Calculus I Review

Nov 28, 2006

Complex Numbers

Perform the operations and write the result in the form $a + bi$.

(a) $(1+3i)^2$

(b) $\dfrac{1+3i}{i} - (6+i)$

(c) $\dfrac{1+3i}{4+i^4} + \dfrac{7+i^3}{4i^2+3}$

Limits

1. Give the $\varepsilon - \delta$ definition for the limit statement; $\lim_{x\to c} f(x) = L$.
2. Use the definition of limit to find a suitable δ that shows
   (a) $\lim_{x\to -2} 3x - x^2 = -10$
   (b) $\lim_{x\to 1} 3x^2 - 4x = -1$

3. Evaluate the limit

(a) $\lim_{x\to\infty} \cos\left(\dfrac{\pi x+6}{\pi+6x}\right)$

(b) $\lim_{x\to\infty} \dfrac{1/x}{\sin(4/x)}$

(c) $\lim_{x\to \pi/3} \left(\dfrac{1-\cos 3x}{\sin 3x}\right)$

(d) $\lim_{x\to 2^+} \left(\dfrac{5}{x^2+x-6} - \dfrac{1}{x-2}\right)$

(e) $\lim_{x\to 0^+} (e^{2x}+x)^{1/(3x)}$

(f) $\lim_{x\to 0} \left(\dfrac{x-\tan 3x}{x+\tan 2x}\right)$

(g) $\lim_{x\to\infty} \left\lfloor \ln\left(e^{\sqrt{x^2+7x}}\right) - x \right\rfloor$

(h) $\lim_{x\to\infty} x \ln\left(\dfrac{x^2}{x^2-1}\right)$

Differentiation

1. State the definition of the derivative of a function then use the definition to find the derivative of the functions.
   (a) $f(x) = 4x^2 - x$
   (b) $f(x) = \sin 5x$
   (c) $f(x) = \dfrac{1}{7-\sqrt{x+3}}$

2. Find the indicated derivative and simplify.

(a) $y = x^3(4-5x^2)$, $\dfrac{d^2y}{dx^2}$

(b) $y = \dfrac{\left(x^2-1\right)^{3/2}}{3x^3}$, $\dfrac{dy}{dx}$

(c) $f(x) = \cos^3\left(\dfrac{2}{x^2+3}\right)$, $f'(x)$

(d) $y = \dfrac{x^{1/3}(9-x)^5}{\sqrt{2x^2+1}}$, $\dfrac{dy}{dx}$

(e) $y = e^{2x}\sec 3x$, $\dfrac{d^2y}{dx^2}$

(f) $y\ln x = x\arctan y$, $\dfrac{dy}{dx}$

(g) $y = x^{\sin x^2}$

(h) $F(x) = \displaystyle\int_1^{\tan x} \sqrt{t^3-1}\ dt$, $F'(x)$

3. Related rates

(a) Assume x and y are differentiable functions of t. Find $\dfrac{dx}{dt}$ given

$x = 2$; $y = 1$; $\dfrac{dy}{dt} = 9$, and $3x^2 + 2y^3 = 57$

(b) A 5-meter-long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at a rate of 0.4 m/sec. Determine how fast the top of the ladder is descending when the foot of the ladder is 3 meters from the house.

(c) The height of a cylinder is increasing at a rate of 2 centimeters per minute. Find the rate of change of the volume of the cylinder with respect to time when the height is 10 centimeters if the radius is constant at 4 cm.

(d) An equilateral triangle has sides that are changing with time. If the length of each side increases at the rate of 0.3 meters per hour, how fast is the area increasing when each side is of length 4 meters.

4 Determine the absolute extrema of the function on the indicated interval.

(a) $f(x) = x\sqrt{2x+6}$; $[-5/2, -1/2]$

(b) $f(x) = 5x^{2/3} + 2x^{5/3}$; $[-8, 8]$

5. State the Mean Value Theorem.

Determine whether the Mean Value Theorem applies to the function on the indicated interval and, if so, find a value 'c' guaranteed by the theorem.

(a) $f(x) = 7 - \dfrac{6}{x}$; $[1,6]$

(b) $f(x) = x\sqrt{9-x}$; $[0,9]$

(c) $f(x) = \sin 2x - 2\sin x$; $[\pi, 2\pi]$

6. For each function below,

(a) determine the domain (unless otherwise specified)

(b) find any intercepts and determine any asymptotes

(c) compute $f'(x)$ and $f''(x)$

(d) Determine the critical numbers, identify the intervals on which *f* is increasing and those on which *f* is decreasing, and find any relative extrema.
(e) identify the intervals on which *f* is concave up and those on which *f* is concave down, then determine the coordinates of any points of inflection.
(f) sketch the graph of the function

$$f(x) = (x-2)^3(3x+14)$$
$$f(x) = \frac{27(x-1)^2}{x^3},\ x > 0$$

7. (a) A rectangular plot is to be bounded on one side by a straight river and enclosed on the other 3 sides by a fence. If the plot is to have an area of 80,000 square feet, use calculus to determine the minimum length of fence that may be used.

(b) Find the point on the graph of $y = \sqrt{x+1}$ closest to the point (3, 0).

8. (a) Find the values of *dy* and Δy for $y = x^3 - 2x$ when $x = 2$ and $\Delta x = 0.1$.

(b) The measurement of the edge of a piece of square floor tile is found to be 12 inches with a possible error of 0.02 inches. Use differentials to approximate the maximum possible error in the calculated area of the tile. Find the relative error and percent error in the area calculation.

(c) The measurement of the circumference of a sphere is given as 54 centimeters. Approximate the percentage error in computing the surface area of the sphere if the possible error in measurement of the circumference is 0.6 centimeters. Round your answer to two decimal places.

