1 Consider the following routing problem in a network. The network is represented by an undirected graph. Let A, B, C be three different nodes. We want to find a simple path from A to C, with the additional requirement that it also goes through node B. Let us call such a path an $A - B - C$ path. By simple path we mean that it is loop-free, that is, no node can be visited more than once as we traverse the path. Thus, you cannot just use a shortest path from A to B and then another one from B to C, since when you combine these paths into a single $A - B - C$ path, then the result may not be a simple (loop-free) path, as the $A - B$ and $B - C$ parts may share some nodes. Propose an algorithm to find a simple $A - B - C$ path. Justify your solution.

2 Find an integer *linear* programming formulation for the design problem described below. Add appropriate comments to your formulation to explain the meaning of the expressions and variables. Make it clear which values are received as input and which are the ones that are computed by the optimization.

A company has n sites that can be used to deploy network equipment. On k of the sites the company wants to build switching hubs and the remaining sites are used for concentrators. At each site either a single hub or a single concentrator will be built, but not both. The cost of building a hub at site i is a_i. The concentrator cost is independent of the site, but depends on how many concentrators are bought altogether. Specifically, a concentrator costs b dollars for the first concentrator and each additional one costs 10% less than the previous one. That is, the first concentrator costs b dollars, the second costs $0.9b$, the third costs 0.9^2b, the fourth costs 0.9^3b, and so on. Once a hub is built at site i, it brings a profit q_i, while a concentrator at site i generates a profit of p_i. All the above parameters are given as input. The company's objective is to maximize the net profit, i.e., the profit that remains after deducting the costs.

3 Consider a network in which each node has exactly d links adjacent to it. That is, in the graph that models the network topology, each vertex has degree d. Each link is operational with probability p, independently of the others. The entire network is considered operational if every node has at least one adjacent operational link.

a.) Give an estimation of the network reliability by reducing it to a combination of parallel and series configurations. Show all your work!

b.) Show through a small example that your result in **a.)** is only an estimation, but not exact. That is, provide a small example in which the estimation does not agree with the exact result.

4 Assume a network has $2n$ nodes ($n \geq 1$). Each node has a unique identifier (ID), which is a number from $\{1, 2, \ldots, 2n\}$. Each node is operational with a given probability p, independently of the others. The entire sytem is considered operational if the number of operational nodes with even ID is strictly more than the number of operational nodes with odd ID. What is the reliability of this system, expressed by the parameters p and n? Show all your work!