MATH 2419: QUESTIONS FROM MATH 1471/2417 #1M

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1. Differentiate the following functions

   (a) $f(x) = \sec^3 5x$

   (b) $f(x) = \ln(5x^3 + 2)$

   (c) $f(x) = e^{-3x} \sin 2x$

   (d) $f(x) = \dfrac{x^3 + 2x + 7}{x^2}$

   (e) $f(x) = \dfrac{5x + 1}{(1 - 2x)^3}$

   (f) $f(x) = \arcsin x + x\sqrt{1 - x^2}$

   (g) $f(x) = \arctan 2x$

   (h) $f(x) = x \arcsin x + \sqrt{1 - x^2}$

   (i) $f(x) = x \arctan x - \frac{1}{2} \ln(1 + x^2)$

   (j) $f(x) = \sin 2x \cos 3x$

2. Evaluate the integral

   (a) $\displaystyle\int e^{-3x}\, dx$

   (b) $\displaystyle\int xe^{-3x}\, dx$

   (c) $\displaystyle\int \frac{1}{\sqrt{1 - 9x^2}}\, dx$

   (d) $\displaystyle\int \frac{1}{1 + 4x^2}\, dx$

   (e) $\displaystyle\int \frac{1}{(x - 1)\sqrt{x^2 - 2x - 8}}\, dx$

   (f) $\displaystyle\int \frac{1}{x(\ln x)^3}\, dx$

(g) $\displaystyle\int \frac{\ln x}{x^2}\,dx$

(h) $\displaystyle\int x^2\sqrt{x-1}\,dx$

(i) $\displaystyle\int \cos^3 x \sin^2 x\,dx$

(j) $\displaystyle\int \sec^4 x \tan^3 x\,dx$

(k) $\displaystyle\int \sin^3 x\,dx$

(l) $\displaystyle\int \sin^4 x\,dx$

(m) $\displaystyle\int \cos^2 2x\,dx$

(n) $\displaystyle\int \frac{1}{(x^2+1)^{3/2}}\,dx$

(o) $\displaystyle\int \frac{1}{(1-x^2)^{5/2}}\,dx$

(p) $\displaystyle\int \frac{x^3}{\sqrt{x^2+9}}\,dx$

(q) $\displaystyle\int \frac{x^2}{(1-x^2)^{3/2}}\,dx$

(r) $\displaystyle\int \frac{1}{\sqrt{x^2-9}}\,dx$

(s) $\displaystyle\int \frac{1}{(x^2-16)^{3/2}}\,dx$

(t) $\displaystyle\int \frac{4x^2+2x-1}{x^2(x+1)}\,dx$

(u) $\displaystyle\int \frac{x^2-1}{x(x^2+1)}\,dx$

3. Evaluate the integral

(a) $\displaystyle\int_2^b \frac{\text{arcsec}x}{x\sqrt{x^2-1}}\,dx \quad b>2$

(b) $\displaystyle\int_0^{\ln 2} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\, dx$

(c) $\displaystyle\int_1^{e} \frac{1 - \ln x}{x}\, dx$

(d) $\displaystyle\int_0^{\ln 2} \frac{2}{e^{-x} + 1}\, dx$

4. Solve the following trigonometric equations

   (a) $\sin 3\theta = 0 \; ; 0 \leq \theta \leq 2\pi$

   (b) $\cos 2\theta = 0 \; ; 0 \leq \theta \leq 2\pi$

   (c) $\sqrt{3} + 2\sin\theta = 0 \; ; 0 \leq \theta \leq 2\pi$

   (d) $1 + 2\cos\theta = 0 \; ; 0 \leq \theta \leq 2\pi$

5. Evaluate the following limits

   (a) $\displaystyle\lim_{x\to\infty} \frac{1/x}{\sin(\pi/x)}$

   (b) $\displaystyle\lim_{x\to\infty} \frac{x}{\ln^2(1+x)}$

   (c) $\displaystyle\lim_{x\to\infty} \frac{x^3}{3^x + x^2}$

   (d) $\displaystyle\lim_{x\to\infty} \frac{1 - e^{1/x}}{-3/x}$

6. Find all ordered pairs (x, y) that satisfy the equations simultaneously

   (a) $x^2 - x = 6 \quad ; \; y^2 + 6y = 0$

   (b) $4x + 3y = 10 \quad ; \; x - 6y = -11$

   (c) $x^2 - 3x - 54 = 0 \quad ; \; y^3 + 2y^2 + y = 0$

   (d) $y - x = 1 \quad ; \; 2y^2 - 7x = 2$

7. Sketch the regions in the xy-plane determined by the inequalities.

   (a) $1 \leq x \leq e \quad ; 0 \leq y \leq \ln x$

   (b) $0 \leq x \leq 2 \quad ; x^2 \leq y \leq \frac{3x}{2} + 1$

(c) $0 \leq y \leq 3 \quad ; y^2 \leq x \leq 9$

(d) $\dfrac{\pi}{4} \leq x \leq \pi \quad ; \cos x \leq y \leq \sin x$

8. In order to perform the following integrations, it is necessary to simplify/rearrange the expressions under the radical sign. Complete the simplification and then integrate using standard techniques.

(a) $\displaystyle\int \sqrt{(2t)^2 + (2)^2 + (1/t)^2}\, dt \quad ; t > 0$

(b) $\displaystyle\int \sqrt{(3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\, dt$

(c) $\displaystyle\int \sqrt{1 + \left(\frac{-2t}{1-t^2}\right)^2}\, dt \quad ; -1 < t < 1$

(d) $\displaystyle\int \sqrt{(\sin t)^2 + (1 - \cos t)^2}\, dt$

(e) $\displaystyle\int \sqrt{(3e^{3t})^2 + (6e^{2t})^2 + (6e^t)^2}\, dt$