Integration

1. Evaluate the integral

(a) $\int_6^8 \frac{x}{\sqrt{100-x^2}}\,dx$

(b) $\int_{-5}^{2} \frac{x}{(x+6)^{2/3}}\,dx$

(c) $\int_0^3 \frac{2x-3}{9+x^2}\,dx$

(d) $\int_1^{e^3} \frac{1-\ln x}{x}\,dx$

(e) $\int 2x \arcsin x\,dx$

(f) $\int \frac{\sqrt{9x^2-1}}{x^4}\,dx$

(g) $\int \frac{1}{x^2\sqrt{x^2+25}}\,dx$

(h) $\int \frac{1}{\left(\sqrt{x^2+1}\right)^5}\,dx$

(i) $\int_0^{\pi/12} (\sin 3x + \cos 3x)^2\,dx$

(j) $\int \tan^2 x \sec^4 x\,dx$

(k) $\int \frac{48}{(x+3)(x-1)^2}\,dx$

(l) $\int \frac{3x^2-6x+9}{(x^2+9)(x-3)}\,dx$

Applications of Integration

Caution: the following problems may require integration techniques from ch 8

1. Area

(a) Find the area of the region bounded by the graphs of $y = \frac{x}{2}$ and $y = \sqrt{x}$ (4/3)

(b) Find the area of the region determined by the inequalities:
$y \geq x^2 + 1$ and $y \leq x + 1$. (1/6)

(c) Find the area of the region bounded above by the graph of $y = \sqrt{4 - x^2}$ and below by the line $y = 1$. $\frac{4\pi}{3} - \sqrt{3}$

(d) Find the area of the region bounded by the graphs of $y = \ln x$, $y = 2$, and $x = 1$.
$e^2 - 3$

2. Volume

(a) Find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the *x*-axis.
$y = \sin x$, $y = 0$, and $0 \leq x \leq \pi/3$

(b) Find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the *x*-axis.
$y = 2 + \sin x$, $y = 0$, $x = 0$, and $x = 2\pi$

(c) Find the volume of the solid of revolution created by rotating the region bounded above by the graph of $y = \sqrt{4 - x^2}$ and below by the line $y = 1$ about the *x*-axis.

Complex Numbers

1. (a) $-8+6i$ (b) $-3-2i$ (c) $\frac{-34}{5}+\frac{8}{5}i$

Limits

2. (a) $\delta=\min(1,\varepsilon/8)$ (b) $\delta=\min(1,\varepsilon/5)$

3. (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{4}$ (c) DNE (d) $\frac{-1}{5}$ (e) 1^{∞}, e (f) $\frac{0}{0}$; $\frac{-3}{2}$ (g) $\infty-\infty$ $\frac{7}{2}$ (h) $\infty 0$; 0

Differentiation

2. (a) $y''=24x-100x^3$ (b) $y'=\frac{(x^2-1)^{1/2}(-2x^2+x+2)}{2x^4}$

(c) $f'(x)=\frac{-12x}{(x^2+3)^2}3\cos^2\left(\frac{2}{x^2+3}\right)\sin\left(\frac{2}{x^2+3}\right)$ (d) $y'=\frac{x^{1/3}(9-x)^5}{\sqrt{2x^2+1}}\left[\frac{1}{3x}-\frac{5}{9-x}-\frac{2x}{2x^2+1}\right]$

(e) $y''=e^{2x}\sec 3x\left[11+9\tan 3x+18\tan^2 3x\right]$ (f) $\frac{dy}{dx}=\frac{\arctan y-y/x}{\ln x-x/(1+y^2)}$

(g) $y'=x^{\sin x^2}\left[2x\cos x^2\ln x+\frac{\sin x^2}{x}\right]$ (h) $F'(x)=\left(\sqrt{\tan^3 x-1}\right)\sec^{2x}$

3. (a) $\frac{dx}{dt}=\frac{-9}{2}$ (b) 0.3 m/sec (c) $\frac{dV}{dt}=32\pi\ cm^3/\min$ (d) $0.6\sqrt{3}\approx 1.039\ m^2/hr$

4. (a) Abs min $-3\sqrt{3}/2$, abs max -1 (b) Abs min -44, abs max 84

5. (a) $c=\sqrt{6}$ (b) $c=6$ (c) $c=4\pi/3$

7. (a) 800 ft. (b) $(5/2,\sqrt{7/2})$

8. (a) $dy=1$, $\Delta y=1.061$ (b) $dA=0.48$ sq in, $dA/A=0.00\overline{3}=0.\overline{33}\%$ (c) 2.22%

Integration

1. (a) 2 (b) $\frac{-27}{4}$ (c) $\ln 2-\frac{\pi}{4}$ (d) $\ln\frac{1}{2}$ (e) $\left(x^2-0.5\right)\arcsin x+0.5x\sqrt{1-x^2}+C$

(f) $9(9x^2-1)^{3/2}+C$ (g) $\frac{-1}{25x}\sqrt{x^2+25}+C$ (h) $\frac{x}{\sqrt{x^2+1}}-\frac{x^3}{(x^2+1)^{3/2}}+C$ (i) $\frac{1}{6}\left(1+\frac{\pi}{2}\right)$

(j) $\frac{\tan^3 x}{3}+\frac{\tan^5 x}{5}+C$ (k) $\frac{12}{1-x}+\ln\left|\left(\frac{x+3}{x-1}\right)^3\right|+C$ (l) $\ln\left|(x-3)(x^2+9)\right|+C$

1. Area (a) 4/3 (b) 1/6 (c) $\frac{4\pi}{3}-\sqrt{3}$ (d) e^2-3