Logic Programming

CS 6371: Advanced Programming Languages

FP vs. LP

- Functional Programming
  - centered around first-class functions
  - strong, parametric polymorphic type system
  - single-assignment
  - operational semantics based on λ-calculus
- Logic Programming
  - centered around *relations*
  - no type system
  - no explicit assignment operation(!)
  - operational semantics based on depth-first search

Relations

- Relation
  - Def: A **relation** is a cartesian product ($A \times B$) of two sets A and B
  - Example: ≤ relation over $\mathbb{N} \times \mathbb{N}$: {(0,0),(0,1),(1,1),(0,2),(1,2),(2,2),...}
- Relations generalize functions
  - Recall: We write functions $f:A \rightarrow B$ as sets of pairs $A \times B$
  - Relations (as defined above) are also sets of pairs
  - Function f encodes relation $\{(x,f(x)) \mid x \in Dom(f)\}$
  - Unlike functions, relations can map same domain element to multiple different range elements

Relational Programming

- Three ways to define a function/relation
  - Imperatively
    - factorial(x) = { z:=1; for i:=1 to x do z:=z*i; return z}
  - Functionally
    - factorial(x) = (if x<=0 then 1 else x*factorial(x-1))
  - Relationally
    - factorial(0,1).
    - factorial(x,y) if factorial(x-1,y/x)
- Note the differences in approach
  - Imperative is an operational recipe
    - you are essentially doing the compiler's job
    - compiler must reverse-engineer your code to optimize it!
  - Functional is a mathematical recipe
    - better, but still somewhat operational
  - Relational defines necessary and sufficient conditions
    - compiler creates a search algorithm for the solution
    - implementation details abstracted away from programmer
    - search algorithm can be highly optimized by language implementation

Prolog Programming

- Programs consist of
  - facts (unconditional truths)
  - rules (conditional truths)
  - queries (cause the program to “run” by initiating search for a solution to a question)
- Example: factorial program

```
factorial(0,1).
factorial(X,Y) :- X2 is X-1, factorial(X2,Y2), Y is X*Y2.
```

```
?-  factorial(5,X).
X = 120
```

Applications

- Originally invited by Robert Kowalski (for theorem-proving) and Alain Colmeraur (for NLP) [1973]
- Now used primarily for
  - artificial intelligence
  - scheduling problems
  - databases (Datalog)
  - model-checking
  - compilers
  - software engineering (verification, etc.)
  - network protocol analysis
  - many other applications…

