Exam 2 Review
Chapter 3, Sections 4.1, 4.4

3/5/08
Sec 3.1
1. Determine the absolute extrema of the function on the given interval.

(a) $f(x) = -\frac{1}{x} + \frac{4}{3x^2}$ $[1, 2]$

(b) $f(x) = \frac{2}{\sqrt{x}} + x$ $[1, 4]$

(c) $f(x) = x + \cos x - \frac{1}{2}\sin 2x$ $[-\pi/2, \pi/2]$

Sec 3.2

1. State the Mean Value Theorem.
2. Show that the function $f(x) = x + \frac{1}{x}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[2,3]$. Find all numbers c in the interval guaranteed by the theorem.
3. Consider the function $f(x) = x^{1/3}$
   (a) Determine whether the mean value theorem applies to the function on the interval $[-1,8]$.
   (b) Sketch the graph of the function and draw the segment connecting the endpoints.
   (c) Is there a tangent to the curve that is parallel to the segment? If so find the point of tangency.

Sec 3.3

1. For each function, find any critical numbers of *f*, identify the <u>open</u> intervals on which the function is increasing and the <u>open</u> intervals on which the function is decreasing. Classify any relative extrema. (give *x* and *y* values).

   (a) $f(x) = \frac{x^2}{x-3}$

   (b) $f(x) = \sin x - x\cos x$ $(-4, 4)$

Sec 3.4

1. Find the open intervals on which the function is concave up and the intervals where it is concave down. Find points of inflection, if any; give only *x* values.
   $f(x) = -36\sqrt{x} - \frac{4}{15}x^{5/2}$
2. If $f(x) = x(x-3)^2$ show that the point of inflection is the midpoint of the segment connecting the relative extrema of *f*.

3. If $f(x) = ax^3 + bx^2$ find constants *a* and *b* so that *f* has a point of inflection at (-2, 16).

Sec 3.5

1. Find each of the limits if possible.

(a) $\lim_{x\to\infty} \tan\left(\dfrac{\pi x^2}{3x-4x^2}\right)$ (b) $\lim_{x\to-\infty} \dfrac{\sqrt{16x+4x^2}}{8-3x}$

(c) $\lim_{t\to\infty} \sqrt{4t^2+3} - 2t$ (d) $\lim_{x\to\infty} \dfrac{\sqrt{x+\sqrt{4x+1}}}{\sqrt{2x+1}}$

Sec 3.6

1. Let $f(x) = x^4 - 2x^3 + 2x - 1$.

(a) If $f'(1) = 0$, find all critical numbers of *f*

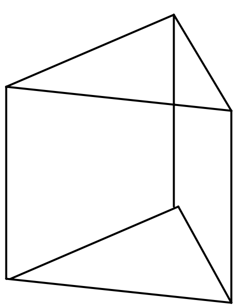
(b) Find the open intervals on which the graph of the function is increasing, the open intervals on which it is decreasing, and classify any relative extrema.

(c) Discuss the concavity of the graph of the function and find any point(s) of inflection.

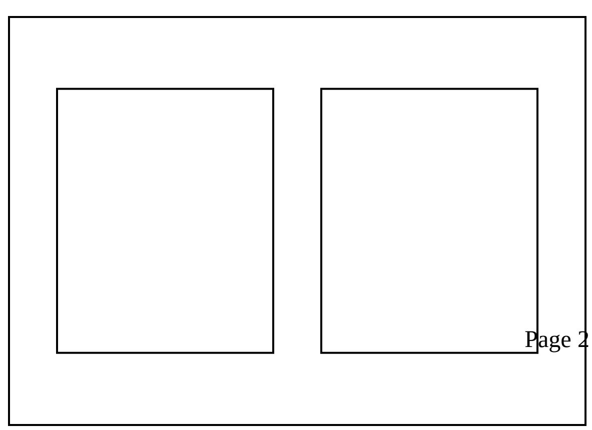
Sec 3.7

1. Find the point on the curve $8y = 40 - x^2$ that is closest to the origin.

2. The cross section of a right triangular prism is an equilateral triangle. Find the smallest surface area possible if the volume of the prism is 16 cubic inches.



3. A rectangular page is to be constructed with two print areas of equal size as shown below. If each print area is to be 200 mm^2, vertical margins of width 4 *mm*, top and bottom margins of of width 6 *mm*, find the dimensions of the print areas that will minimize the area of rectangular page. What are the dimensions of the page?



