Performance of Concatenated Channel Codes and Orthogonal Space-Time Block Codes

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Abstract

In this paper, we analyze the performance of channel codes concatenated with orthogonal space-time block codes. We present general analysis for independent fading as well as spatially and temporally correlated fading. Simulations verify the validity of our analysis.

I. Introduction and System Model

Orthogonal space-time block codes (STBC) provide diversity but have little or no coding gain, therefore in practice it is often necessary to use them in conjunction with channel coding. The overall decoder consists of two separable decoding operations. The adoption of this structure, e.g., in the WCDMA standard, has motivated us to analyze its performance under realistic conditions. To improve the performance of the resultant code, we introduce an interleaver between the outer and inner codes which provides higher time diversity. The placement of the (random/uniform) interleaver also facilitates the analysis of the block fading channel (which is required for linear decoding of orthogonal STBC's). This work is different from the previous results of [1], [2], [3] where either block fading or interleaving or both have been absent. Moreover, we present performance analysis of the concatenated scheme in the presence of spatially and/or temporally correlated channel.

It is well known that the multiple-input multiple output (MIMO) channel, driven by an orthogonal STBC, can be represented by an equivalent single-input single-output (SISO) channel, assuming an optimal combining of the received signals. The equivalent SNR is $\gamma = \bar{\gamma}||\mathbf{H}||^2$ where $||\cdot||$ denotes the Frobenius norm, $\bar{\gamma} = R_c E_b / n_T N_0$ is the average SNR per information bit per transmit antenna, and R_c is the code rate and n_T is the number of transmit antennas. The resultant instantaneous SNR per bit, γ, follows non-central chi-square distribution with degree of freedom $2n_T n_R$ [1]. Hence, the outer code sees a block fading channel with the statistics that have been altered by the mapping induced by the STBC encoder and decoder.

II. Performance Analysis

We employ an approach similar to [4] to evaluate the pairwise error probability (PEP) of an outer codeword. In this approach, the PEP is evaluated under a block fading pattern and then, using the concept of uniform interleaving, the PEP is averaged over all the possible fading patterns. We develop the PEP of a codeword with distance d under a given fading pattern of Rician channel with parameter K. Due to space limit the presentation is abbreviated and proofs as well as analysis for non-binary modulations are omitted. For proofs, the interested reader is referred to [**?**].

To evaluate $P(d|\mathbf{f})$, we need to determine how much error weight is present in each fading block. To characterize that, we build a histogram of weights as follows: assume the number of blocks that have weight m is f_m, and consider the vector $\mathbf{f} = (f_0, \ldots, f_w)$ where $w = \min(\ell, d)$, and ℓ is the block fading length. A given vector $\mathbf{f}$ is a valid histogram if $\sum f_m = F$ and $\sum m f_m = d$, and F is the number of fading blocks in a codeword. We assume full channel state information at the receiver. Denoting the moment generating function (MGF) of γ as $\Phi_\gamma(.)$, we have

$$P(d|\mathbf{f}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{m=1}^{w} \left[\Phi_\gamma\left(-\frac{m}{\sin^2\theta}\right)\right]^{f_m} d\theta. \quad (1)$$

