Unbiased junction equations

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{n(x_{n0})p(-x_{p0})}{n_i^2}\right)$$

$$= \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$= kT\ln\left(\frac{p(-x_{p0})}{p(x_{n0})}\right)$$

$$= kT\ln\left(\frac{n(x_{n0})}{n(-x_{p0})}\right)$$

$$= \frac{qN_D}{2\varepsilon}(x_{n0})^2 + \frac{qN_A}{2\varepsilon}(x_{p0})^2$$

$$= \frac{1}{2}\frac{q}{\varepsilon}\frac{N_A N_D}{(N_A + N_D)}w^2$$

$$x_{n0} = \left(\frac{2\varepsilon N_A V_{bi}}{qN_D(N_A + N_D)}\right)^{1/2}$$

$$= w\frac{N_A}{(N_A + N_D)}$$

$$x_{p0} = \frac{x_{n0}N_D}{N_A} = \left(\frac{2\varepsilon V_{bi} N_D}{qN_A(N_A + N_D)}\right)^{1/2}$$

$$= w\frac{N_D}{(N_A + N_D)}$$

$$w = x_{n0} + x_{p0}$$

$$= \left(\frac{2\varepsilon V_{bi}(N_A + N_D)}{qN_A N_D}\right)^{1/2}$$

$$E_{xp} = \int_{-x_{p0}}^{x} -\frac{qN_A}{\varepsilon}dx$$

$$= -\frac{qN_A}{\varepsilon}(x + x_{p0})$$

$$E_{xn} = \int_{x}^{x_{n0}} \frac{qN_D}{\varepsilon}dx$$

$$= \frac{qN_D}{\varepsilon}(x_{n0} - x)$$

$$Q_p = Aqx_{n0}N_D$$
$$Q_n = -Aqx_{p0}N_A$$
$$Q_p + Q_n = 0$$
$$\Downarrow$$
$$x_{n0}N_D = x_{p0}N_A$$

Time independent Biased junctions

$$\Delta n_p = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{+(x+x_{p0})/L_n}$$
$$\Delta p_n = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{-(x-x_{n0})/L_p}$$

Minority currents density

$$J_N = qD_n \frac{\partial \Delta n_p(x)}{\partial x}$$
$$= \frac{qD_n}{L_n}\Delta n_p(x)$$
$$J_P = -qD_p \frac{\partial \Delta p_n(x)}{\partial x}$$
$$= \frac{qD_p}{L_p}\Delta p_n(x)$$

Total current is

$$I = I_0\left(e^{qV_A/kT} - 1\right)$$
$$I_0 = qn_i^2 A\left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right)$$

$$I_{P_{total}} = \frac{Q_p}{\tau_p}$$
$$I_{n_{total}} = \frac{Q_n}{\tau_n}$$

$$w = x_{n0} + x_{p0}$$
$$= \left(\frac{2\varepsilon(V_{bi} - V_A)(N_A + N_D)}{qN_A N_D}\right)^{1/2}$$

Total injected charge

$$Q_p = qAL_p p_{n0}\left(e^{qV_A/kT} - 1\right)$$

$$Q_n = qAL_n n_{p0}\left(e^{qV_A/kT} - 1\right)$$

Time varying junctions

i_D = Total current through the diode

I_D = DC current through the diode

i_d = AC current through the diode

$$i_D(t) = I_D + i_d(t)$$

v_A = Total voltage across the diode

V_A = DC voltage across the diode

v_a = AC voltage across the diode

$$v_A(t) = V_A + v_a(t)$$

$$i_D(t) = \frac{Q(t)}{\tau} + \frac{dQ(t)}{dt}$$

$$v_A = \frac{kT}{q}\ln\left(\frac{I_F\tau_p}{qAL_p p_{n0}}\exp\left[-t/\tau_p\right] + 1\right)$$

Storage delay time

$$t_{sd} = \tau_p \ln\left[\frac{I_f + I_r}{I_r}\right] \quad t_{sd} = \tau_p\left(\mathrm{erf}^{-1}\left[\frac{I_r}{I_f + I_r}\right]\right)^2$$

Turn on transient

$$v_A = \frac{kT}{q}\ln\left[\frac{I_F}{I_0}\left(1 - e^{-t/\tau_p}\right) + 1\right]$$

Diode capacitance/resistance

Reverse Bias Capacitance

$$C = \varepsilon A\left(\frac{qN_A N_D}{2\varepsilon(N_A + N_D)(V_{bi} - v_A)}\right)^{1/2}$$

$$= \frac{\varepsilon A}{w}$$

$$\equiv C_j$$

$$G_0 = \frac{q}{kT}(I + I_0)$$

$$\approx 0 \text{ in reverse bias}$$

Forward Bias (again low frequency)

$$Q_p = qAL_p p_{n0}\left(e^{qv_A/kT} - 1\right)$$
$$\approx qAL_p p_{n0} e^{qv_A/kT}$$
$$C = \frac{q}{kT} I\tau_p \left[= \frac{q}{kT}(I - I_0)\tau_p \text{ - if done without approximations} \right]$$
$$\equiv C_S$$
$$C_S \approx G_0 \tau_p$$