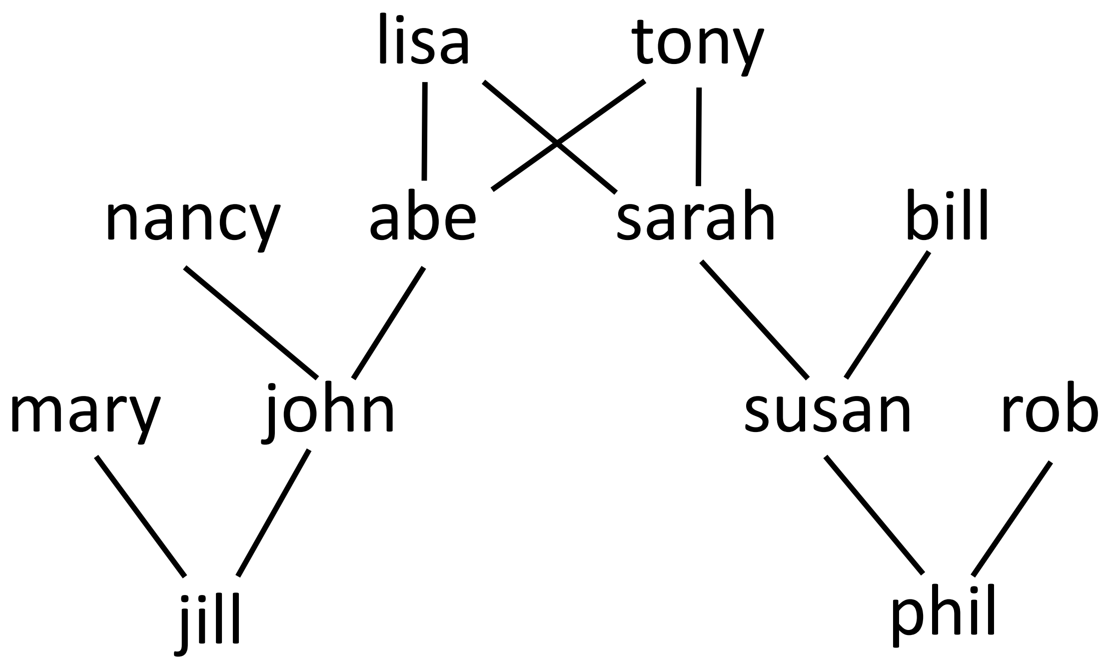
Running Prolog

- One Prolog programming assignment (given next time)
- Two installation options
  - Use CS Dept Unix machines to do assignment, or
  - Install SWI Prolog on your machine (see link on course web page)
- Programming
  - create a text file named “lastname.pl”
  - text file contains facts and rules (no queries)
- Running your program
  - type “pl” at the Unix prompt
  - type “consult(lastname).” at Prolog prompt
  - enter queries at Prolog prompt
  - to reload after changing programs, just type “make.”
  - exit by typing control-C then “e”

Prolog Syntax

- Each program line has one of two forms:
  - $p(t_1,\ldots,t_n)$.
  - $p(t_1,\ldots,t_n)$:- $p_1(t_1,\ldots,t_i)$, $p_2(t_1,\ldots,t_j)$, …, $p_m(t_1,\ldots,t_k)$.
  - Don't forget the period ending each line!
  - p is a "predicate" consisting of lower-case letters (e.g., "factorial")
  - $t_1,\ldots,t_n$ are "terms"
- Terms can be…
  - integer constants (1, -13, etc.)
  - atoms (non-numerical constants)
    - consist of lower-case letters or surrounded by single-quotes
    - Examples: x, abc, 'Foo'
  - variables (start with Capital letters)
    - Examples: X, Foo
  - structures (tree-like data structures)
    - Examples: foo(3,12), foo(foo(13),foo(16,12))
    - syntax like predicates but not the same as predicates!
    - no type system, so be careful!

Example: Family Tree



Prolog Representation of Family Tree

```
father(tony,abe).
father(tony,sarah).
father(abe,john).
father(bill,susan).
father(john,jill).
father(rob,phil).
mother(lisa,abe).
mother(lisa,sarah).
mother(nancy,john).
mother(sarah,susan).
mother(mary,jill).
mother(susan,phil).
```

Reasoning about Family Trees

- Parent
  - parent(X,Y) :-

Reasoning about Family Trees

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  - parent(X,Y) :- father(X,Y).
  - parent(X,Y) :- mother(X,Y).
- Grandparent
  - gp(X,Y) :-

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  - gp(X,Y) :- parent(X,Z), parent(Z,Y).
- Great-grandparent
  - ggp(X,Y) :-

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- Ancestor
  - ancestor(X,Y) :- parent(X,Y).
  - ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).

Query Examples

```
?- father(abe,john).
true.

?- father(tony,X).
X = abe ;                    <user presses semicolon>
X = sarah.

?- parent(X,susan).
X = bill ;                   <user presses semicolon>
X = sarah ;                  <user presses semicolon>
false.

?-
```

Queries

- typed at Prolog prompt (not in external files)
- consist of a predicate possibly containing variables
  - if no variables, result is either “true” or “false”
  - otherwise, result is an instantiation of variables or “false”
- no solutions, one solution, or many solutions
  - no solution: “false”
  - after printing one solution, Prolog waits for user input
  - type RETURN to stop search; Prolog says “true”
  - type SEMICOLON to find more solutions; Prolog either finds another and waits for more input or says “false”
- convergence not guaranteed!
  - queries can diverge (i.e., loop infinitely)
  - type control-C to interrupt, then “a” to abort

Search Procedure

- How does Prolog search for query solutions?
- Three internal data structures
  - search tree in which each node has…
  - a list of goals (predicates)
  - a set of variable bindings (instantiations)
- Two important concepts
  - unification – find instantiation of vars to make equal terms (if such instantiation exists)
  - back-tracking – revisiting past decisions after a failed goal is reached

