices. After observing these prices, the retailer commits to paying for the prebook quantity (or rejects the contract). Next the supplier, after receiving the prebook order from the retailer, sets the production quantity. Finally, demand and profits are realized for both players. We discuss more details of the APD contract in Section 4.

Figure 1 depicts the decision sequence for the three treatments in our study.
Figure 1. Decision sequence for each contract in the experiment.

In all three treatments a rejection results in both parties earning 60. In theory this outside option is not binding for the push and pull contracts given our parameters\(^1\), but it is binding under the APD contract, because the supplier has the ability to extract the entire channel profit (which we will show when we outline the APD theory). Our setting makes the APD prediction somewhat more realistic, because the proposer should now extract a majority, but not 100%, of the channel profit.

We used the same demand distribution, per unit revenue, and cost parameters in all three treatments. Specifically, customer demand is an integer uniformly distributed between 0 and 100, U[0,100], the retailer receives revenue of \( r = 15 \) for each unit sold, and the supplier incurs a cost of \( c = 3 \) for producing each unit.

In each treatment we provided both participants with a decision support tool. The player in the proposer role could test wholesale prices between 3 and 15 (the unit cost and seller revenue per unit) using a scroll bar. For each test wholesale price, the computer would show them the stocking quantity that would maximize the other player’s expected profit. We made it clear that this amount is best in terms of average profit for the other player for this test wholesale price, but that they were playing with a human who may not necessarily stock this amount. Similarly, for the player setting a stocking quantity, once a wholesale price was offered to them, they could test different stocking quantities with a scroll bar (from 0 to 100). Each time they moved the scroll bar a line graph would

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\(^1\) This value is slightly below the minimum of any party’s profits, in any contract, in equilibrium. The experimental profit predictions will be illustrated in the next section.
display their actual potential profit calculated for every realization of demand (from 0 to 100). We provided this decision support tool to ensure that participants could comprehend the task and also allow the standard theory a viable chance of being confirmed. We include screenshots for the participants’ decisions in the sample instructions in the online Appendix.

In total 120 human subjects participated in the study, 40 in each treatment. We randomly assigned subjects to a role (retailer or supplier) at the beginning of each treatment. To reduce the complexity of the game, roles remained fixed for the duration of the session. Subjects made decisions in 30 rounds. Retailers and suppliers were placed into a cohort of 6 to 8 participants, and a single retailer was randomly re-matched with a single supplier within the cohort in each round, replicating a one-shot game. To mitigate reputation effects, subjects were unaware that their cohort size was 6 to 8 participants, and simply told that they would be randomly re-matched with someone else in the session. Each experimental treatment had 6 cohorts. Because subjects were placed into a fixed cohort for an entire session, we use the cohort as the main statistical unit of analysis.

We conducted the experiment at a public northeast U.S. university in 2010. Participants in all treatments were students, mostly undergraduates, from a variety of majors. Before each session, we allowed the subjects to read the instructions themselves for a few minutes. Following this, we read the instructions verbally and answered any questions (to assure common knowledge about the rules of the game). Each individual participated in a single session only. We recruited participants through an online system, and offered them cash as the only incentive to induce them to participate. Subjects earned a $5 show-up fee plus an additional amount that was proportional to their total profits from the experiment. Average compensation for the participants, including the show-up fee, was $25. Each session lasted approximately 1 to 1.5 hours and we programmed the software using the zTree system (Fischbacher 2007).

2.2. Theoretical Benchmarks for Push and Pull Settings

In all treatments, a retailer $R$ receives revenue $r$, for each unit sold, incurs no fixed ordering costs, and loses sales if demand exceeds inventory. A supplier $S$ produces inventory at a fixed per unit cost of $c$. Customer demand $D$ is a continuous random variable with cdf $F(\cdot)$ and pdf $f(\cdot)$. There is full information of all cost parameters, and we assume that retailers and suppliers are risk-neutral expected-profit maximizers. Lastly, we measure efficiency by the percent ratio between the decentralized supply chain expected profit and the centralized supply chain expected profit.

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2 In the APD treatment this support was the same. Suppliers here had to set two wholesale prices, then retailers could test the prebook quantity with a graph, and finally suppliers could also test production amounts with a similar graph before entering their decision.
For the push contract, a supplier offers a per unit wholesale price \( w \) to a retailer. The retailer sets a stocking quantity \( q \), for a given \( w \) that maximizes its expected profit, \( \pi_r(q) = rS(q) - wq \). Let \( S(q) = E[\min(q,D)] \) represent the expected sales for a stocking quantity \( q \), and \( q^*_{\text{push}} \) be the quantity that maximizes the retailer's expected profit under the push contract. In this case, the best response stocking quantity for the retailer is:

\[
q^*_{\text{push}} = F^{-1}\left( \frac{r - W^*_{\text{push}}}{r} \right).
\]

(1)

The supplier's decision under a push contract is \( w \), where \( W^*_{\text{push}} \) maximizes the supplier's push contract profit \( \pi_s(w) = (w - c)q \), and must satisfy:

\[
F^{-1}\left( \frac{r - W^*_{\text{push}}}{r} \right) = \frac{w^*_{\text{push}} - c}{r \times f\left( F^{-1}\left( \frac{r - W^*_{\text{push}}}{r} \right) \right)}.
\]

(2)

For demand following a \( U[0,100] \), \( W^*_{\text{push}} \) simplifies to:

\[
w^*_{\text{push}} = \frac{r + c}{2}.
\]

(3)

Under the pull contract, the decisions of the retailer and supplier are reversed; the retailer offers a per unit wholesale price \( w \) and the supplier then sets a stocking quantity \( q \) that maximizes its expected profit, \( E[\pi_s(q)] = wS(q) - cq \). Let \( q^*_{\text{pull}} \) be the stocking quantity that maximizes the supplier's expected profit. Then \( q^*_{\text{pull}} \) must satisfy:

\[
q^*_{\text{pull}} = F^{-1}\left( \frac{W^* - c}{w} \right).
\]

(4)

The retailer's decision under the pull contract is \( w \). Let \( W^*_{\text{pull}} \) be defined as the wholesale price that maximizes the retailer's expected profit in the pull contract, \( W^*_{\text{pull}} = \arg\max E[\pi_R(w)] \), where \( E[\pi_R(w)] = (r - w)S(q) \), \( \pi_R(w) \) is unimodal in \( w \) if demand has the increasing generalized failure rate property (Cachon 2004), so the optimal solution can be characterized using the first order condition. For the case of demand following a \( U[0,100] \), \( W^*_{\text{pull}} \) must satisfy,

\[
(W^*_{\text{pull}})^3 = c^2(2r - W^*_{\text{pull}})
\]

which we compute numerically for our experimental parameters.
2.3. Experimental Predictions

Table 1 summarizes the theoretical predictions for retailer and supplier profits, and supply chain efficiency given our experimental parameters \( r = 15, \ c = 3, \) demand \( U[0,100] \) and the outside option of 60). Participants were allowed to enter their decisions up to two decimal places for the wholesale prices and integers for stocking quantities.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale Price</td>
<td>9.00</td>
<td>6.00</td>
</tr>
<tr>
<td>Quantity</td>
<td>40.00</td>
<td>50.00</td>
</tr>
<tr>
<td>Retailer Profit</td>
<td>120.00</td>
<td>337.50</td>
</tr>
<tr>
<td>Supplier Profit</td>
<td>240.00</td>
<td>75.00</td>
</tr>
<tr>
<td>Channel Efficiency</td>
<td>75.00%</td>
<td>85.94%</td>
</tr>
</tbody>
</table>

3. Results of the Push and Pull Treatments

We begin by presenting the results from the push and pull treatments. We first show expected profits for the channel and both parties separately, and then proceed to examine wholesale prices, stocking quantities, and rejections. Following this, we present a behavioral model that organizes our data and structurally estimate its parameters. Ultimately, we can determine whether our behavioral model generates predictions in line with observed decisions.

