Small-Step Operational Semantics of TIMP

1 Syntax of TIMP

commands	$c ::= \texttt{skip} \mid c_1; c_2 \mid v\texttt{:=}e \mid \texttt{if}\ e\ \texttt{then}\ c_1\ \texttt{else}\ c_2$
	$\mid \texttt{while}\ e\ \texttt{do}\ c \mid \texttt{int}\ v \mid \texttt{bool}\ v$
expressions	$e ::= n \mid \texttt{true} \mid \texttt{false} \mid v \mid e_1\ aop\ e_2 \mid e_1\ bop\ e_2 \mid e_1\texttt{<=}e_2 \mid \texttt{!}e$
arithmetic operators	$aop ::= \texttt{+} \mid \texttt{-} \mid \texttt{*}$
boolean operators	$bop ::= \texttt{\&\&} \mid \texttt{||}$
variable names	v
integer constants	n
boolean values	$p ::= T \mid F$
stores	$\sigma : v \rightharpoonup \mathbb{Z} \cup p$

2 Small-step Semantics of TIMP

2.1 Commands

$$\langle \texttt{int}\ v, \sigma\rangle \to_1 \langle \texttt{skip}, \sigma\rangle \tag{1}$$

$$\langle \texttt{bool}\ v, \sigma\rangle \to_1 \langle \texttt{skip}, \sigma\rangle \tag{2}$$

$$\frac{\langle c_1, \sigma\rangle \to_1 \langle c_1', \sigma'\rangle}{\langle c_1; c_2, \sigma\rangle \to_1 \langle c_1'; c_2, \sigma'\rangle} \tag{3}$$

$$\frac{\langle e, \sigma\rangle \to_1 \langle e', \sigma'\rangle}{\langle v\texttt{:=}e, \sigma\rangle \to_1 \langle v\texttt{:=}e', \sigma'\rangle} \tag{4}$$

$$\langle v\texttt{:=}n, \sigma\rangle \to_1 \langle \texttt{skip}, \sigma[v \mapsto n]\rangle \tag{5}$$

$$\langle v\texttt{:=}p, \sigma\rangle \to_1 \langle \texttt{skip}, \sigma[v \mapsto p]\rangle \tag{6}$$

$$\frac{\langle e, \sigma\rangle \to_1 \langle e', \sigma\rangle}{\langle \texttt{if}\ e\ \texttt{then}\ c_1\ \texttt{else}\ c_2, \sigma\rangle \to_1 \langle \texttt{if}\ e'\ \texttt{then}\ c_1\ \texttt{else}\ c_2, \sigma'\rangle} \tag{7}$$

$$\langle \texttt{if true then}\ c_1\ \texttt{else}\ c_2, \sigma\rangle \to_1 \langle c_1, \sigma\rangle \tag{8}$$

$$\langle \texttt{if false then}\ c_1\ \texttt{else}\ c_2, \sigma\rangle \to_1 \langle c_2, \sigma\rangle \tag{9}$$

$$\langle \texttt{while}\ e\ \texttt{do}\ c, \sigma\rangle \to_1 \langle \texttt{if}\ e\ \texttt{then}\ (c; \texttt{while}\ e\ \texttt{do}\ c)\ \texttt{else skip}, \sigma\rangle \tag{10}$$

2.2 Expressions

$$\frac{\sigma(v) = n}{\langle v, \sigma\rangle \rightarrow_1 \langle n, \sigma\rangle} \tag{11}$$

$$\frac{\sigma(v) = T}{\langle v, \sigma\rangle \rightarrow_1 \langle \texttt{true}, \sigma\rangle} \tag{12}$$

$$\frac{\sigma(v) = F}{\langle v, \sigma\rangle \rightarrow_1 \langle \texttt{false}, \sigma\rangle} \tag{13}$$

$$\frac{\langle e_1, \sigma\rangle \rightarrow_1 \langle e_1', \sigma'\rangle \qquad op \in aop \cup bop \cup \{\texttt{<=}\}}{\langle e_1\ op\ e_2, \sigma\rangle \rightarrow_1 \langle e_1'\ op\ e_2, \sigma'\rangle} \tag{14}$$

$$\frac{\langle e_2, \sigma\rangle \rightarrow_1 \langle e_2', \sigma'\rangle \qquad op \in aop \cup \{\texttt{<=}\}}{\langle n\ op\ e_2, \sigma\rangle \rightarrow_1 \langle n\ op\ e_2', \sigma'\rangle} \tag{15}$$

$$\langle n_1 \texttt{ + } n_2, \sigma\rangle \rightarrow_1 \langle n_1 + n_2, \sigma\rangle \tag{16}$$

$$\langle n_1 \texttt{ - } n_2, \sigma\rangle \rightarrow_1 \langle n_1 - n_2, \sigma\rangle \tag{17}$$

$$\langle n_1 \texttt{ * } n_2, \sigma\rangle \rightarrow_1 \langle n_1 n_2, \sigma\rangle \tag{18}$$

$$\frac{n_1 \leq n_2}{\langle n_1 \texttt{<=} n_2, \sigma\rangle \rightarrow_1 \langle \texttt{true}, \sigma\rangle} \tag{19}$$

$$\frac{n_1 > n_2}{\langle n_1 \texttt{<=} n_2, \sigma\rangle \rightarrow_1 \langle \texttt{false}, \sigma\rangle} \tag{20}$$

$$\langle \texttt{true \&\& } e_2, \sigma\rangle \rightarrow_1 \langle e_2, \sigma\rangle \tag{21}$$

$$\langle \texttt{false \&\& } e_2, \sigma\rangle \rightarrow_1 \langle \texttt{false}, \sigma\rangle \tag{22}$$

$$\langle \texttt{true || } e_2, \sigma\rangle \rightarrow_1 \langle \texttt{true}, \sigma\rangle \tag{23}$$

$$\langle \texttt{false || } e_2, \sigma\rangle \rightarrow_1 \langle e_2, \sigma\rangle \tag{24}$$

$$\frac{\langle e, \sigma\rangle \rightarrow_1 \langle e', \sigma\rangle}{\langle !e, \sigma\rangle \rightarrow_1 \langle !e', \sigma'\rangle} \tag{25}$$

$$\langle !\texttt{true}, \sigma\rangle \rightarrow_1 \langle \texttt{false}, \sigma\rangle \tag{26}$$

$$\langle !\texttt{false}, \sigma\rangle \rightarrow_1 \langle \texttt{true}, \sigma\rangle \tag{27}$$