

# Math 2417 PRECALCULUS QUESTIONS

## #1E

January 7, 2009

1.  $f(x) = \frac{9 - 2x^2}{\sqrt{9 - x^2}}$

- (a) For what x values is  $f(x) = 0$ ?
- (b) For what x values is  $f(x)$  undefined?
- (c) For what x values is  $f(x) < 0$ ?
- (d) For what x values is  $f(x) > 0$ ?

2.  $f(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

- (a) For what x values is  $f(x) = 0$ ?
- (b) For what x values is  $f(x)$  undefined?
- (c) For what x values is  $f(x) < 0$ ?
- (d) For what x values is  $f(x) > 0$ ?

3.  $f(x) = x \left[ \frac{1}{2}(1 - x^2)^{-1/2}(-2x) \right] + \sqrt{1 - x^2}$

- (a) For what x values is  $f(x) = 0$ ?
- (b) For what x values is  $f(x)$  undefined?
- (c) For what x values is  $f(x) < 0$ ?
- (d) For what x values is  $f(x) > 0$ ?

4.  $f(x) = x \left[ \frac{-1}{2}(x^2 + 1)^{-3/2}(2x) \right] + (x^2 + 1)^{-1/2}$

- (a) For what x values is  $f(x) = 0$ ?
- (b) For what x values is  $f(x)$  undefined?
- (c) For what x values is  $f(x) < 0$ ?
- (d) For what x values is  $f(x) > 0$ ?

5.  $f(x) = 2xe^{-x^2} - 2x^3e^{-x^2}$
- For what  $x$  values is  $f(x) = 0$ ?
  - For what  $x$  values is  $f(x)$  undefined?
  - For what  $x$  values is  $f(x) < 0$ ?
  - For what  $x$  values is  $f(x) > 0$ ?
6.  $f(x) = x^2 \ln(x - 1) - 5x \ln(x - 1)$
- Determine the domain of  $f(x)$
  - For what  $x$  values is  $f(x) = 0$ ?
  - For what  $x$  values is  $f(x) < 0$ ?
  - For what  $x$  values is  $f(x) > 0$ ?
7.  $f(x) = 15(3x + 2)^4(1 - 2x)^3 - 6(3x + 2)^5(1 - 2x)^2$
- Factor as completely as possible
  - For what  $x$  values is  $f(x) = 0$ ?
  - For what  $x$  values is  $f(x) < 0$ ?
  - For what  $x$  values is  $f(x) > 0$ ?
8.  $f(x) = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3}$
- For what  $x$  values is  $f(x) = 0$ ?
  - For what  $x$  values is  $f(x)$  undefined?
  - For what  $x$  values is  $f(x) < 0$ ?
  - For what  $x$  values is  $f(x) > 0$ ?
9.  $f(x) = \sec x \sin x : 0 < x < 2\pi$
- For what  $x$  values is  $f(x) = 0$ ?
  - For what  $x$  values is  $f(x)$  undefined?
  - For what  $x$  values is  $f(x) < 0$ ?
  - For what  $x$  values is  $f(x) > 0$ ?

10.  $f(x) = \sqrt{1 - \sin^2 x} : 0 < x < 2\pi$

- (a) For what x values is  $f(x) = 0$ ?
- (b) For what x values is  $f(x)$  undefined?
- (c) For what x values is  $f(x) < 0$ ?
- (d) For what x values is  $f(x) > 0$ ?

11.  $f(x) = \frac{\sin^2 x}{1 - \cos x} : 0 < x < 2\pi$

- (a) For what x values is  $f(x) = 0$ ?
- (b) For what x values is  $f(x)$  undefined?
- (c) For what x values is  $f(x) < 0$ ?
- (d) For what x values is  $f(x) > 0$ ?