3.1. Channel Efficiency and Expected Profits

We calculate the expected profit for each decision and report it as the “observed profit.” Figure 2 displays the predicted and observed supply chain profits along with the corresponding channel efficiency (located at the top of each column) for the push and pull contracts. There is no significant difference between observed and predicted supply chain profits \( p = 0.173 \) for push and \( p = 0.173 \) for pull). In Figure 2 we also see that the observed supply chain profits increase as the channel switches from the push contract to the pull contract \( p = 0.025 \). These results suggest that the normative prediction of improving channel efficiency by shifting the inventory risk from the retailer to the supplier, for a simple wholesale price contract, is correct.

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3 All one-sample tests are Wilcoxon signed-rank test and all two-sample tests are the Mann-Whitney U-test.
Moving to each party’s profits, we see that retailers in the pull contract, in Figure 3(a), earn significantly less than theory predicts ($p = 0.028$). However, retailers earn the same as theory predicts in the push contract ($p = 0.463$). Directionally consistent with the standard theory, when selecting between the two contracts, a retailer earns the most profit in the pull contract ($p < 0.01$).

The observed supplier profits shown in Figure 3(b), are below theory in the push contract ($p = 0.075$). For the pull contract, we observe suppliers earn significantly more than the theoretical prediction ($p = 0.028$). Comparing the supplier profits between the two contracts, the profits under the push contract are significantly higher than under the pull contract ($p = 0.0104$).
These initial results indicate that channel efficiency increases when moving from a push to a pull contract, and retailers prefer pull contracts while suppliers prefer push contracts. Both of these observations qualitatively agree with standard theory. However, actual levels of profits for both parties systematically deviate from the predicted values in that the profit split is somewhat more equitable. In this, our results for the push contract are similar to Keser and Paleologo (2004), the only other paper that reports on lab experiments with uncertain demand (they only have a push contract) in which both sides are human. They too find that wholesale prices are below optimal (see next section) and the profit distribution is more equitable than the standard theory predicts.

3.2. Decisions

Table 2 summarizes the average wholesale prices and stocking quantities for the push and pull contracts. For both contracts, proposers set wholesale prices that are significantly different from the theoretical predictions ($p = 0.028$ for both push and pull). Specifically, for both contracts, the party setting the wholesale price made offers that were more generous than theory predicts. In the push contract, the average wholesale price is below the prediction, while in the pull contract, it is above the prediction.

In order to interpret the observed stocking quantities correctly, we calculate the optimal stocking quantities conditioned on the proposers' wholesale prices, and then average them for the predicted values. The second row of values in Table 2 shows these results. There are significant differences between observed and best reply values in the pull quantity ($p = 0.046$). In the push contract we find that the observed quantity is lower than predicted, but not statistically different ($p = 0.463$).
Table 2. Average wholesale prices and quantities for agreements, and overall rejection rates.

<table>
<thead>
<tr>
<th></th>
<th>Push Predicted</th>
<th>Push Observed</th>
<th>Pull Predicted</th>
<th>Pull Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale Price</td>
<td>9.00**</td>
<td>8.26</td>
<td>6.00**</td>
<td>8.02</td>
</tr>
<tr>
<td></td>
<td>[0.16]</td>
<td></td>
<td>[0.28]</td>
<td></td>
</tr>
<tr>
<td>Quantity</td>
<td>w</td>
<td>44.93</td>
<td>42.60</td>
<td>61.26**</td>
</tr>
<tr>
<td></td>
<td>[1.04]</td>
<td></td>
<td>[1.30]</td>
<td></td>
</tr>
<tr>
<td>Rejection Rate</td>
<td>w</td>
<td>2.04%**</td>
<td>8.15%</td>
<td>4.77%</td>
</tr>
<tr>
<td></td>
<td>[1.05]</td>
<td></td>
<td>[2.00]</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets. Significance of Wilcoxon signed-rank test given by *** p-value<0.01, and ** p-value<0.05.

Recall that the party setting the stocking quantity, when receiving a wholesale price, had the option to reject the contract, such that both parties earn an outside option of 60. In Table 2 we provide the predicted rejection rates, conditioned on the observed wholesale prices, along with the observed rejection rates. In the push contract, retailers rejected significantly more than predicted ($p = 0.028$), whereas, in the pull contract, there are no significant differences in the predicted and observed rejections rates. Despite these minor differences, when calculating the percentage of time that the party stocking the inventory made the correct accept/reject decision, we observe that the correct decision was made 92.8% of the time in the push contract, and 94.7% of the time in the pull contract. Lastly, rejections do not appear to change over time in either contract (based on a logit regression with random effects with the decision period as the independent variable, plus a constant). We believe these results suggest that, with the provided decision support tools, subjects were generally able to comprehend the task.

Table 3. Efficiency impacts in the push and pull contracts.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency given correct accept/reject and quantity</td>
<td>78.65%</td>
<td>90.96%</td>
</tr>
<tr>
<td>Efficiency lost from incorrect rejection</td>
<td>3.15%</td>
<td>0.94%</td>
</tr>
<tr>
<td>Efficiency lost from incorrect acceptance</td>
<td>-0.16%</td>
<td>-1.40%</td>
</tr>
<tr>
<td>Efficiency lost from incorrect quantity</td>
<td>5.49%</td>
<td>7.59%</td>
</tr>
<tr>
<td>Observed efficiency</td>
<td>70.18%</td>
<td>83.83%</td>
</tr>
</tbody>
</table>

Focusing only on the players who set the stocking quantity, Table 3 shows how much supply chain efficiency was lost based on (a) incorrect rejections, (b) incorrect acceptances, and (c) incorrect stocking quantities, with respect to the theoretical predictions. In the push contract, 3.15% of the predicted efficiency was lost from rejecting favorable offers, while 5.49% was lost from suboptimal
stocking quantities. In the pull contract, only 0.94% efficiency was lost due to rejecting favorable offers, but 7.59% was lost due to low stocking quantities. Considering that both accept/reject decisions and quantities drive efficiency losses, we will consider both of these effects in our behavioral models that we develop in the next section.

### 3.3. Behavioral Models

Our goal in this section is to formulate a parsimonious behavioral model that can explain the regularities we observe in our data. These regularities are: (1) wholesale prices that are below predicted in the push contract and above predicted in the pull contract, (2) suboptimal order quantities (primarily in the pull contract), and (3) incorrect responder rejections. We consider behavioral models that have been proposed in recent literature: loss aversion from leftover inventory (Becker-Peth, Katok and Thonemann 2013, Ho, Lim and Cui 2010), inequality aversion (Cui, Raju and Zhang 2007), anchoring towards the mean (Schweitzer and Cachon 2000, Benzion, Cohen, Peled and Shavit 2008), and random errors in accept/reject decisions (Su 2008).

Because of the nature of these behavioral regularities, it is natural to assume that the proposer may have none of these deviations. In particular, losses and anchoring cannot happen since proposers do not hold inventory, and inequality aversion is unlikely to play a major role because proposers work under advantageous inequality (it has been shown that advantageous inequality aversion is virtually non-existent in the laboratory, see DeBruyn and Bolton 2008, Katok and Pavlov 2013). Also, Katok and Wu (2009) find that when suppliers in a push wholesale contract are matched with automated retailers programmed to place optimal orders, suppliers learn to set wholesale prices optimally very quickly. Considering that the behavioral regularities we investigate are unlikely to be present for proposers, we begin by making a simplifying assumption that proposers are fully rational. Later, we can validate whether this assumption holds by comparing predicted wholesale prices, given any behavioral biases in stocking quantities, to observed wholesale prices. We introduce the following notation for our behavioral parameters:

- $\beta \geq 1$: the degree of loss aversion that the party stocking the inventory experiences from having leftover inventory. $\beta > 1$ implies loss aversion, and $\beta = 1$ corresponds to rational behavior (see Becker-Peth et al. 2013, and Ho et al. 2010 for a related but slightly different formulation).
- $\alpha \geq 0$: the degree of disadvantageous inequality aversion (see Cui et al. 2007). We assume that decision-makers do not have disutility from advantageous inequality.
• $0 \leq \delta \leq 1$ : the degree of anchoring towards the mean (see Benzion et al. 2008 and Becker-Peth et al. 2013 for a similar approach to the one we apply).

