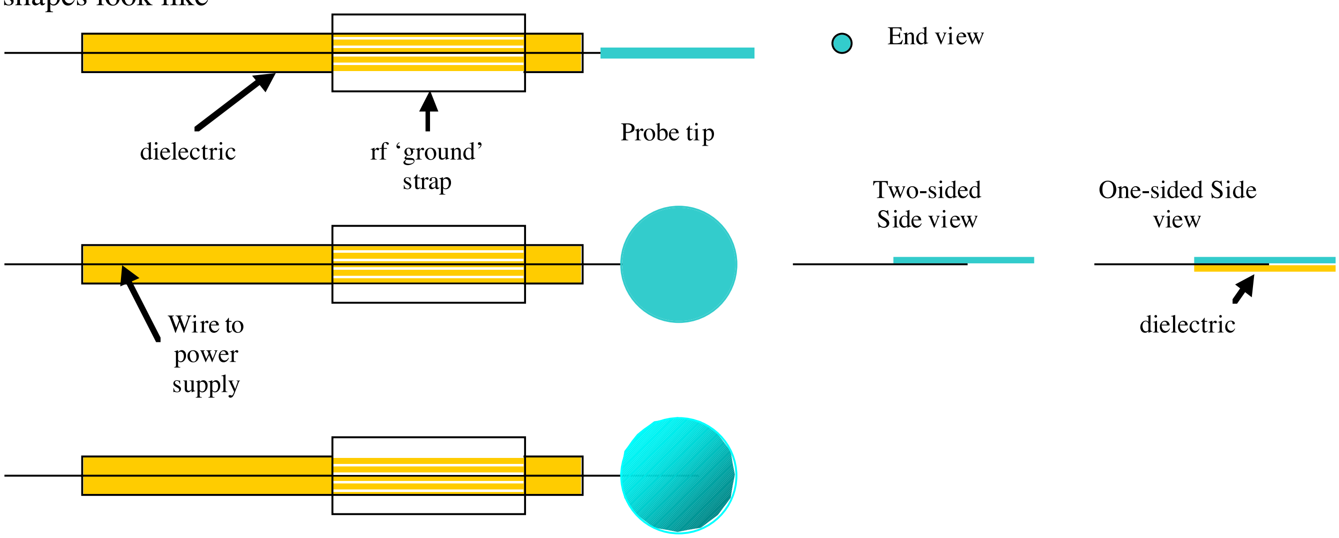
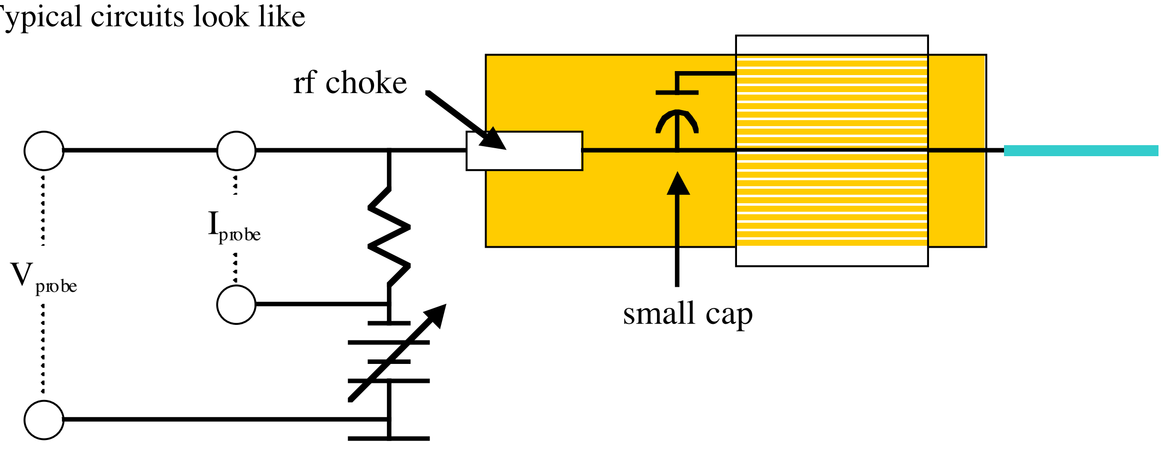
Lecture 9 Langmuir Probes

New homework problems:
Lieberman 6.6, 6.7 and 6.8   Due April 4th, 2001.