12. Solve the following trigonometric equations.

- (a)  $\sin 2x + \cos x = 0; 0 < x < 2\pi$
- (b)  $\cos 2x = 0 : 0 < x < 2\pi$
- (c)  $\cos 2x + \sin x = 1 : 0 \leq x \leq 2\pi$
- (d)  $2 \cos 2x - 4 \cos x + 3 = 0; 0 \leq x \leq 2\pi$
- (e)  $\cos 2x + \sin^2 x = 1 : 0 \leq x \leq 2\pi$

13. Simplify

(a)  $\frac{(e^{3x})^2(e^{2x})^4}{(e^{4x})^3}$

(b)  $\ln(e^{x^2+2x+1})$

(c) Rewrite as the sum/difference of logarithms:  $\ln\left(\frac{x^2 \sin x}{\sec x}\right)$

(d) Express as a single logarithm.  $5 \ln(x + 3) - \frac{1}{2} \ln(x^2 - 9)$

14. Simplify  $\frac{1}{2} \left[ x \left( \frac{-x}{\sqrt{1-x^2}} \right) + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}} \right]$

15. Simplify  $f(x) = \frac{1}{\sqrt{1 + \tan^2 x}} - \cos x : 0 < x < \frac{\pi}{2}$

16. Simplify  $f(x) = \frac{1}{\sqrt{1 + \tan^2 x}} - \cos x : \pi < x < \frac{3\pi}{2}$

17. Simplify  $f(x) = \frac{\sin^2 x}{1 - \cos x} : 0 < x < 2\pi$

18. Complete the square for the following expressions

(a)  $4x - x^2$

(b)  $6x - x^2 - 8$

(c)  $3 + 2x - x^2$

(d)  $9x^2 - 18x + 5$

(e)  $16x - 16x^2 - 3$

(f)  $-x^2 - 4x$

(g)  $-x^2 + 2x$

(h)  $x^2 + 6x + 13$

(i)  $2 \sin^2 x + \sin x - 1$

19. Form the difference quotient  $\frac{f(x+t) - f(x)}{t}$  and simplify the answer for the following functions.

(a)  $f(x) = 6 - 2x - x^2$

(b)  $f(x) = \frac{1}{x}$

(c)  $f(x) = \frac{1}{2x+3}$

(d)  $f(x) = \sqrt{3x+2}$

(e)  $f(x) = 3x^2 - 5x + 7$

20. Classify the graph of the equation as

- A circle
- A parabola
- An ellipse or
- An hyperbola

Give the essential details e.g. a circle whose center is at  $(23,-13)$  with radius 39 and draw the graph.

- (a)  $x^2 + 6x + y^2 + 10y = 15$   
 (b)  $4x^2 + 9y^2 + 16x - 54y + 61 = 0$   
 (c)  $x^2 - 4x - y^2 + 6y - 6 = 0$

21. An open box is to be made from a rectangular piece of metal (2ft by 3 ft) by cutting equal squares of side  $x$  from each corner and turning up the sides ( see figure 1). Find

- (a) An expression for the volume of the box in terms of the variable  $x$ .  
 (b) An expression for the surface area of the box in terms of the variable  $x$

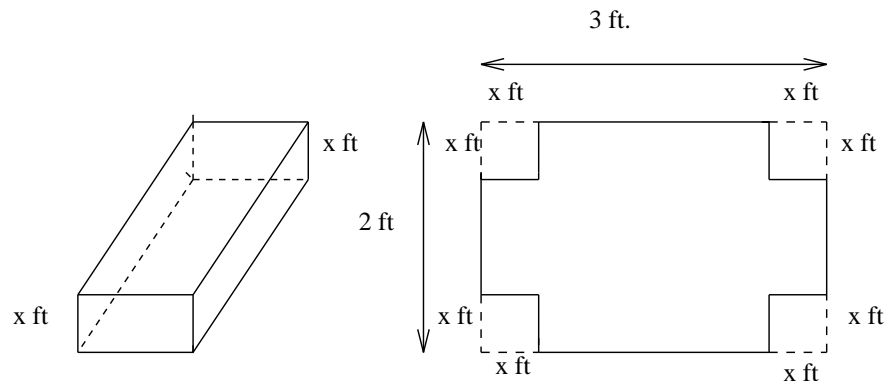


Figure 1: Figure for # 21

22. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals ( see figure 2). Find an expression for the total area of the two fields in terms of the variable 'x'.