We consider each of the above behavioral issues separately, but will add random errors in rejections when we discuss our parameter estimation.

### 3.3.1 Push Behavioral Models

In Table 4 we outline each of the behavioral models for the push setting, for demand uniformly distributed between 0 and 100. We relegate the derivations of these equations and other details to the Appendix.

Table 4. Expected utility functions for suppliers, along with their predicted stocking quantities, for the push contract.

<table>
<thead>
<tr>
<th>Behavioral Issue</th>
<th>Utility Function</th>
<th>Stocking Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loss Aversion</strong></td>
<td>$u_R = (r-w)S(q) - \beta w(q - S(q))$</td>
<td>$q^* = 100 \left( \frac{r-w}{r+w(\beta-1)} \right)$</td>
</tr>
<tr>
<td><strong>Inequality Aversion</strong></td>
<td>$u_R = rS(q) - wq - \alpha (\pi_S(w) - \pi_R(q))^+$</td>
<td>$q^* = \begin{cases} 100 \left( \frac{r-w+\alpha(r+c-2w)}{r(1+\alpha)} \right) &amp; , w \geq \bar{w}; \ 100 \left( \frac{2(r+c-2w)}{r-2w} \right) &amp; , \bar{w} &lt; w &lt; \bar{w}; \ 100 \left( \frac{r-w}{r} \right) &amp; , w \leq \bar{w}. \end{cases}$</td>
</tr>
<tr>
<td><strong>Anchoring</strong></td>
<td>$u_R = rS(q) - wq$</td>
<td>$q^* = (1-\delta) \left( 100 \left( \frac{r-w}{r} \right) \right) + \delta \mu$</td>
</tr>
</tbody>
</table>

In the table, $u_R$ denotes retailer’s utility, $S(q) = E[\min(q,D)] = q - q^2 / 200$, $\pi_S(w) = (w-c)q$, $\pi_R(q) = rS(q) - wq$, $\mu = 50$ (mean demand), $\bar{w} = \frac{21+18\alpha}{3+2\alpha}$, and $\bar{w} = 7$.

### 3.3.2 Pull Behavioral Models

As with the push contract, we outline each of the behavioral models for the pull setting, for demand uniformly distributed between 0 and 100, in Table 5 (please see the Appendix for additional details).
Table 5. Expected utility functions for retailers, along with their predicted stocking quantities, for the pull contract.

<table>
<thead>
<tr>
<th>Loss Aversion</th>
<th>$u_S = (w-c)S(q) - \beta c(q - S(q))$</th>
<th>$q^* = 100 \left( \frac{w-c}{w+c(\beta-1)} \right)$</th>
</tr>
</thead>
</table>
| Inequality Aversion | $u_S = wS(q) - cq - \alpha (\pi_R(w) - \pi_S(q))^*$ | $q^* = \begin{cases} 
100 \left( \frac{w-c}{w} \right), & w \geq \bar{w}; \\
100 \left( \frac{2(r+c-2w)}{r-2w} \right), & w < w < \bar{w}; \\
100 \left( \frac{w-c - \alpha(r+c-2w)}{w-\alpha(r-2w)} \right), & w \leq \bar{w}.
\end{cases}$ |
| Anchoring | $u_S = wS(q) - cq$ | $q^* = (1-\delta) \left( 100 \left( \frac{w-c}{w} \right) \right) + \delta \mu$ |

In the table, $\pi_R(w) = (r-w)S(q)$, $\pi_S(q) = wS(q) - cq$, $\mu = 50$, $\bar{w} = 9.80$, and

$$w = \frac{3(5 + 22\alpha + \sqrt{65 + 60\alpha + 4\alpha^2})}{4 + 8\alpha}.$$ 

Under both the push and pull contracts, the party offering a wholesale price takes into account the responder’s behavior, and selects the wholesale price that maximizes her expected profit. We find that the optimal wholesale price $w^*$, when dealing with a loss-averse responder ($\beta > 1$), is lower under push, and higher under pull, than the original predictions, $w^*_{\text{push}}$ and $w^*_{\text{pull}}$, which correspond to $\beta = 1$. The effect is somewhat similar when facing an inequality-averse responder—the profit maximizing wholesale price is lower under push and higher under pull, than the optimal wholesale price for a responder with no fairness concerns. We summarize these results in Table 6.
Table 6. Directional predicted wholesale prices in the push and pull contracts.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss Aversion ($\beta &gt; 1$)</td>
<td>$w^* &lt; w^*_{push}$</td>
<td>$w^* &gt; w^*_{pull}$</td>
</tr>
<tr>
<td>Inequality Aversion ($\alpha &gt; 0$)</td>
<td>$w^* &lt; w^*_{push}$</td>
<td>$w^* &gt; w^*_{pull}$</td>
</tr>
</tbody>
</table>

3.4. Structural Estimation of the Behavioral Model

We fit the stocking quantity decisions to find the levels of anchoring, loss aversion, and inequality aversion that match our data best using maximum likelihood estimation (MLE). This allows us to compare model fits and determine which behavioral factors are likely to be responsible for the regularities we observe. The estimates will also allow us to predict the proposer’s optimal wholesale price, given this behavior by the party setting stocking quantities, and compare them to the wholesale prices we observe in our experiment.

For stocking quantities, we assume errors follow a normal distribution with left side truncation at 0 and right side truncation at 100. Let $\varphi(\cdot)$ denote the density of the truncated normal distribution with mean $q^*(\cdot)$ and variance $\sigma^2$.

As previously noted, subjects also exhibited errors with respect to their accept/reject decisions. Therefore, we assume that the utility of the party stocking the inventory has an extreme value error term so that the probability of accepting a wholesale price follows a logistic form with precision parameter $\tau$:

$$\frac{\exp\{u_p / \tau\}}{\exp\{u_p / \tau\} + \exp\{60 / \tau\}}$$

Let $V$ denote the party setting the stocking quantity, and recall that the outside option is 60 in our experiment. Note, that as $\tau \to 0$, the party stocking the inventory accepts any offer that results in her expected utility exceeding the utility from the outside option of 60. Similarly, $\tau \to \infty$ results in accepting with probability 1/2.

The joint-likelihood function, where $t$ is a single decision period and $T$ denotes the total number of periods, is given by:

$$L(\alpha, \beta, \delta, \tau, \sigma) = \prod_{t=1}^{T} \varphi(q_t)^{A_t} \Pr(Accept)^{A_t} (1 - \Pr(Accept))^{1-A_t}$$

where $A_t = 1$ if the proposed wholesale price was accepted in period $t$ and 0 otherwise.

In Table 7 we present the MLEs for the normative model, errors, errors with anchoring, errors with loss aversion, and errors with inequality aversion.
Table 7. Results of the structural estimations for each of the outlined models.

<table>
<thead>
<tr>
<th>Model</th>
<th>LL</th>
<th>BIC</th>
<th>δ</th>
<th>β</th>
<th>α</th>
<th>τ</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normative⁴</td>
<td>-5434.85</td>
<td>-4974.82</td>
<td>-4958.38</td>
<td>-4951.83</td>
<td>-4973.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Errors</td>
<td>10876.79</td>
<td>9963.82</td>
<td>9938.03</td>
<td>9924.93</td>
<td>9967.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Errors +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anchoring</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loss Aversion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inequality Aversion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets.

