Lecture 3: Mutual Recursion & Tail Recursion

CS 6371: Advanced Programming Languages
January 21, 2014

#let rec length =
  function [] -> 0 | _::t -> (length t)+1;;
length : 'a list -> int = <fun>

“function ... -> ...” is an abbreviation for “fun x -> (match x with ... -> ...)”

#type staff = Programmer | Manager of dept
and dept = Outsourced | Staffed of staff;;
Type staff defined.
#type staff2str s =
  (match s with
    Programmer -> "Peon"
  | Manager d ->
    "Dictator["^(dept2str d)^"]")
and dept2str d =
  (match d with Outsourced -> "Exiled"
    | Staffed s -> staff2str s);
staff2str : staff -> string = <fun>
dept2str : dept -> string = <fun>

Mutually recursive types are separated by the word “and”. Notice that there is no “;;” before the “and” and there is no second “type” keyword. You can string as many mutually recursive types together as you wish with “and”.

#type 'a btree = BNull
| BNode of ('a * 'a btree * 'a btree);;
Type btree defined.
BNode (3,BNull,BNull);;
- : int btree = BNode (3, BNull, BNull)
BNode ("foo",BNull,BNull);;
- : string btree = BNode ("foo",BNull,BNull)
BNode("foo",BNode(3,BNull,BNull),BNull);;
Toplevel input:
> BNode("foo",BNode(3,BNull,BNull),BNull);;
> ^^^^^^^^^^^^^^^^^^^^^^^
This expression has type int btree,
but is used with type string btree.

Polymorphic variants define a type constructor that is parameterized by a type variable.

Here’s an example of a function that converts a polymorphic binary tree to a polymorphic list (with list elements given in prefix order). The “@” operator concatenates two lists. This differs from the “::” operator, which inserts an element onto the head of a list.

#let fold_left f b l =
  (match l with
    [] -> b
  | h::t -> fold_left f (f b h) t);;
fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a = <fun>
fold_left (fun x y -> x+y) 0 [1;2;3];;
- : int = 6

“Fold” is an extremely important list operation in functional programming. (fold_left f b [w;x;y;z]) computes the formula f(f(f(f(f(b,w),x),y),z)). Parameter ‘b’ is called the “base case”.

From “fold” one can derive many useful list functions, such as existence and forall functions that check if a given condition holds for any or all
<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td><code>fold_left</code></td>
<td>Applies function ( f ) starting with the leftmost element. That is, ((\text{fold}_\text{left} \ f \ [w;x;y;z]) \ b) computes (f(w,f(f(f(w,b),b),b),b)).</td>
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