Langmuir probes come in a large number of shapes and varieties. Varieties include single probes (which are the typical Langmuir probe) double probes and emissive probes (sometimes called emissive Langmuir probes). Single probes cut as planar, spherical and cylindrical probes. Electronics varies radically dependent on the plasma that it is being used in. Typical probe shapes look like



Typical circuits look like



For a probe used in a dc discharge, the rf components are removed.

While Langmuir probes are simple in concept, correct use of probes requires through understanding and care. Here we will examine a set of simplified equations that partially describe Langmuir probes. IT SHOULD BE NOTED THAT THIS THEORY, WHILE CLOSE, IS OFTEN NOT THE CORRECT MODEL IN MOST EXPERIMENTAL APPLICATIONS. In our simplified model of a Langmuir probe we can determine the current of ions and electrons to the probe tip – as a function probe voltage. The ion current is simply

$$j_i = e\Gamma_i = \text{const}$$
$$= \mathrm{e}n_S v_B$$
$$= en_S\sqrt{\frac{kT_e}{M_i}}$$

where n_S is the density at the presheath-sheath boundary. This of course assumes that the ion flux is conserved, e.g they are collisionless and that ions are not produced in the sheath around the probe. Further, it assumes that the probe is biased sufficiently below the plasma potential to develop a presheath. Thus, for any significant negative probe voltage (w.r.t. the plasma potential) that density is 0.61 n_0. If the probe is biased above the plasma potential, ions will not flow to the probe. In the intermediate range, we have to go back to our model of the presheath. In such a case, we know that

$$\frac{1}{2}m\Delta v^2 = -q\Delta\phi$$
$$\Downarrow$$
$$\frac{1}{2}M_i\left(v_{probe}^2 - \overbrace{v_{plasma}^2}^{\approx 0}\right) = -q\left(\phi_{probe} - \phi_{plasma}\right)$$
$$v_{probe} = \sqrt{\frac{2q}{M_i}\left(\phi_{plasma} - \phi_{probe}\right)}$$

Thus

$$j_i = q\Gamma_i$$
$$= qn_{probe}v_{probe} = qn_{probe}\sqrt{\frac{2q}{M_i}\left(\phi_{plasma} - \phi_{probe}\right)}$$

The density can be determined from Boltzmann's relation

$$n_{probe} = n_0 e^{e\left(\phi_{plasma} - \phi_{probe}\right)/kT_e}$$

giving

$$j_i = qn_0 e^{e\left(\phi_{plasma} - \phi_{probe}\right)/kT_e}\sqrt{\frac{2q}{M_i}\left(\phi_{plasma} - \phi_{probe}\right)}$$

Putting all of our parts together leaves

$$j_i = \begin{cases} 0 & \phi_{probe} \geq \phi_{plasma} \\ qn_0 e^{e\left(\phi_{plasma} - \phi_{probe}\right)/kT_e}\sqrt{\frac{2q}{M_i}\left(\phi_{plasma} - \phi_{probe}\right)} & \phi_{Bohm} < \phi_{probe} < \phi_{plasma} \\ 0.61qn_0\sqrt{\frac{kT_e}{M_i}} & \phi_{probe} \leq \phi_{Bohm} < \phi_{plasma} \end{cases}$$

$$\phi_{Bohm} = \phi_{plasma} - \frac{1}{2q}kT_e$$

Now we need to consider the electrons. This is simpler than the ions as we have already done much of math when we examined floating sheaths. The current density of electrons to the probe tip is