According to the Bayesian Information Criterion (BIC), errors in the accept/reject decision improves the fit a great deal, as does loss aversion (and anchoring to a lesser extent). This is in line with our experimental data in that subjects did not always make correct accept/reject decisions, and parties set stocking quantities too low, as if the cost of unsold inventory was greater than its true value.

To compare the three models with errors more rigorously, we conducted a Vuong test (Vuong 1989). The results show that both the loss aversion model and anchoring model are significantly better for our setting than the inequality aversion model, (both $p<0.001$). Although inequality aversion plays an important role in studies with simpler settings that resemble ultimatum games (see Katok and Pavlov 2013), it can be “washed out” by factors such as competition (see Bolton and Ockenfels, 2000). It may be that in our setting, the complexity of the contracting game, with uncertain demand, makes inequality aversion less salient.

Comparing anchoring directly to loss aversion, we find that loss aversion is not significantly better than anchoring, although it is close to being weakly significant ($p=0.1139$). Recall however, that anchoring on the mean is not observed in the push contract (observed quantity of 42.60 versus conditional prediction of 44.93), so it does not explain the data well across treatments. Therefore, we focus on the loss aversion model in the out-of-sample test section (presented in Section 4), which

⁴ For the normative estimation, we set $\tau$ as low as possible such that it would yield real log-likelihood results, $\tau = 4.2$, emulating a rational decision maker.
provides a more consistent fit across treatments. Also, loss aversion has provided a good fit in similar, previous studies, such as Ho et al. (2010) and Becker-Peth et al. (2013).

We calculated the order quantities, given the maximum-likelihood estimates for the loss aversion with errors model in Table 7, and find that they match our data quite well when compared to the observed quantities in Table 2: \( q = 41.39 \) versus \( q = 42.60 \) for the push contract, and \( q = 57.63 \) versus \( q = 55.34 \) for the pull contract.

We also evaluated whether the errors and loss aversion model fit the observed rejections rates. Figure 4 plots the predicted rejection levels along with the data for the push and pull contracts. While there are some deviations in both directions, it appears that the model provides a reasonable fit on average.

![Figure 4](image URL)  
(a) Push  
(b) Pull

Figure 4. Predicted and observed probability of rejection given different wholesale prices for the push and pull contracts.

Lastly, recall that we assumed fully rational parties offering wholesale prices, who merely best reply to the party stocking the inventory. We can test how close this assumption is to reality by comparing the predicted best reply wholesale prices, given the estimates for the errors plus loss aversion model, to the average observed wholesale prices in Table 2: \( w = 8.33 \) versus \( w = 8.26 \) for push and \( w = 7.34 \) versus \( w = 8.02 \) for pull. The predicted wholesale prices are remarkably close to our data, indicating that while the full rationality assumption may not be perfectly satisfied, this simplification still results in a favorable way to organize the data.

4. **Advance-Purchase Discount Contract**

The push and pull wholesale price contracts cannot coordinate the channel due to double marginalization. However, Cachon (2004) shows that the advance purchase discount (APD) contract can coordinate the channel by distributing inventory risk between the supplier and the retailer. Next
we will review the theory for the APD contract under our behavioral model, and develop a number of experimental hypotheses, which we will proceed to test in a separate, out-of-sample, experiment.

4.1. APD Behavioral Model

Under the APD contract, a supplier begins by proposing two wholesale prices; a regular wholesale price $w$, and a discount wholesale price $w_d$. It is reasonable, although not necessary, to assume $w \geq w_d$ (see Özer and Wei 2006 for a slightly different setting where this is relaxed). A retailer then sets a prebook quantity $q_R$, where the retailer commits to purchasing the entire prebook quantity regardless of demand, and pays $w_d$ for each unit of the prebook quantity. Following this, a supplier sets a production amount $q_S$, where $q_S \geq q_R$. We will outline the APD contract for our behavioral model, where the standard theory is the special case of $\beta_R = \beta_S = 1$. Table 8 shows the expected utility functions and the corresponding optimal order quantities (please see the Appendix for corresponding derivations of the optimal quantities).

Table 8. Expected utility functions for suppliers and retailers, along with their optimal stocking quantities in the APD contract.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$u_S(q_s) = w_d q_R + w (S(q_S,q_R) - q_R) - c S(q_S,q_R) - \beta_S c (q_S - S(q_S,q_R))$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q_S^* = \begin{cases} 100 \left( \frac{w - c}{w + c (\beta_S - 1)} \right), &amp; w \geq \frac{c (100 + q_R (\beta_S - 1))}{100 - q_R} \ q_R, &amp; w &lt; \frac{c (100 + q_R (\beta_S - 1))}{100 - q_R} \end{cases}$.</td>
</tr>
<tr>
<td>Retailer</td>
<td>$u_R(q_R) = (r - w_d) S(q_R) + (r - w) (S(q_S) - S(q_R)) - \beta w_d (q_R - S(q_R))$.</td>
</tr>
<tr>
<td></td>
<td>$q_R^* = 100 \left( \frac{w - w_d}{w + w_d (\beta_R - 1)} \right)$.</td>
</tr>
</tbody>
</table>

In the table, $S(q_S,q_R)$ corresponds to the expected number of units the supplier sells when the retailer prebooks $q_R$ units and the supplier produces $q_S$ units, $S(q_S,q_R) = E \left[ \min \left[ \max (q_S, D), q_S \right] \right]$, $S(q_R) = E \left[ \min (q_R, D) \right]$, and $S(q_S) = E \left[ \min (q_S, D) \right]$. The first term in the expected utility for the supplier represents immediate revenue from the retailer's prebook quantity, the second term represents the additional revenue from selling any units above the prebook quantity, the third term is the supplier's production cost for units sold, and the last
term represents the cost and disutility from any potential leftover units. Similarly, the first term in
the expected utility for the retailer represents the profits from prebook sales, the second term the
additional profits from units sold beyond the prebook order, and the third term is the cost and
disutility of any leftover prebooked units.

Lastly, we allow errors to affect the APD contract the same way that they affect the push and pull
contracts. The retailer, faced with a proposed set of wholesale prices \( w \) and \( w_d \), accepts with
probability:

\[
\frac{\exp\left\{u_R(q_R) / \tau\right\}}{\exp\left\{u_R(q_R) / \tau\right\} + \exp\{60 / \tau\}}.
\]  

(7)

A few comments are in order regarding the APD contract. Consider the special case of the
standard theory, such that \( \beta_R = \beta_S = 1 \) and \( \tau \to 0 \). Under this setting, the supplier can achieve
100% channel efficiency by setting \( w = r \) (thus inducing herself to produce the first-best order).
Assuming the retailer plays the best response, then \( w_d \) determines the division of channel profits
between the two parties. For example, if both parties set \( q_R^* = q_R^* \) and \( q_S = q_S^* \), then the supplier can
extract 100% of the channel profits by setting \( w_d = w \). On the other hand, if the supplier sets \( w_d = c \),
the retailer would earn 100% of the channel profits because the retailer would be induced to set \( q_R \)
to the first-best order quantity, which the supplier would produce.

For our experimental setting, the standard theory predicts \( w = 15.00 \), \( w_d = 10.75 \), \( q_S = 80.00 \), and
\( q_R = 28.30 \). It also predicts 100% channel efficiency, where the split of profits is 419.79 for the
supplier and 60.21 for the retailer. Note that the standard theory results in \( r > w_d \) for our
experiment, as the retailer will only accept if their expected profit is greater than 60, the value of the
outside option.

