EEMF 6321 HW 2 Solutions 2/28/2017

1. Si Bipolar NPN

	E	B	C
N	6.4×10^{18}	3.0×10^{17}	2.0×10^{15}
L_D (μm)	30	8	55
D $\frac{cm^2}{s}$	12	40	12

$W_B = 2.0\ \mu m$

a) Find V_{bi}:

$$V_{bi}[BE] = \frac{kT}{q}\ln\left(\frac{N_E N_B}{n_i^2}\right) = 0.0259V \times \ln\left(1.33\times10^{16}\right)$$

$$= 0.0259\times 37.13 = \underline{0.962\,V}$$

$$V_{bi}[BC] = \frac{kT}{q}\ln\left(\frac{N_B N_C}{n_i^2}\right) = 0.0259\times\ln\left(4.17\times10^{12}\right)$$

$$= 0.0259\times 29.06 = \underline{0.753\,V}$$

b) Find W_{BE} and W_{BC}

There is a simplified formula, which I should have given you, for the total depletion-layer width:

$$W = \left[\frac{2K_S\epsilon_0}{q}\frac{(N_A+N_D)}{N_A N_D}\times V_{bi}\right]^{\frac{1}{2}}$$

$$W_{BE} = \left[\frac{2\times11.8\times8.854\times10^{-14}F/cm}{1.6\times10^{-19}C}\frac{(6.4\times10^{18}+3\times10^{17})}{6.4\times10^{18}\cdot3\times10^{17}}cm^3\times0.962V\right]^{\frac{1}{2}}$$

$$= \left[4.38\times10^{-11}cm^2\right]^{\frac{1}{2}} = 6.62\times10^{-6}\,cm = \underline{66.2\,nm}$$

$$W_{BC} = \left[\frac{2 \times 11.8 \times 8.854 \times 10^{-14} F/cm}{1.6 \times 10^{-19} C} \times 0.753V \times \frac{(3.0 \times 10^{17} + 2.0 \times 10^{15})}{3.0 \times 10^{17} \times 2.0 \times 10^{15}} cm^3 \right]^{1/2}$$

$$= \left[495 \times 10^{-9} cm^2 \right]^{1/2} = 7.05 \times 10^{-5} \ cm = \underline{0.705 \ \mu m}$$

c) Find the independent pre-exponential factors.

First, the factor $C = qAn_i^2$ is common to all of them.

$$C = qAn_i^2 = 1.6 \times 10^{-19} C \times 40 \times 10^{-8} cm^2 \times \left[1.2 \times 10^{10} cm^{-3} \right]^2$$
$$= 9.216 \times 10^{-6} \ C \ cm^{-4}$$

$$I_{EnBS} = \frac{qAn_i^2 D_B}{N_B W_B} = \frac{9.216 \times 10^{-6} C cm^{-4} \times 40 \ cm^2/s}{3.0 \times 10^{17} cm^{-3} \times 2.0 \times 10^{-4} cm} = \underline{6.14 \times 10^{-18} A}$$

$$I_{EpS} = \frac{qAn_i^2 D_E}{N_E L_E} = \frac{9.216 \times 10^{-6} C cm^{-4} \times 12 \ cm^2/s}{6.4 \times 10^{18} cm^{-3} \times 30 \times 10^{-4} cm} = \underline{5.76 \times 10^{-21} A}$$

$$I_{CpS} = \frac{qAn_i^2 D_C}{N_C L_C} = \frac{9.216 \times 10^{-6} C cm^{-4} \times 12 \ cm^2/s}{2 \times 10^{16} cm^{-3} \times 55 \times 10^{-4} cm} = \underline{1.005 \times 10^{-18} \ A}$$

d.) Estimate forward α and β

$$\beta_F = \frac{I_C}{I_B} \approx \frac{I_{EnBS}}{I_{EpS}} = \frac{6.14 \times 10^{-18} \ A}{5.76 \times 10^{-21} \ A} = \underline{1066}$$

$$\alpha_F = \frac{\beta_F}{\beta_F + 1} = \underline{0.999}$$

e) Estimate reverse α and β

$$\beta_R = \frac{I_E}{I_B} \approx \frac{I_{EnBS}}{I_{cpS}} = \frac{6.14\times10^{-18}}{1.005\times10^{-18}} = \underline{6.11}$$

$$\alpha_R = \frac{\beta_R}{\beta_R+1} = \frac{6.11}{7.11} = \underline{0.86}$$

f) Evaluate Ebers-Moll I_S

$$I_S = I_{EnBS} = \underline{6.14\times10^{-18}\,A}$$

g) Include base recombination:

We need $\cosh(W_B/L_B)$ and $\sinh(W_B/L_B)$

$$\frac{W_B}{L_B} = \frac{2.0\,\mu m}{8.0\,\mu m} = 0.25 \qquad e^{+0.25} = 1.284 \qquad e^{-0.25} = 0.7788$$

$$\cosh = \tfrac{1}{2}(e^{+x}+e^{-x}) = 1.03$$

$$\sinh = \tfrac{1}{2}(e^{+x}-e^{-x}) = 0.253$$

$$I_{EnCS} = C\frac{D_B}{N_B L_B \sinh(\ )} = \frac{9.216\times10^{-6}\,C\,cm^2 \times 40\,cm^2/s}{3.0\times10^{17}\,cm^3 \times 8.0\times10^{-4}\,cm \times 0.253}$$

$$= \underline{6.07\times10^{-18}\,A}$$

$$I_{EnBS} = \frac{C\,D_B \cosh(\ )}{N_B L_B \sinh(\ )} = \frac{9.216\times10^{-6}\,C\,cm^{-2} \times 40\,cm^2/s \times 1.03}{3.0\times10^{17} \times 8.0\times10^{-4}\,cm \times 0.253}$$

$$= \underline{6.28\times10^{-18}\,A}$$

h) $\alpha_T = \frac{1}{\cosh(W_B/L_B)} = \frac{1}{1.03} = \underline{0.971}$

i) $\alpha_F = \gamma \cdot \alpha_T = 0.999 \times 0.971 = \underline{0.970}$

$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.970}{0.030} = \underline{323}$

2. Ebers-Moll Model: $I_S = 1.0 \times 10^{-16} A$

$\alpha_F = 0.99$

$\alpha_R = 0.7$

a) For $V_{BE} = 0.75V$ and $V_{CE} = 2.0V$

$\Rightarrow V_{BC} = V_{BE} - V_{CE} = -1.25V$

$\Rightarrow$ Forward-active mode, I_R will be negligible

$e^{qV_{BE}/kT} = e^{0.75/0.0259} = e^{28.96} = 3.768 \times 10^{12}$

$I_F = \frac{1.0 \times 10^{-16}}{0.99} \times 3.77 \times 10^{12} = 3.806 \times 10^{-3} A = 3.806 mA$

$I_E = I_F - \cancel{\alpha_R I_R} = 3.81 mA$

$I_C = \alpha_F I_F = 3.77 mA$

$I_B = (1-\alpha_F) I_F = 0.01 \times 3.81 mA = 38.1 \mu A$

b) $V_{BE} = 0.75$, $V_{CE} = 0.1$

$$V_{BC} = V_{BE} - V_{CE} = 0.75 - 0.1 = 0.65$$

I_F is same as before

$$I_R = \frac{1\times10^{-15}}{0.7} e^{0.65/0.0259} = 1.43\times10^{-15} A \times \overbrace{e^{25.096}}^{7.93\times10^{10}}$$

$$= 1.13\times10^{-4} A = 0.113 \text{ mA}$$

$$\alpha_R I_R = 0.7 \times 0.113 \text{mA} = 7.93\times10^{-5} = 79.3 \mu A$$

$$I_E = I_F - \alpha_R I_R = 3.806 - .079 = \underline{3.727} \text{ mA}$$

$$I_C = \alpha_F I_F - I_R = 3.77 - 0.113 = \underline{3.66 \text{mA}}$$

$$I_B = (1-\alpha_F) I_F + (1-\alpha_R) I_R = 38.1 \mu A + 0.3\times113 \mu A$$

$$= 38.1 + 33.9 = \underline{72.0 \mu A}$$