$$j_e = e\Gamma_e = \mathrm{e}\int_{v_{\min}}^{\infty} v_x f(v)dv_x dv_y dv_z$$

where

$$v_{\min} = \sqrt{\frac{2e\Phi_{probe}}{m_e}}$$

$$\Phi_{probe} = \phi_{plasma} - \phi_{probe}$$

is set by energy requirements

Thus

$$j_e = en_e\left(\frac{m_e}{2\pi kT_e}\right)^{3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\sqrt{\frac{2e\Phi_{probe}}{m_e}}}^{\infty} v_x \exp\left[\frac{-m_e\left(v_x^2+v_y^2+v_z^2\right)}{2kT_e}\right]dv_x dv_y dv_z$$

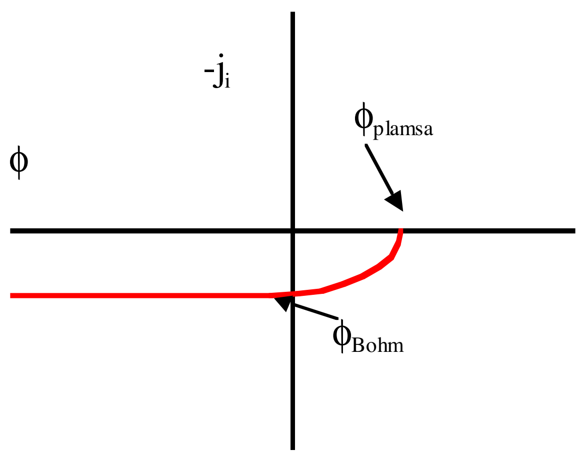
$$= en_e\left(\frac{m_e}{2\pi kT_e}\right)^{1/2}\int_{\sqrt{\frac{2e\Phi_{probe}}{m_e}}}^{\infty} v_x \exp\left[\frac{-m_e\left(v_x^2\right)}{2kT_e}\right]dv_x$$

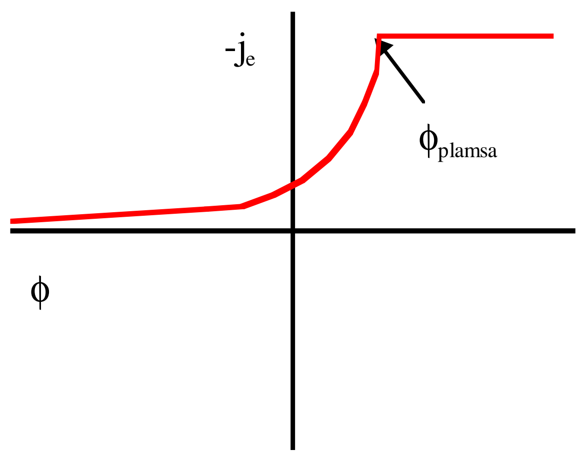
$$= en_e\frac{1}{2}\left(\frac{2kT_e}{\pi m_e}\right)^{1/2}\int_{\sqrt{\frac{2e\Phi_{probe}}{m_e}}}^{\infty} \exp\left[\frac{-m_e\left(v_x^2\right)}{2kT_e}\right]d\left(\frac{m_e v_x^2}{2kT_e}\right)$$

$$= en_e\frac{1}{2}\left(\frac{2kT_e}{\pi m_e}\right)^{1/2}\int_{\sqrt{\frac{2e\Phi_{\mathrm{Pr}obe}}{m_e}}}^{\infty} \exp[-U]d(U)$$