4.2. Out-of-Sample-Test

We now investigate how our behavioral model impacts the APD contract, and generate a number of
experimental hypotheses. Our goal here is not to identify the best fitting model for the APD contract,
rather, to evaluate how a favorable model in push and pull extends to alternative structures, such as
an APD. We begin with our first formal hypothesis for the APD contract:

**Hypothesis 1 (Model):** The loss aversion plus errors model will fit the data better than the standard
theory.
In some ways the APD contract can be considered a combination of the push and pull contracts; the retailer’s prebook, and its associated cost of unsold inventory, is essentially a push contract. Additionally, the difference between the supplier’s production amount and the prebook, and its cost of unsold inventory, is similar to a pull contract. Therefore, in order to develop a hypothesis regarding whether the loss aversion parameters for the two parties may differ in the APD contract, we fit the errors plus loss aversion model to the push and pull data separately. These results are displayed in Table 9.

Table 9. Results of the structural estimations of the errors plus loss aversion model for push and pull contracts separately.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>-2428.19</td>
<td>-2515.04</td>
</tr>
<tr>
<td>BIC</td>
<td>4875.57</td>
<td>5049.28</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.083</td>
<td>1.194</td>
</tr>
<tr>
<td></td>
<td>[0.108]</td>
<td>[0.096]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>24.08</td>
<td>14.80</td>
</tr>
<tr>
<td></td>
<td>[5.22]</td>
<td>[1.98]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>16.14</td>
<td>16.71</td>
</tr>
<tr>
<td></td>
<td>[1.47]</td>
<td>[1.36]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets.

We find that the loss aversion parameter is higher in the pull contract than push contract, and that the loss aversion parameter in the push contract, while positive, is not significantly different from one. This leads to our second formal hypothesis.

**HYPOTHESIS 2 (QUANTITIES):** $\beta_R = 1$ and $\beta_S = 1.20$, such that $\beta_R < \beta_S$, causing the retailer to set prebook quantities that match the standard theory, but causing the supplier to set production levels below the normative benchmark.

Given that we have predictions about prebook quantities and production amounts, we now determine the optimal wholesale prices for the supplier. Cachon (2004) notes that in a fully rational model, the supplier maximizes his expected utility by coordinating the channel. Therefore, in this case he sets $w = r$, and then sets $w_d$ in a way that splits the channel profits in some way. However, when the party setting the stocking quantity is loss averse and makes errors, $w = r$ may not maximize the supplier’s expected utility. Therefore, the supplier’s optimal wholesale prices $w$ and $w_d$ can be computed by replacing the optimal order quantities in his utility function, and using the first order conditions to solve for the two wholesale prices simultaneously. The resulting expression is a third
degree polynomial, and therefore closed-form solutions are not readily obtainable. However, for any specific set of parameters and demand distributions, one can compute the optimal wholesale prices.

Figure 5. The effect of loss aversion and precision parameters on optimal wholesale prices in the APD contract.

Figure 5(a) plots the optimal \((w, w_d)\) pairs given our experimental parameters, when loss aversion is equal for both parties and cases of \(\tau = 1\) and \(\tau = 20\). Figure 5(b) depicts a similar plot for \(\tau = 20\), but allows the two levels of loss aversion to vary; \(\beta_R = 1.194\) and a range of \(\beta_S\). In Figure 5(a), the optimal wholesale prices converge for small levels of loss aversion when the loss aversion is restricted to be the same across both parties.\(^5\)

Figure 5(a) also illustrates that for larger \(\tau\), the two wholesale prices may converge at slightly higher levels of loss aversion. However, even for \(\tau = 20\) the level of loss aversion required for convergence (approximately 1.04) is significantly below the level of loss aversion we observe in our push and pull data (\(\beta = 1.154\) overall and \(\beta = 1.194\) for pull).

In Figure 5(b) we see that the wholesale price convergence also exists when the retailer’s level of loss aversion exceeds that of the supplier’s. This is intuitive; if the retailer is sufficiently loss averse, they will be reluctant to stock a large prebook amount. As a result, the supplier will operate much like a pull contract with a single wholesale price. However, Figure 5(b) also shows that when the supplier’s level of loss aversion becomes somewhat higher than the retailer’s, the wholesale prices split in a way such that \(w = 15\) again, but drives the discount wholesale price lower than standard theory (recall standard theory predicts a discount wholesale price of 10.75). This is also expected, by

\(^5\) We find that this convergence exists for a number of different demand distributions, such as Normal and Beta.
setting a lower discount wholesale price, a loss averse supplier attempts to push more of the
inventory risk onto the retailer. This also leads to a more equitable split of channel profits.

In regards to predicting the wholesale prices and efficiency of the APD contract, we can take our
structural estimates from push and pull separately, and those observations mentioned above, and
articulate them into our final two hypotheses:

**HYPOTHESIS 3 (WHOLESALE PRICES):** $\beta_R = 1$ and $\beta_S = 1.20$, such that $\beta_R < \beta_S$ leading to the regular
wholesale price and the discount wholesale price being equal to each other, approximately $w = w_d = 12.60$, resulting in a more equitable split of profits.

**HYPOTHESIS 4 (EFFICIENCY):** The presence of loss aversion on the supplier’s side, $\beta_S > 1.00$, will
drive production amounts down, resulting in channel efficiency below 100%.

### 4.3. APD Results

We begin by first presenting summary statistics for supply chain efficiency and profits. Figure 6
presents the supply chain results from all three treatments. Unlike the earlier supply chain efficiency
results, the APD contract performs far below its theoretical prediction. In fact, the supply chain
efficiency is virtually identical between the APD contract and pull contract. This suggests, counter to
the standard theory, that moving from a pull contract to an APD contract does not improve overall efficiency.

![Figure 6. Predicted and observed supply chain profits for the push, pull, and APD contracts.](image)

Figure 7(a) and (b) display the observed profits for all three treatments, separately for retailers
and suppliers. First, looking the APD results, we observe that profits are split in a more equitable
way than the standard theory predicts. Second, comparing APD to the push contract, we observe an
interesting phenomenon: the APD contract weakly Pareto dominates the push contract. Specifically,
retailers are better off under the APD contract compared to the push contract, and suppliers are no worse off.

![Retailer and Supplier Profit Graphs]

Figure 7. Predicted and observed retailer and supplier profits for the push, pull, and APD contracts.

We now turn to evaluating our formal hypotheses. Table 10 presents the results of the structural estimation for the APD treatment. We conduct three estimations. The first is the standard theory, the second applies the structural estimates, from the push and pull experiment of our loss aversion plus errors model to the APD data (from Table 8, where we apply $\tau = 20$ as the single value), and the third is the loss aversion plus errors model where the parameters are allowed to vary. Immediately one can observe the considerable improvement in fit over the normative benchmark, even when restricting the parameters to the push and pull estimation (a log-likelihood of $-5073.03$ versus the standard theory of $-5325.19$). If we allow the parameters to vary, we find an even stronger fit over the normative model (likelihood ratio test yields $\chi^2 = 661.96$, $p < 0.001$). This confirms our first hypothesis, that the loss aversion better describes the data than the standard theory.

We can also get a preliminary sense of our other hypotheses from the estimates in Table 10. In particular, each of the remaining three hypotheses are based on the separate push and pull predictions, $\beta_R = 1$ and $\beta_S = 1.20$, and $\beta_R < \beta_S$. In short, we have partial confirmation for all three of these hypotheses. Specifically, we do observe that the retailer’s estimate of loss aversion is close to 1, and that the supplier’s estimate of loss aversion is different and significantly higher than 1, $\beta_R < \beta_S$. However, the magnitude of the supplier’s estimate is much larger than anticipated (1.845 versus predicted of 1.20).