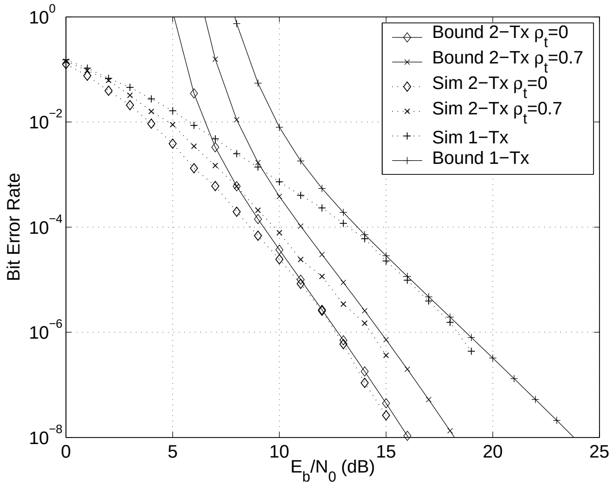

Fig. 1. Convolutional code, 2-Tx, 1-Rx, $F_dT_s = 0.1$, $K = 0$ dB

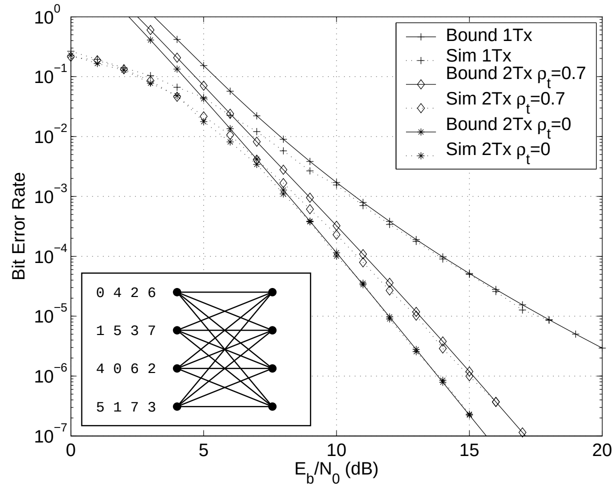

Fig. 2. TCM, 2-Tx and 1-Rx, block i.i.d. fading $K = 5$ dB

We employ a well-accepted spatial correlation model which introduces correlation in the transmit and receive sides by two matrix correlation R_{Tx} and R_{Rx}. Therefore, the correlated channel is $\mathbf{H} = \mathbf{R}_{\text{Tx}}^{1/2}\ \tilde{\mathbf{H}}\ \mathbf{R}_{\text{Rx}}^{1/2}$. Denoting the eigenvalues of R_{Tx} and R_{Rx} with $\lambda^{(t)}$ and $\lambda^{(r)}$, we have

$$\Phi_\gamma(s) = \prod_{i=1}^{n_T}\prod_{j=1}^{n_R} \frac{(1+K)}{1+K-s\lambda_i^{(\text{t})}\lambda_j^{(\text{r})}\bar{\gamma}} \exp\left(\frac{Ks\lambda_i^{(\text{t})}\lambda_j^{(\text{r})}\bar{\gamma}}{1+K-s\lambda_i^{(\text{t})}\lambda_j^{(\text{r})}\bar{\gamma}}\right) \quad . \tag{2}$$

In the case of temporal and spatial correlation, we consider the spatial correlation as before and assume it is stationary. The temporal correlation is only considered between the d time instances contributing in the distance. Let $\mathbf{R}_t(i,j)$ be the correlation between the MIMO channel of two time instances i and j, and denote the eigenvalues of $\mathbf{R}_t$ by μ, then

$$\Phi_\gamma(s) = \prod_{k=1}^{d}\prod_{i=1}^{n_T}\prod_{j=1}^{n_R} \frac{(1+K)}{1+K-s\mu_k\lambda_i^{(\text{t})}\lambda_j^{(\text{r})}\bar{\gamma}} \exp\left(\frac{K\,s\,\mu_k\,\lambda_i^{(\text{t})}\lambda_j^{(\text{r})}\bar{\gamma}}{1+K-s\mu_k\lambda_i^{(\text{t})}\lambda_j^{(\text{r})}\bar{\gamma}}\right) \quad . \tag{3}$$

III. Results

We employ a four-state rate–$1/2$ code with generator function $(5,7)_8$. Each frame contains $k = 100$ information bits (200 BPSK symbols). This code is concatenated with Alamouti STBC and transmitted through a spatially and temporally correlated Rayleigh channel. The results are shown in Figure 1. When the correlation between transmit antennas is $\rho_t = 0.7$ the loss in coding gain is about 2dB at BER$= 10^{-6}$.

Our TCM experiments use an 8PSK-code from [5] (whose trellis is shown in Figure 2) concatenated with Alamouti scheme. The frame length is 130 symbols (260 information bits). The performance of this code under Rician $K = 5$dB channel appear in Figure 2. The performance loss due to antenna correlation of $\rho_t = 0.7$ is about 1.5dB at BER $= 10^{-6}$.

References

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