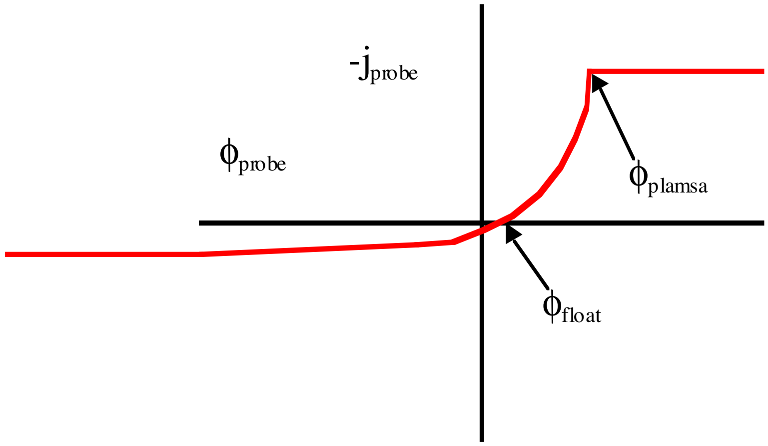
$$= en_e\left(\frac{kT_e}{2\pi m_e}\right)^{1/2}\exp\left[\frac{-e\Phi_{probe}}{kT_e}\right] \qquad \left\{= n_e\frac{1}{4}\overbrace{\langle v\rangle}^{\text{average speed}}\exp\left[\frac{-e\Phi_{probe}}{kT_e}\right]\right\}$$

We can draw graphs of what we think the current-voltage, or 'I-V,' curve should look like.