It is also worth mentioning that the estimate for the errors is considerably larger than in the push and pull data. This is somewhat intuitive, as the APD contract has a higher degree of complexity than the push and pull contracts.
### Table 10. MLE results for the behavioral model in the APD contract

<table>
<thead>
<tr>
<th></th>
<th>Normative</th>
<th>Errors + Loss aversion (using push and pull estimates)</th>
<th>Errors + Loss aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>-5325.19</td>
<td>-5073.03</td>
<td>-4994.71</td>
</tr>
<tr>
<td>BIC</td>
<td>10664.56</td>
<td>10160.24</td>
<td>10024.87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_R$</td>
<td>-</td>
<td>1.083</td>
<td>1.001</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>-</td>
<td>1.194</td>
<td>1.845</td>
</tr>
<tr>
<td>$\tau$</td>
<td>-</td>
<td>20.00</td>
<td>37.89</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>17.72</td>
<td>18.01</td>
<td>17.72</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>27.84</td>
<td>24.80</td>
<td>19.67</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets.

Using the estimates from the final column in Table 10, we can generate predicted quantities, wholesale prices, and profits for the behavioral model. Table 11 presents this information, along with the observed data, conditional theory, and standard theory. A few comments are in order. The column labeled “Standard Theory,” highlights the original experimental predictions, based on the special case of $\beta_S = \beta_R = 1$, outlined in Section 4.1. The column labeled “Conditional Theory” represents the standard theory’s best reply, when conditioned on decisions. Specifically, prebook quantities and production amounts are conditioned on observed wholesale prices, and corresponding profits are conditioned on these quantities. We provide these columns for informational purposes, and point out that essentially all of the observed decisions and profits are significantly different from both the standard theory and conditional theory (the only comparison that is not significant is the prebook amount).

---

6 This is to ensure a fair comparison between the standard theory and behavioral model. Specifically, in order to generate profit predictions for the behavioral model, we must use the best response prebook quantities and production amounts for the observed wholesale prices.
Table 11. Observed, behavioral theory, standard theory, and conditional theory, for the APD contract.

<table>
<thead>
<tr>
<th></th>
<th>Observed</th>
<th>Behavioral Theory</th>
<th>Standard Theory</th>
<th>Conditional Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale Price</td>
<td>11.76</td>
<td>15.00**</td>
<td>15.00**</td>
<td>15.00**</td>
</tr>
<tr>
<td></td>
<td>[0.26]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount Price</td>
<td>7.62</td>
<td>8.71**</td>
<td>10.75**</td>
<td>10.75**</td>
</tr>
<tr>
<td></td>
<td>[0.24]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prebook ( (w, w_d) )</td>
<td>36.75</td>
<td>34.73</td>
<td>28.30**</td>
<td>34.75</td>
</tr>
<tr>
<td></td>
<td>[2.46]</td>
<td>[1.61]</td>
<td></td>
<td>[1.61]</td>
</tr>
<tr>
<td>Production ( (w, w_d) )</td>
<td>58.99</td>
<td>60.02</td>
<td>80.00**</td>
<td>73.16</td>
</tr>
<tr>
<td></td>
<td>[3.99]</td>
<td>[0.94]</td>
<td></td>
<td>[0.84]</td>
</tr>
<tr>
<td>Channel Efficiency</td>
<td>84.1%</td>
<td>86.8%**</td>
<td>100%**</td>
<td>94.0%**</td>
</tr>
<tr>
<td></td>
<td>[1.23]</td>
<td>[0.63]</td>
<td></td>
<td>[1.71]</td>
</tr>
<tr>
<td>Supplier Profit</td>
<td>215.41</td>
<td>208.29</td>
<td>420.00**</td>
<td>246.08**</td>
</tr>
<tr>
<td></td>
<td>[10.96]</td>
<td>[11.55]</td>
<td></td>
<td>[13.74]</td>
</tr>
<tr>
<td>Retailer Profit</td>
<td>188.33</td>
<td>208.55**</td>
<td>60.00**</td>
<td>205.17**</td>
</tr>
<tr>
<td></td>
<td>[10.24]</td>
<td>[10.49]</td>
<td></td>
<td>[9.17]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets. Significance of Wilcoxon signed-rank test comparing the models to the observed values given by *** p-value<0.01 and ** p-value<0.05.

Let us return to our formal hypotheses. In regards to our second hypothesis that deals with stocking quantities, we do in fact observe that prebook amounts match the standard theory, due to the loss aversion parameter for retailers not being different from 1.00. Furthermore, production amounts are significantly lower than the standard theory predicts, thus confirming this hypothesis.

While our second hypothesis is directionally correct, note that the production levels are lower than originally predicted, due to the loss aversion parameter for the supplier being 1.845 (rather than 1.20). However, when calculating production levels based on the MLEs, the behavioral model is very accurate compared to the data (60.02 versus 58.99).

Turning to our third hypothesis, which deals with wholesale prices, we find little supporting evidence. As previously noted, depending on the magnitude of the difference between the loss aversion parameters, one could expect the wholesale prices to converge, or split similar to the standard theory. Even though the retailer’s loss aversion parameter was qualitatively predicted correctly, the supplier’s loss aversion parameter was so large that the predicted wholesale prices split back to \( w=15 \) and \( w_d=8.71 \) (refer to Figure 5(b)). Therefore, we reject our third hypothesis, but will return to these wholesale prices in the next subsection.

Lastly, we focus on our fourth hypothesis that deals with efficiency. The predicted presence of loss aversion for the supplier should drive the expected supply chain efficiency below 100%, which is what we observe, thus confirming our final hypothesis. More precisely, the observed supply chain
efficiency (as observed earlier in Figure 6 and Table 11) is 84.1%, whereas the loss aversion plus errors model predicts efficiency of 86.8%.

In summary, we find qualitative support for three of our four hypotheses, but not hypothesis 3, dealing with wholesale prices. In this case, even when considering the structural estimates, the observed decisions are slightly different from behavioral predictions. The following section offers some additional explanations about what may be driving overall wholesale prices.

4.4. Explaining Wholesale Prices in the APD Contract

We offer three informal explanations as to why wholesale prices are lower than the behavioral predictions: (1) random errors, (2) the flatness of the supplier’s expected utility function, and (3) learning.

First, because the optimal wholesale price is 15 (equal to $r$), if suppliers make random errors, we would expect the average observed $w$ to be below 15, simply due to truncation. Second, the supplier’s expected utility function is relatively flat. Figure 8 displays the contour plot of the supplier’s expected utility function, for the behavioral parameters we estimated and assuming correct stocking quantities. The plot suggests that the supplier’s expected utility is quite flat for a range of wholesale prices.

![Figure 8. A contour plot of the supplier’s expected utility function in the APD contract for the MLEs ($\beta_R = 1.001, \beta_S = 1.845, \tau = 37.89$). The arrows denote the average observed wholesale prices and behavioral predictions.](image)

Third, in Figure 9 we plot average wholesale prices over time. It is apparent from the figure that initially both wholesale prices increase rather quickly—suppliers learn to design more profitable contracts. This is not surprising, as the APD contract is more complex than the push and pull
contracts. However, it appears that the discount wholesale price stabilizes over time, while the regular wholesale price continues to increase for the duration of the session. We ran a set of linear regressions with period as the independent variable, the two wholesale prices as dependent variables (in separate regressions), and random effects for individuals. For the regular wholesale price the coefficient on period is positive and significant for the entire session, as well as for various sub-sessions (period > 10, p < 0.001; period > 20, p = 0.034). In contrast, for the discounted wholesale price, while the period coefficient is positive and significant for the overall session (p < 0.001), it is not significant for the sub-session in which period > 10 (p = 0.143) or period > 20 (p = 0.67). So the discounted wholesale price quickly settles down around $w_d = 8$, while the regular wholesale price continues to increase.