Sec 3.9

1. Find dy if dx = -.04 and x = 4 where $y = \frac{\sqrt{x}}{\left(x^2+9\right)^2}$.
2. The range of a gun is given by the formula $x = \frac{V^2 \sin 2\alpha}{32}$ where V is the muzzle velocity and α is the angle of elevation. To hit a certain target, the charge was designed to produce a muzzle velocity of $V = 640$ ft/sec. If $\alpha = 15°$ compute the theoretical range. Approximate the percent error in the range if the velocity is off by 1%. Bambi is within 100 ft of the target, is he at risk?
3. Use differentials to estimate $\sqrt[3]{8.3}$ to three significant digits. Compare with the result from your calculator.
4. The length of a side of an equilateral triangle has measure 6.6 ± 0.02 cm. Use differentials to estimate the error in the calculated area of the triangle.

Sec 4.1

1. Solve the differential equation

   (a) $\frac{dy}{dx} = \frac{2}{x^3} - \frac{4}{x^5}$; $(1,-3)$

2. Evaluate the integral

   (a) $\int \frac{2+\cos^2 x}{1-\sin^2 x}\,dx$ (b) $\int \frac{(\sqrt{x}-x)(3x+1)}{\sqrt{x}}\,dx$

Sec 4.4

1. Find the area of the region bounded by the x-axis, the function and the vertical lines.

   (a) $f(x) = x^{1/2} + \frac{1}{x^{1/2}}$, $x=1$, $x=16$

   (b) $y = 3+\sqrt{3}\sec^2 x$, $x = 0$, $x = \frac{\pi}{3}$
2. Find the average value of the functions in problem 1.
3. Use the second fundamental theorem of Calculus to find $F'(x)$

   (a) $F(x) = \int_2^{\cos 5x} \frac{1}{3+t^3}\,dt$ (b) $F(x) = \int_{x^3}^{x^2} \frac{1}{1+t^5}\,dt$

Selected Answers

Sec 3.1

1. (a) Max. $f(1)=\frac{1}{3}$, Min. $f(2)=\frac{-1}{6}$

   (b) Max. of 5 at x = 4, Min. of 3 at x = 1

   (c) max. of $\frac{\pi}{2}$ at $x=\frac{\pi}{2}$, min. of- $\frac{\pi}{2}$ at $x= -\frac{\pi}{2}$

Sec 3.2

2. $\sqrt{6}$ 3. the slope is $\frac{1}{3}$; the point $(1,1)$

Sec 3.3

1. (a) Rel max: $f(0)=0$ Rel min: $f(6)=12$

   (b) CN's $-\pi,0,\pi$, Increasing on $(-\pi,\pi)$, Decreasing on $(-4,-\pi)\cup(\pi,4)$

Sec 3.4

1. conc. up on (0,3); conc. down on $(3,\infty)$, POI $(3,-\frac{192\sqrt{3}}{5})$ 3. $a=1$ $b=6$

Sec 3.5

1. (a) -1 (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{2}}$

Sec 3.6 1. (a) $x=-0.5$ and 1 (b) Decreasing on $(-\infty,-0.5)$ Increasing on $(-0.5,\infty)$, rel min $(-0.5, f(-0.5))$ (c) Concave upward on $(-\infty,0)\cup(1,\infty)$ concave downward on $(0,1)$ POI $(1, f(1))=(-1,0)$ and $(0, f(0))=(0,-1)$

Sec 3.7

1. Find the point on the curve $8y=40-x^2$ that is closest to the origin.
2. $24\sqrt{3}$ square inches
3. print area: $10mm\times 20mm$, page: $32mm\times 32mm$

Sec 3.9

1. $\frac{0.19}{25^3}$ 2. theoretical range 6400 ft. $\frac{dx}{x}=2\frac{dV}{V}=2\%$. definitely.
3. 2.025 4. $\pm 0.114\,cm^2$

Sec 4.1

1. $y=\frac{-1}{x^2}+\frac{1}{x^4}-3$ 2. (a) $2\tan x+x+C$ (b) $\frac{3}{2}x^2+x-\frac{6}{5}x^{5/2}-\frac{2}{3}x^{3/2}+C$

Sec 4.4

1. (a) 48 (b) $\pi+3$ 3. (a) $F'(x)=\left(\frac{-5\sin 5x}{3+\cos^3 5x}\right)$ (b) $F'(x)=\left(\frac{2x}{1+x^{10}}\right)-\left(\frac{3x^2}{1+x^{15}}\right)$