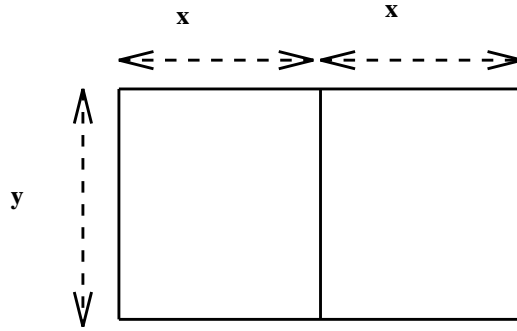


Figure 2: Figure for # 22

23. A farmer plans to fence in a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. Find an expression for the length of fence in terms of the variable 'x' in Figure 3. Assume that no fencing is required along the riverfront.

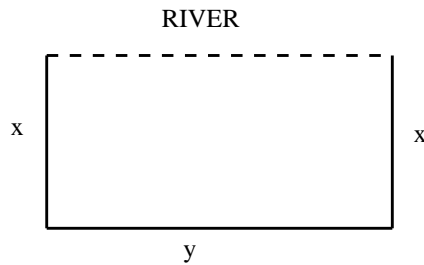


Figure 3: Figure for # 23

24. At a sand and gravel plant, sand is falling off a conveyor belt and onto a conical pile. The diameter of the base of the cone is three times the altitude. Find an expression for the volume of the cone in terms of  $h$ , the height of the cone.
25. Find an expression for the volume of a cone in terms of the base radius  $r$ , if the altitude of this particular cone is three times the radius.

26. A page is to contain 30 square inches of print. The margins on each side are 1 inch. Find an expression for the area of the page in terms of the variable 'x'(see figure 4)

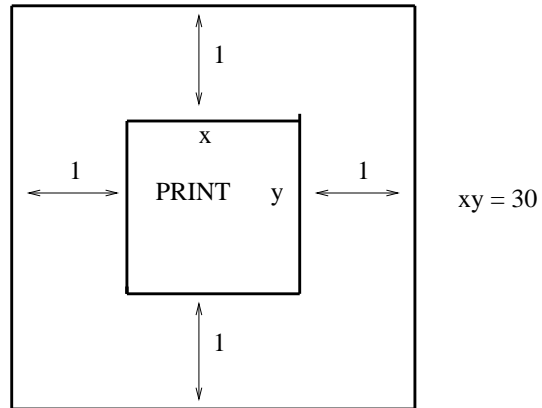


Figure 4: Figure for # 26

27. A page contains 96 square inches. Printing is to be done on the page such that there is a 1 inch margin on either side and a 2 inch margin at the top and a 1 inch margin at the bottom of the page. Find an expression for the printed area in terms of the variable 'x'(see figure 5)

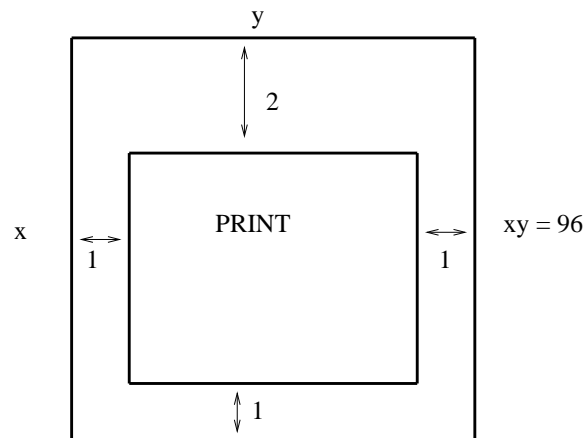


Figure 5: Figure for # 27

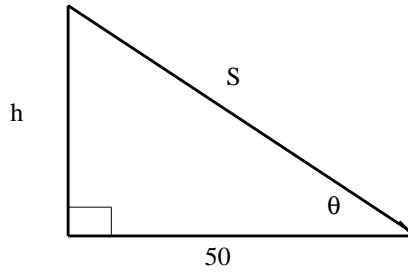


Figure 6: Figure for # 28

28. The following questions refer to Figure 6

- (a) Write an expression for  $S$  in terms of  $h$
- (b) Write an expression for  $S$  in terms of  $\theta$
- (c) Write an expression for  $h$  in terms of  $S$
- (d) Write an expression for  $h$  in terms of  $\theta$