Metal-Semiconductor Junctions

$$V_{bi} = \phi_M - \phi_S$$
$$\phi_B = (\phi_M - \chi).$$
$$I_{total} = I_0\left(e^{qv_A/kT} - 1\right)$$

Useful equations

deBroglie momentum $p = h/\lambda = \hbar k.$

Heisenberg uncertainty principle
$$\Delta p \Delta x \ge \hbar$$
$$\Delta E \Delta t \ge \hbar$$

Photon energy $E_{photon} = h\nu,$

Bohr Model
$$r_n = \frac{K\hbar^2 n^2}{me^2}$$
$$= 0.529\text{Å}n^2$$
$$E_{Bohr} = -\frac{1}{2}\frac{me^4}{K^2\hbar^2 n^2}$$
$$= -13.56\text{eV}/n^2$$

Schrödinger's equation
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi(\mathbf{r},t) = -\frac{\hbar}{j}\partial_t \Psi(\mathbf{r},t)$$

Maxwellian distribution
$$f(v) = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[\frac{-m(v)^2}{2kT}\right].$$
$$f(\mathcal{E}) = n\frac{1}{kT}\exp\left[\frac{-\mathcal{E}}{kT}\right]$$

Equation of motion

$$\mathbf{F}_n = -e\mathbf{E} = m_n^*\mathbf{a} \quad \text{(electron)}$$
$$\mathbf{F}_p = +e\mathbf{E} = m_p^*\mathbf{a} \quad \text{(hole)}$$

$$\mathrm{f}(\mathcal{E}) = \frac{1}{1+\exp\left[\frac{(\mathcal{E}-\mathcal{E}_F)}{kT}\right]}$$

<u>Fermi-Dirac</u> function.

$$N_c(\mathcal{E})d\mathcal{E} = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2}\sqrt{\mathcal{E}-\mathcal{E}_c}\,d\mathcal{E} \quad \text{Conduction band}$$

$$N_v(\mathcal{E})d\mathcal{E} = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2}\sqrt{\mathcal{E}_v-\mathcal{E}}\,d\mathcal{E} \quad \text{Valance band}$$

STATE DENSITY
state distribution function.

$$n(\mathcal{E})d\mathcal{E} = f(\mathcal{E})N_c(\mathcal{E})d\mathcal{E} = \frac{1}{1+\exp\left[\frac{(\mathcal{E}-\mathcal{E}_F)}{kT}\right]}\frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2}\sqrt{\mathcal{E}-\mathcal{E}_c}\,d\mathcal{E} \quad \text{Electrons in the Conduction band}$$

$$p(\mathcal{E})d\mathcal{E} = (1-f(\mathcal{E}))N_v(\mathcal{E})d\mathcal{E} = \left(1-\frac{1}{1+\exp\left[\frac{(\mathcal{E}-\mathcal{E}_F)}{kT}\right]}\right)\frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2}\sqrt{\mathcal{E}_v-\mathcal{E}}\,d\mathcal{E} \quad \text{Holes in the Valance band}$$

$$n_0 \approx N_c\exp\left[\frac{-(\mathcal{E}_c-\mathcal{E}_F)}{kT}\right] = n_i\exp\left[\frac{(\mathcal{E}_F-\mathcal{E}_i)}{kT}\right]$$

$$p_0 \approx N_v\exp\left[\frac{(\mathcal{E}_v-\mathcal{E}_F)}{kT}\right] = n_i\exp\left[\frac{-(\mathcal{E}_F-\mathcal{E}_i)}{kT}\right]$$

total number of electrons and holes

$$n_i^2 = np$$

$$N_c = 2\left(\frac{m_n^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{m_p^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$\mathcal{E}_i = \frac{(\mathcal{E}_v+\mathcal{E}_c)}{2} + \frac{kT}{2}\ln\left(\left(\frac{m_p^*}{m_n^*}\right)^{3/2}\right)$$

Intrinsic Energy

Mobility	$$\mu_n = -\frac{\langle v\rangle}{E} = \frac{\tau\lvert q\rvert}{m_n^*}$$ $$\mu_p = \frac{\langle v\rangle}{E} = \frac{\tau\lvert q\rvert}{m_p^*}$$
Conduction	$$\begin{aligned}\mathbf{J} &= nq\langle\mathbf{v}\rangle \\ &= q\left(n_0\mu_n + p_0\mu_p\right)\mathbf{E} \\ &= \sigma\mathbf{E} \\ &= \mathbf{E}/\rho\end{aligned}$$
Diffusion	$$D_n = \mu_n \frac{kT}{q_n}$$ $$D_p = \mu_p \frac{kT}{q_p}$$
Diffusion length	$$L_n = \sqrt{D_n\tau_n}$$
Diffusion time	$$\tau = \frac{1}{\alpha_r\left(p_0 + n_0\right)}$$