![Figure 9. Average wholesale prices over the duration of the APD treatment.](image)

What effect does this learning have on the suppliers’ expected utility? If both players behaved according to the behavioral model (in other words, if the retailer and supplier set quantities according to their best response functions), then the supplier’s expected utility would have increased from 136 at the start of the session to 236 by the end of the session—an increase of about 74%. In fact, by the end of the session the supplier’s expected utility of 236 would be relatively close to the predicted expected utility of 269, only a 13% difference (utility given $w = 15$, $w_d = 8.71$, and correct stocking quantities).

### 4.5. Robustness Check

In comparing the push and pull contracts, the relative bargaining power of the two players is somewhat constant (up to the “retailer” and “supplier” labels), as one player offers prices and the other player sets a stocking quantity. In the APD contract, the relative bargaining power changes compared to both push and pull, because now the supplier has the ability to extract all profit from the
channel by taking on some of the inventory risk. As a robustness check we also collected data on an APD contract (called APD alternative) with the same ability to split inventory risk, but different bargaining power structure. Under this APD alternative, the supplier proposes \( w_d \), then the retailer offers \( w \) and \( q_R \), and finally the supplier decides on \( q_S \). This way, both parties set a wholesale price and stock a quantity. The APD alternative cannot fully coordinate the channel, but given our experimental parameters, it can achieve 94% efficiency, with the retailer earning about 78% of the profits.

With this structure, suppliers set \( w_d \) too high, and nearly at an identical level as the wholesale price in the push contract. The retailers then respond with a \( w \) that is also too high, and nearly identical to the pull contract. Retailers set \( q_R \) higher than expected, thus assuming more inventory risk than they should. Suppliers still produce less than predicted, agreeing with our earlier loss aversion estimates. In short, the overall the performance of this APD alternative contract is similar to the pull and original APD contracts in terms of efficiency (about 80%), and to pull in terms of the profit split (retailer earns 55% of the profit). A more complete analysis of the APD-alternative contract is available upon request.

5. Conclusion

In this study we evaluate three wholesale price contracts, each differing in how inventory risk is allocated across the supply chain. Managers, who rely on human judgment in making these strategic decisions, design supply contracts. Therefore, understanding how people make decisions that involve inventory risk is a key step to helping managers design behaviorally robust contracts. We begin by testing the performance of the push and pull contracts in the laboratory and find that, consistent with the standard theory, the pull contract results in higher channel efficiency. However, standard theory fails to capture some important quantitative predictions, specifically, that orders are lower than they should be, rejection rates are positive, and the wholesale prices are far from the normative benchmark. We proceed to estimate and compare several behavioral models that have been used in the literature: random errors alone and random errors combined with loss aversion, anchoring, and inequality aversion. Ultimately a simple model with random errors and loss aversion fits the data quite well.

We further test our model through an out-of-sample test with the APD contract. In this additional experiment, we find that our behavioral model provides accurate predictions of the most critical decision for channel efficiency, production amounts. It also makes the correct qualitative prediction
that average discount wholesale prices should be significantly lower than average regular wholesale prices. However, it fails to correctly predict the levels of wholesale prices. There are three suggestive explanations for this. First, errors for the regular wholesale price are one sided; the standard theory predicts a regular wholesale price equal to the seller revenue per unit. Second, the supplier’s expected utility function is quite flat in the region of the wholesale prices we observe. Third, wholesale prices increase throughout the session, so that suppliers are able to increase their expected utility by roughly 74% from the start to the end of the session. Ultimately, by the end of the session, suppliers’ expected utility is within 13% of the utility achieved by the optimal wholesale prices.

A limitation of our work is that our data does not allow us to separately estimate the effect of the different behavioral irregularities. This is because many of these motivations have a similar effect on order quantities and best-response wholesale prices. Separating the effect of these behavioral factors is an important direction for future research. One possibility might be to create a competitive market where there are an unequal number of multiple suppliers and retailers, and one side is therefore at a disadvantage, similar to Leider and Lovejoy (2013).

A key managerial implication of our work pertains to which inventory structure performs best for the supply chain and the parties involved. Our experimental results indicate that when supply chains are choosing which inventory arrangement to use, retailers should prefer the pull contract, and suppliers should prefer the push or APD contract. More importantly, when considering the push and APD contracts, retailers are better off under the APD contract, and suppliers are no worse off, making the APD contract an attractive option that also has an equitable split of profits.

At first glance implementing a pull or APD contract might not seem feasible for certain companies. For example, consider a brick-and-mortar retailer. These retailers may have the need to carry at least some product on shelf. Having product on shelf does not necessarily preclude a supply chain from operating under a pull or APD contract. For the pull contract, the supply chain might consider vendor-managed-inventory, where the product is physically on shelf, but the supplier manages the inventory decisions and may retain ownership until point-of-sale. Furthermore, the APD contract elegantly addresses this problem as well; the prebook quantity allows the retailer to carry at least some product on shelf, where the retailer can order more if needed.

In conclusion, our study suggests that retailers and suppliers should carefully evaluate their inventory risk arrangements, as the location of this risk in the supply chain can have serious consequences on profits for both parties and the overall supply chain.
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References


Appendix: Behavioral Model Derivations

Here we present the derivation of the optimal quantities for the push, pull, and APD contracts.

A.1 Push Contract

For the push contract under loss aversion, the problem the retailer must solve, to find the optimal quantity, is as follows:

\[ \max_{q>0} u_R = (r + w(\beta - 1))(q - \int_0^q F(x)dx) - \beta w q. \]  

(8)

The solution for this problem is given by:

\[ q^* = F^{-1}\left(\frac{r - w}{r + w(\beta - 1)}\right). \]  

(9)

When \( F \) is \([0,1]\) the problem, and the corresponding solution, yield:

\[ \max_{q>0} u_R = (r + w(\beta - 1))(q - \frac{q^2}{2}) - \beta w q, \]  

(10)

\[ q^* = \frac{r - w}{r + w(\beta - 1)}. \]  

(11)

In turn, the problem the retailer must solve in a push contract under inequality aversion is:

\[ \max_{q>0} u_R = r\left(q - \int_0^q F(x)dx\right) - w q - \alpha(\pi_S(w) - \pi_R(q))^+. \]  

(12)

In this setting we need to consider two cases to find the optimal quantities:

i. when \( q(2w - c - r) + r\int_0^q F(x)dx \geq 0, \) \((\pi_S(w) - \pi_R(q))^+ = (2w - c)q - r\left(q - \int_0^q F(x)dx\right)\), and

ii. when \( q(2w - c - r) + r\int_0^q F(x)dx \leq 0, \) \((\pi_S(w) - \pi_R(q))^+ = 0.\)

Note that in both cases, when \( q(2w - c - r) + r\int_0^q F(x)dx = 0, \) \((\pi_S(w) - \pi_R(q))^+ = 0.\)

Using conditions (i) and (ii) we can solve each separately to find the optimal quantities, and then establish the range of wholesale prices for which they remain optimal. That way, we can define what the optimal quantity should be, given a wholesale price. However, the solution for the optimal
quantity and the wholesale price thresholds, for a general distribution, cannot be found explicitly. Therefore, when \( F \) is a \( U[0,1] \) the results are:

Condition (i) holds for \( w \geq \bar{w} \), and corresponds to \( q \geq \frac{2(r+c-2w)}{r} \). The problem in this case is:

\[
\max_{q \geq 0} u_R = r \left( q - \frac{q^2}{2} \right) - wq - \alpha \left( 2wq - cq - r \left( q - \frac{q^2}{2} \right) \right),
\]

\text{s.t.} \quad q \geq \frac{2(r+c-2w)}{r}, \quad (13)

\[
q^* = \frac{r - w + \alpha(r + c - 2w)}{r(1 + \alpha)}, \quad (14)
\]

\[
\bar{w} = \frac{r(1 + \alpha) + c(2 + \alpha)}{3 + 2\alpha}. \quad (15)
\]

Condition (ii) holds for \( w \leq \bar{w} \), and corresponds to \( q \leq \frac{2(r+c-2w)}{r} \). The problem in this case is follows:

\[
\max_{q \leq 0} u_R = r \left( q - \frac{q^2}{2} \right) - wq,
\]

\text{s.t.} \quad q \leq \frac{2(r+c-2w)}{r}, \quad (16)

\[
q^* = \frac{r - w}{r}, \quad (17)
\]

\[
w = \frac{r + 2c}{3}. \quad (18)
\]

When \( w < \bar{w} < \bar{w} \):

\[
q^* = \frac{2(r+c-2w)}{r}. \quad (19)
\]

Note that if \( w \geq \frac{r + \alpha(r + c)}{1 + 2\alpha} > \bar{w} \), then the optimal quantity is equal to zero, since it cannot be negative.