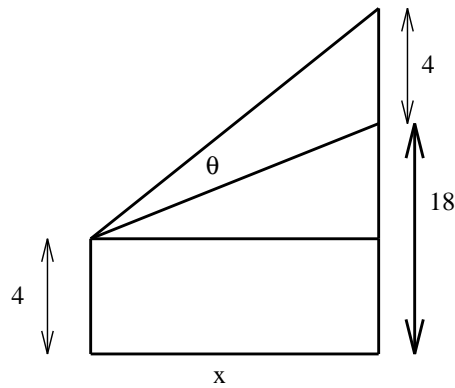


Figure 7: Figure for # 29

29. The following question refers to Figure 7. Show that  $\tan \theta = \frac{4x}{x^2 + 252}$

30.  $\tan \theta = x$ ;  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ . Find an expression for  $\sec \theta$

31.  $\sin \theta = x$ ;  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$  Find an expression for  $\sin 2\theta$

32. Solve the following system of equations

(a)  $y = \frac{1}{1+x^2} : y = \frac{x^2}{2}$

(b)  $x = y^2 : y = x - 2$

(c)  $y = 2x^2 : y = x^4 - 2x^2$

(d)  $y = \sqrt{3x} + 1 : y = x + 1$

(e)  $y = x^2 : y = x^3 - 3x^2 + 3x$

(f)  $y = x^2 - 4x + 3 : y = 3 + 4x - x^2$

(g)  $y = \sin 2x : y = \cos x ; 0 \leq x \leq \pi$

(h)  $y = 2 \sin x : y = \tan x ; \frac{-\pi}{2} < x < \frac{\pi}{2}$

33. Write out expansions for the following trigonometric functions.

(a)  $\sin(A + B)$

(b)  $\sin(A - B)$

(c)  $\cos(A + B)$

(d)  $\cos(A - B)$

(e)  $\tan(A + B)$

(f)  $\tan(A - B)$

(g)  $\sin 2x$

(h)  $\cos 2x$  ( 3 variations)

(i) Using the results of 33a and 33c, establish the result quoted for 33e

(j) Imitating the procedures followed in 33i, find an expansion for  $\cot(A + B)$  in terms of  $\cot A$  and  $\cot B$

34. Establish the following trigonometric identities

(a)  $\operatorname{cosec} x - \cot x = \tan\left(\frac{x}{2}\right)$

(b)  $\operatorname{cosec} x - \cot x = \frac{1}{\operatorname{cosec} x + \cot x}$

(c)  $-\ln(\cot \theta - \tan \theta) = \ln\left(\frac{1}{2}\right) + \ln \tan(2\theta)$