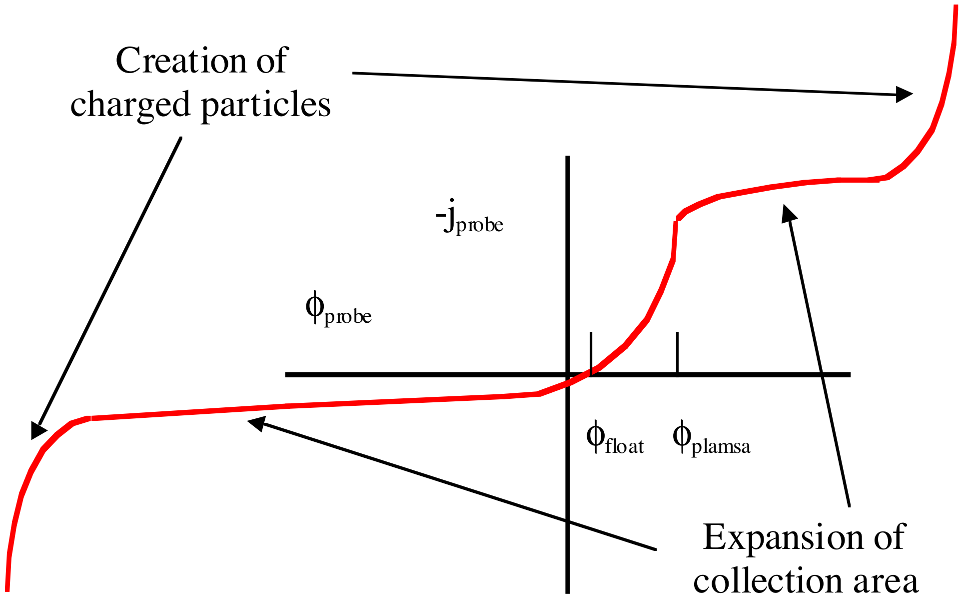




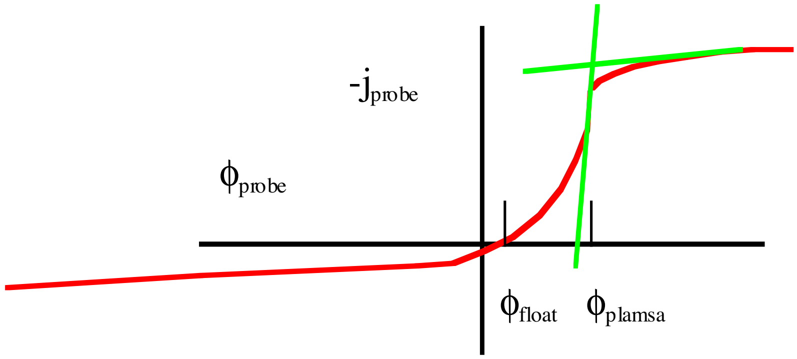
Combining these graphs we get



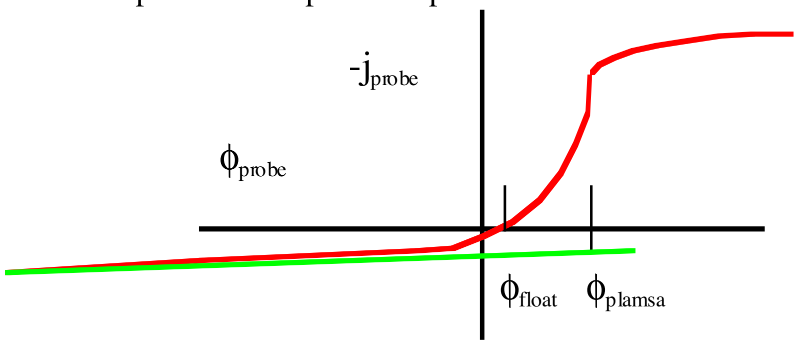
Unfortunately a 'true' I-V trace looks different than what we have drawn. They tend to look more like



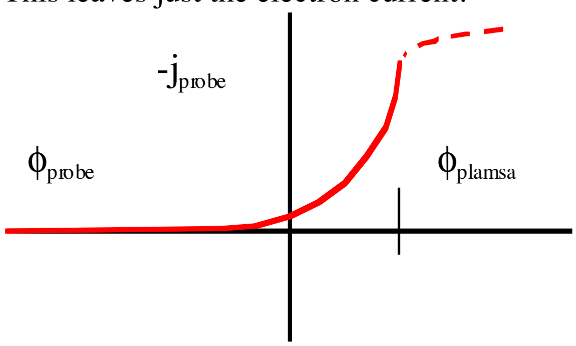
The creation of charged particles is due to ionization events that occur due to the relative bias. The expansion of the collection area is due to the growth of the sheath width as a function of bias. (This can be examined theoretically. (See page 140 "How Langmuir Probes work" N. Hershkowitz, in Plasma Diagnostics edited by O. Auciello and D.L. Flamm) Simple analysis can still be performed on this complex curve. First, we can easily determine the floating potential. (IF you can't figure out ask one of your classmates.) Now, how do we get the rest of the information? Typically the first thing that we wish to obtain is the plasma potential. We note that there is a transition in the I-V trace as we cross the ϕ_{plasma}. We can determine the potential either by fitting lines to the curve above and below and finding the crossing point or by taking the first or second derivative.



The next thing that is usually obtained is to remove the ion component of the curve. Again, this is done by fitting a line to the ion component of the curve, well below the floating potential - and thus has little electron current - and subtracting that line from the curve. (The exact curvature of the line depends on the probe shape. This can be calculated with orbital theory.)



This leaves just the electron current.



The only important part of the curve is that below the plasma potential. We can now determine the electron temperature from of equation for this curve.

$$j_e = en_e\left(\frac{kT_e}{2\pi m_e}\right)^{1/2}\exp\left[\frac{-e\Phi_{probe}}{kT_e}\right]$$

$$= j_{e0}\exp\left[\frac{-e\Phi_{probe}}{kT_e}\right]$$

By taking the natural log of the curve, normalized by the current at the plasma potential we get

$$\ln\left(\frac{j_e}{j_{e0}}\right) = \frac{-e\Phi_{probe}}{kT_e}$$

This means that the inverse of the slope of this line is kT_e. We can now use this to determine the ion density from

$$I_i = j_i A_{probe} = 0.61 q n_0 A_{probe} \sqrt{\frac{kT_e}{M_i}}$$

To eliminate the expansion of the sheath, the current is usually that current that is fit for the ions at the plasma potential.

NOTE THIS IS THE SIMPLE PROBE THEORY. RESEARCH IN THIS AREA IS STILL ON GOING AND PH.D. DISSERTATIONS HAVE BEEN WRITTEN ON THIS SUBJECT.

We will now turn to emissive probes.