Summarizing, we have the following results for the push contract under inequality aversion:
A.2 Pull Contract

For the pull contract under loss aversion, the problem the supplier must solve, to determine the optimal quantity is:

\[
q^* = \begin{cases} 
\frac{r - w + \alpha(r + c - 2w)}{r(1 + \alpha)}, & w \geq \bar{w}; \\
\frac{2(r + c - 2w)}{r}, & \bar{w} < w < \bar{w}; \\
\frac{r - w}{r}, & w \leq \bar{w}.
\end{cases}
\]

The solution for this problem yields:

\[
q^* = F^{-1}\left(\frac{w - c}{w + c(\beta - 1)}\right). \tag{21}
\]

When \(F\) is U[0,1] the corresponding problem and solution are given by:

\[
\max_{q \geq 0} u_S = (w + c(\beta - 1)) \left(q - \int_0^w F(x)dx\right) - \beta cq. \tag{22}
\]

\[
q^* = \frac{w - c}{w + c(\beta - 1)}. \tag{23}
\]

In the case of a pull contract under inequality aversion, the problem the supplier needs to solve is defined as follows:

\[
\max_{q \geq 0} u_S = w \left(q - \int_0^w F(x)dx\right) - cq - \alpha(\pi_R(w) - \pi_S(q))^+. \tag{24}
\]

Similar to the push contract, in this setting we need to consider two cases to find the optimal quantities:

i. when \((r - 2w)\left(q - \int_0^w F(x)dx\right) + cq \geq 0, \ (\pi_R(w) - \pi_S(q))^+ = (r - 2w)\left(q - \int_0^q F(x)dx\right) + cq\), and

ii. when \((r - 2w)\left(q - \int_0^w F(x)dx\right) + cq \leq 0, \ (\pi_R(w) - \pi_S(q))^+ = 0\).

Note that in both cases when \((r - 2w)\left(q - \int_0^w F(x)dx\right) + cq = 0\), \((\pi_R(w) - \pi_S(q))^+ = 0\).

As in the push contract, using conditions (i) and (ii) we can solve each separately to find the optimal quantities, and then establish the range of wholesale prices for which they remain optimal. However,
as with the push contract, they cannot be found explicitly for a general distribution. When \( F \) is \( U[0,1] \) the results are as follows:

Condition (i) holds for \( w \leq \bar{w} \), and corresponds to \( q \leq \frac{2(r+c-2w)}{r-2w} \). The problem in this case is:

\[
\max_{q \geq 0} u_S = w\left(q - \frac{q^2}{2}\right) - cq - \alpha \left(r - 2w\right)\left(q - \frac{q^2}{2}\right) + cq,
\]

\( \text{s.t.} \quad q \leq \frac{2(r+c-2w)}{r-2w} \),

\[ q^* = \frac{w-c}{w} \left(r - \alpha \left(r - 2w\right) \right), \quad (26) \]

\[ w = \frac{4\alpha r + r + 2\alpha c + \sqrt{(4\alpha r + r + 2\alpha c)^2 + 8r(1 + 2\alpha)(c(1 - \alpha) - \alpha r)}}{4(1 + 2\alpha)}. \quad (27) \]

Condition (ii) holds for \( w \geq \bar{w} \) and corresponds to \( q \geq \frac{2(r+c-2w)}{r-2w} \). The problem in this case is:

\[
\max_{q \geq 0} u_S = w\left(q - \frac{q^2}{2}\right) - cq,
\]

\( \text{s.t.} \quad q \geq \frac{2(r+c-2w)}{r-2w} \),

\[ q^* = \frac{w-c}{w} \left(r - \alpha \left(r - 2w\right) \right), \quad (29) \]

\[ \bar{w} = \frac{r + \sqrt{r^2 + 8cr}}{4}. \quad (30) \]

When \( \bar{w} < w < \bar{w} \):

\[ q^* = \frac{2(r+c-2w)}{r-2w}. \quad (31) \]

Note again that if \( w \leq \frac{c + \alpha(r + c)}{1 + 2\alpha} < \bar{w} \), then the optimal quantity is equal to zero, since it cannot be negative.

Summarizing, we have the following results for the pull contract under inequality aversion:
Figure 10 plots the optimal quantities in the push and pull contract under the standard theory, loss aversion with $\beta = 1.15$, and inequality aversion with $\alpha = 1$, and $r = 15$ and $c = 3$.

\begin{align*}
q^* &= \begin{cases}
\frac{w-c}{w}, & w \geq \bar{w}; \\
\frac{2(r+c-2w)}{r-2w}, & w < w < \bar{w}; \\
\frac{w-c - \alpha(r+c-2w)}{w-\alpha(r-2w)}, & w \leq \bar{w}.
\end{cases}
\end{align*}

A.3 APD Contract

The problem the supplier needs to solve to determine the optimal quantity $q_s^*$ given the retailer’s proposed quantity is as follows.

$$\max_{q_S \geq q_R} u_S(w, w_d, q_s) = w_d q_R + w(S(q_S, q_R) - q_R) - cS(q_S, q_R) - \beta_S c(q_s - S(q_s, q_R))$$

where $S(q_s, q_R) = E \left[ \min \left( q_R, D \right), q_s \right] = q_R F(q_R) + \int_{q_R}^{q_s} x f(x) dx + q_S \left( 1 - F(q_s) \right)$. 

The corresponding optimal production quantity is given by:
\[
q_s^* = \begin{cases} 
F^{-1}\left(\frac{w - c}{w + c(b_s - 1)}\right), & w \geq \frac{c(1 + F(q_s)(b_s - 1))}{1 - F(q_s)}; \\
q_r, & w < \frac{c(1 + F(q_s)(b_s - 1))}{1 - F(q_s)}.
\end{cases}
\quad (34)
\]

When \( F \) corresponds to a U[0,1], then:

\[
q_s^* = \begin{cases} 
\frac{w - c}{w + c(b_s - 1)}, & w \geq \frac{c(1 + q_s (b_s - 1))}{1 - q_s}; \\
q_r, & w < \frac{c(1 + q_s (b_s - 1))}{1 - q_s}.
\end{cases}
\quad (35)
\]

We next consider the problem that the retailer must solve to set the prebook quantity, given the proposed wholesale prices by the supplier, which can be stated as follows:

\[
\max_{q_R \geq 0} u_R(q_R) = (r - w_d)S(q_R) + (r - w)(S(q_s) - S(q_R)) - \beta w_d (q_R - S(q_R)),
\]

where \( S(q_R) = E[\min(q_R, D)] \), and \( S(q_s) = E[\min(q_s, D)] \).

The corresponding optimal prebook quantity is given by:

\[
q_r^* = F^{-1}\left(\frac{w - w_d}{w + w_d(b_r - 1)}\right).
\quad (37)
\]

When \( F \) is a U[0,1] then:

\[
q_r^* = \frac{w - w_d}{w + w_d(b_r - 1)}.
\quad (38)
\]