(d)  $\frac{1}{\cot \theta - \tan \theta} = \frac{1}{2} \tan 2\theta$

35. Without reference to a calculator or a table of trigonometric functions, write down exact values for

(a)  $\sin \frac{4\pi}{3}$

(b)  $\tan \frac{5\pi}{6}$

(c)  $\cos \frac{5\pi}{4}$

(d)  $\sin \frac{2\pi}{3}$

(e)  $\cos \frac{7\pi}{6}$

(f)  $\sec^2 \left( \frac{\pi}{4} \right)$

36. Without the aid of a calculator, evaluate the following expressions.

(a)  $\operatorname{arcsec}(2)$

(b)  $\operatorname{arcsec}(2/\sqrt{3})$

(c)  $\operatorname{arcsec}(-2)$

(d)  $\operatorname{arcsec}(-2/\sqrt{3})$

(e)  $\arctan(1)$

(f)  $\arctan(-1)$

(g)  $\arcsin(-\sqrt{3}/2)$

(h)  $\arcsin(-1/\sqrt{2})$

37. Write an algebraic expression that is equivalent to the given expression

(a)  $\cos(\arcsin(2x))$

(b)  $\operatorname{cosec}[\arctan(x/\sqrt{2})]$

(c)  $\sin(\arctan x)$

(d)  $\sec[\arcsin(x-1)]$

38. Solve the following absolute value inequalities

(a)  $|x-2| < 5$

(b)  $|7-2x| \leq 5$

(c)  $|2x-5| \leq 3$

(d)  $|8-2x| < 2$

(e)  $|x+3| > 1$

39. Example If  $|x - 3| < 1$ , then find the smallest  $k$  such that  $|2 - 3x| < k$   
Solution  $|x - 3| < 1$  implies  $-1 < x - 3 < 1$  or by adding 3 to all entries in the inequality we have  $2 < x < 4$ . Once you have established a string of inequalities for  $x$  alone, then proceed to generate the required term by multiplying by appropriate quantities or by adding/subtracting the appropriate constants. In this case, multiply across by  $-3$ , giving  $-6 > -3x > -12$  where the direction of the inequality string has been reversed due to multiplication by a negative number. Rewriting, we have  $-12 < -3x < -6$ . Now add 2 to all terms, giving  $-10 < 2 - 3x < -4$ . Finally, we must put the inequality string in the form that will enable us to rewrite it as an absolute value inequality, namely  $-10 < 2 - 3x < -4 < 10$  or  $|2 - 3x| < 10$ . Why is 10 the minimum value that satisfies the conditions of the problem? Clearly we could have written  $-20 < -10 < 2 - 3x < -4 < 10 < 20$  leading to the absolute value inequality  $|2 - 3x| < 20$ . From this we conclude that any number greater than 10 can be used but that 10 is the smallest number that will satisfy the inequality.
40. If  $|x - 3| < 1$ , then find the smallest  $k$  such that
- $|2x + 3| < k$
  - $|3 - 5x| < k$
  - $|5x + 7| < k$
41. If  $|x - 5| < 1$ , then find the smallest  $k$  such that
- $|x - 9| < k$
  - $|2 - 3x| < k$
  - $|4x - 5| < k$
42. U.T.D is hosting a computer conference. At 11.30 am, half of the participants remain at their terminals working on an interesting problem while the other half leave for lunch. At 12.30 pm, 60 have returned from lunch and logged on to their terminals while 18 others have logged off and gone to lunch. At this time, there are three times as many people using the computer terminals as there are people at lunch. How many people are attending the conference?
43. An athlete from U.T.D is running across a bridge over the Red River. When she is two fifths of the way across, she hears a train coming from behind her. She quickly calculates that if she runs constantly at 6 mph towards the other end of the bridge, she will reach that end of the bridge at the same time as the train. On the other hand, if she turns

- around and runs back at 6 mph towards the train, she will reach the start of the bridge at the same instant as the train. What is the speed of the train?
44. A farmer prepares a fertilizer solution which contains 2 per cent by weight of a chemical and 98 per cent ,by weight, water.The total weight of the solution is 250 pounds. Some time later due to evaporation, the solution contains only 96 per cent water. What is the weight of the solution now?
  45. A bridge is directed north/south across the Red River. Runners are in position at each end of the bridge. Greg is located at the south end and Tom is at the north end. Assume that both runners start at the same time and run towards the opposite end of the bridge at constant but different speeds.When they pass each other, Greg is 600 ft from the south end of the bridge. When Tom and Greg reach the ends of the bridge, they immediately reverse direction and run towards their starting points. This time they pass each other when Greg is 200 ft from the north end and running towards the south end of the bridge. What is the length of the bridge?
  46. At a social event in the U.T.D. library at which fewer than two dozen people showed up, two thirds of the men were married to three fifths of the women. Assuming that all spouses were present, how many were single?
  47. Two U.T.D. students drive for 100 miles along a straight highway, one in a red car ,the other in a blue car. The speed limit for the first 50 miles is 55 mph. For the second 50 miles, the speed limit is 65 mph. The student in the red car stops for gas during the first half of the trip. The student in the blue car stops for gas during the second half of the trip. Each stop is of 15 minutes duration. Assuming that both cars travel at the speed limit, will the student in the red car arrive before the student driving the blue car? Explain your answer.
  48. Suppose a farmer buys 100 animals for \$ 100.Only cows, pigs, or chickens are to be considered. There must be at least one animal of each type. No other animals are to be included. Suppose that cows cost \$10 each, the pigs cost \$ 3 each and the chickens cost 50 cents each. How many of each did the farmer buy?
  49. A large, square , brick wall is two bricks higher and two bricks narrower than another wall. How many more bricks are in the first wall?
  50. (a) 70 cows take 24 days to eat all of the grass in a pasture. The same pasture sustains 30 cows for 60 days. How many cows would take 96 days to eat all of the grass.

- (b) Find another combination of days and number of cows to eat all of the grass. Remember that your answers must be integers