Search Procedure

- Initially
  - search tree has just a root
  - goal list consists only of the query
  - set of variable bindings is empty
- **Step 1:** Scan file from **top to bottom** for a fact or rule whose lhs potentially matches the current goal
  - for each such fact/rule, add a child node to the search tree
  - descend to the leftmost child
- **Step 2:** Unify the top goal with this rule's lhs, yielding more variable bindings
- **Step 3:** Add rhs predicates to goal list, **left to right**
- Return to Step 1
- Steps 1 or 2 may fail
  - no matching rule or failed unification
  - if so, backtrack to parent node and try next child
  - if root node fails, stop and return "false"

Search Example

ancestor(tony,phil)

ancestor(X_1,Y_1) :- parent(X_1,Y_1).

X_1=tony, Y_1=phil

parent(tony,phil)

ancestor(X_1,Y_1) :- parent(X_1,Z_1),
ancestor(Z_1,Y_1).

parent(X_2,Y_2) :-
father(X_2,Y_2).

X_1=X_2=tony, Y_1=Y_2=phil

father(tony,phil)

FAILS

parent(X_2,Y_2) :-
mother(X_2,Y_2).

X_1=X_2=tony, Y_1=Y_2=phil

mother(tony,phil)

FAILS

Search Example

ancestor(tony,phil)

⋮

ancestor(X_1,Y_1) :- parent(X_1,Z_1),
ancestor(Z_1,Y_1).

X_1=tony, Y_1=phil
parent(tony,Z_1)
ancestor(Z_1,phil)

parent(X_2,Y_2) :-
father(X_2,Y_2).

X_1=X_2=tony, Y_1=phil, Y_2=Z_1
father(tony,Z_1)
ancestor(Z_1,phil)

parent(X_2,Y_2) :-
mother(X_2,Y_2).

Search Example

X_1=X_2=tony, Y_1=phil, Y_2=Z_1
father(tony,Z_1)
ancestor(Z_1,phil)

father(tony,abe).

X_1=X_2=tony, Y_1=phil, Y_2=Z_1=abe
ancestor(abe,phil)

...

EVENTUALLY FAILS

father(tony,sarah).

X_1=X_2=tony, Y_1=phil, Y_2=Z_1=sarah
ancestor(sarah,phil)

...

ancestor(susan,phil)

ancestor(X_3,Y_3) :- parent(X_3,Y_3).

X_3=susan, Y_3=phil
parent(susan,phil)

father(susan,phil).

FAILS

mother(susan,phil).

SUCCEEDS

ancestor(X_3,Y_3) :- parent(X_3,Z_3),
ancestor(Z_3,Y_3).

Important Points

- Order matters!
  - order of facts/rules in file
  - order of predicates on rhs of rules
  - order ONLY AFFECTS TERMINATION
  - does not affect outcomes
- Tips for good ordering
  - put facts before rules (base case before recursive case)
  - put "easy" predicates before harder ones

Effects of Reordering

- Our definition of ancestor:
  - ancestor(X,Y) :- parent(X,Y).
  - ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
- What would happen if we reversed the lines?
  - ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
  - ancestor(X,Y) :- parent(X,Y).
- What about if we reversed the conjuncts in the rule?
  - ancestor(X,Y) :- parent(X,Y).
  - ancestor(X,Y) :- ancestor(Z,Y), parent(X,Z).
- What about both changes together?
  - ancestor(X,Y) :- ancestor(Z,Y), parent(X,Z).
  - ancestor(X,Y) :- parent(X,Y).

Equality Predicates

- “=” means “unifiable”
  - attempts a unification
  - Example #1: f(X,a)=f(b,Y). (succeeds with X=b, Y=a)
  - Example #2: X=a, X=b. (fails)
  - Example #3: X=a, a=X. (succeeds with X=a)
- “==” means “physically equal”
  - tests existing bindings (no new unification!)
  - Example #1: a==b (false)
  - Example #2: X==Z (false)
  - Example #3: X=Z, X==Z (true)
  - Example #4: X==a (false)
  - Example #5: X=a, X==a (true)
- “\==” is negation of “==”
  - Example: Siblings
  - sibling(X,Y) :- parent(Z,X), parent(Z,Y), X \== Y.

