Homework #7

Question 1:
Consider the following program, that takes as its input `x` and `y` and computes x^y. (This program is similar to the one in class, except it increments t rather than decrementing it).

```
{y >= 0}
t := 1;
z := 1;
while t =< y
  begin
    z := z*x;
    t := t+1;
  end
  {z = x^y}
```

1. Define a loop invariant? What is the loop invariant for the while loop in the above program?
2. Prove that the above program correctly computes what it is supposed to compute using Hoare's method.

Question 2

The following program computes gcd(x,y). Using the Hoare Axiomatization, prove that this program is partially correct wrt the precondition I(x) & I(y) and the postcondition g = gcd(x,y). Begin by finding a suitable loop invariant q (hint: the invariant is a relationship between gcd(x,y) and gcd(s,g)). In your proof, you can make use of the following law of the greatest common divisor: $1 \leq X \leq Y \rightarrow gcd(X, Y) = gcd(YmodX, X)$

```
{I(x) & I(y) & x > 0 & y > 0}
  if x > y then
    begin
      g := x;
      s := y;
    end
  else
    begin
      g := y;
      s := x;
    end
      {q}
  while s ≠ 0 do
    begin
      t := s;
      s := g mod s;
      g := t;
    end
{g = gcd(x, y)}
```

Question 3

Problem 7 in Chapter 6

Question 4

A. Compute the series of non-recursive functions $F^0(\Phi), F^1(\Phi)F^2(\Phi), F^3(\Phi), \ldots$ for the following mutually recursive functions (note: $\Phi = \lambda n.\bot$):

```
f(m,n) = g(n) → m | f(m-2,n-2)
g(m) = m = 0 → false | m = 1 → true | g(m-2)
```

B. Find the least fixpoints of `f` and `g`. [Hint: First find the fixpont of `g`, then use it to find the fixpont of `f`.]

Hint: LFP of `g`:

```
g(m) = even(m) → false | odd(m) → true | ⊥
```

Now use the graph of `g`, to find fixpoint of `f`.