Inequalities

- Numerical inequalities
  - X < Y, X > Y, X =< Y, X >= Y
  - these succeed ONLY when both X and Y are already bound to integers
  - no unification occurs
  - no arithmetic expressions allowed!
    - example: X+3 < X+4 (syntax error!)
- Non-numerical comparisons
  - X @< Y, X @> Y, X @=< Y, X @>= Y
  - compare arbitrary atoms according to a “standard” ordering
  - Example: bar @< foo (succeeds)
  - X and Y must be bound

Arithmetic

- “is” keyword
  - Syntax: X is 3+5
  - single variable on left
  - arithmetic expression on right
  - no non-unified variables on right!
- Examples:
  - X=5, X is 4+2 (false)
  - X is Y+3 (abort with error)
  - X=5, Y is X+3 (succeeds with Y=8)
- Equality does NOT solve arithmetic
  - X=3+5 (binds X to the literal STRUCTURE “3+5”)
- The “is” keyword is NOT an assignment operation
  - X is X+1 (ALWAYS FAILS!)
  - X=X+1 (ALWAYS FAILS!)

Lists

- Syntax
  - [] is the empty list
  - [H|T] is a list with head H and tail T
    - recall: list tail is a list of all elements except the head
    - tail can be empty!
  - [X,Y|Z] is a list with first two elements X and Y and remaining elements Z
- Example: Summing a list
  - sum(L,S) should succeed if S is the sum of the elements in list L

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  - [X,Y|Z] is a list with first two elements X and Y and remaining elements Z
- Example: Summing a list
  - sum([],0).
  - sum([H|T],S) :- sum(T,S1), S is H+S1.

More List Examples

- Appending to a list
  - append(L1,L2,L3) succeeds when L3 is list L1 appended by list L2

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  - append([H1|T1],L2,[H1|T3]) :- append(T1,L2,T3).
- List member selection
  - pick(X,L1,L2) succeeds when X is a member of list L1 and L2 is list L1 without X

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- List member selection
  - pick(X,L1,L2) succeeds when X is a member of list L1 and L2 is list L1 without X
  - pick(X,[X|T],T).
  - pick(X,[Y|T1],[Y|T2]) :- X \== Y, pick(X,T1,T2).

Logical Arithmetic

- Encode natural numbers as structures:
  - zero is 0
  - one is s(0)
  - two is s(s(0))
  - num(0).
  - num(s(N)) :- num(N).
- Compute the sum of two logically encoded natural numbers

Logical Arithmetic

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  - num(s(N)) :- num(N).
- Compute the sum of two logically encoded natural numbers
  - lplus(0,Y,Y).
  - lplus(s(X),Y,s(Z)) :- lplus(X,Y,Z).

Logical Arithmetic

- Logical subtraction

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  - lminus(X,Y,Z) :- lplus(Y,Z,X).
- Logical multiplication

Logical Arithmetic

- Logical subtraction
  - lminus(X,Y,Z) :- lplus(Y,Z,X).
- Logical multiplication
  - ltimes(0,Y,0).
  - ltimes(s(X),Y,Z) :- ltimes(X,Y,XY), lplus(XY,Y,Z).

Negation

- \+ P
  - succeeds when predicate P terminates with failure
  - NOT the same as logical negation!
  - think of it more like "P is disprovable"
  - loops when P loops
  - exacerbates order-sensitivity issues
  - avoid spurious uses

Misc. Operators

- semicolon is “or”
  - parent(X,Y) :- (father(X,Y); mother(X,Y)), X \== Y
  - never needed but can make rules shorter
- Underscore is a wildcard
  - len([],0).
  - len([_|T],X) :- len(T,Y), X is Y+1.
  - If you write “[H|T]” instead of “[_|T]”, you’ll get a warning because H is defined but never used.
  - Warnings are to help you identify typos (e.g., mistyped variable names).
- Other operators available as well
  - see online Prolog documentation (linked from website)
  - not needed for this class, but you can use them if you wish