

1 Ion Diffusion Noise

1.1 Description

A rare type of noise found in some semiconductor materials, is due to the diffusion of a trap. This is a low-frequency noise which is called Ion Diffusion Noise or sometimes just called Diffusion Noise. It is created by impurity ions diffusing along dislocation lines of the crystal lattice. This type of noise was “seen” in early GaAs passive and active devices when Si atoms were used to dope Gallium Arsenide *n*-type. In this time frame, the anneal cycle only activated approximately 50% of the donors so the remaining Si atoms were situated interstitially and able to move through the lattice.

There are two mathematical formulations for Ion Diffusion Noise; the Green’s function method employed by K. van Vliet and the Langevin method used by A. van der Ziel. I will follow the second method to develop the spectral equation for this noise type.

Consider a fluctuating quantity $x(t)$ that is the result of a fluctuating concentration $C(\mathbf{r}, t)$ of particles, where \mathbf{r} is the radius vector. Assume that the concentration is governed by a diffusion mechanism, and let $C(\mathbf{r}, t)$ satisfy the diffusion equation $D\nabla^2 C - \dot{C} = 0$. The Langevin method adds a source term, $h(\mathbf{r}, t)$, to this equation. The source term is a random function that describes high frequency interactions with the environment which is exhibited as fluctuations. The differential equation to solve is

$$\frac{\partial C(\mathbf{r}, t)}{\partial t} = D \nabla^2 C(\mathbf{r}, t) + h(\mathbf{r}, t) , \quad (1)$$

where D is the diffusion constant. To solve equation (1), Fourier analysis will be used after applying periodic boundary conditions and expanding the dependent variables into spatial harmonics of vector \mathbf{k} . The vector components are given by:

$$k_i = \frac{2\pi m_i}{L_i} ; (m_i = 0, \pm 1, \pm 2 \dots) \quad \text{and } i = 1, 2, 3 ; \quad (2)$$

where L_i is the dimension over which the diffusion takes place for each coordinate direction.

For simplicity, only the one-dimensional model will be considered. Let the distance over which the diffusion takes place be ℓ in our 1-D model. Also, let the average number of particles be, $\bar{n} = \bar{C}\ell$, where \bar{C} is the average value of the concentration. Now the exact power spectrum for the one-dimensional case can be written as;

$$S_x(\omega) = \frac{2\varepsilon \bar{n} \ell}{\pi} \int_{-\infty}^{\infty} \left[\frac{\sin(\frac{1}{2}k\ell)}{\frac{1}{2}k\ell} \right]^2 \frac{D k^2}{D^2 k^4 + \omega^2} dk , \quad (3)$$

where ε is a factor for the case when Δn is not a Poisson distribution. To evaluate the integral in equation (3), the integrand will be approximated by introducing a parameter k_t . For the case where k is less than k_t , then $\sin(\frac{1}{2}k\ell) = \frac{1}{2}k\ell$, and for k greater than k_t , $\sin(\frac{1}{2}k\ell) = 1/\sqrt{2}$ so that $\sin^2(\frac{1}{2}k\ell)/(\frac{1}{2}k\ell)^2 = 2/k^2\ell^2$. Since the two approximations must yield a continuous solution, $k_t = \sqrt{2}/\ell$. By substitution and considering symmetry, the following expression for the spectrum can be obtained.

$$S_x(\omega) = \frac{4\varepsilon \bar{n} \ell}{\pi} \int_0^{k_t} \frac{D k^2}{D^2 k^4 + \omega^2} dk + \frac{8\varepsilon \bar{n}}{\pi \ell} \int_{k_t}^{\infty} \frac{D}{D^2 k^4 + \omega^2} dk . \quad (4)$$

In equation (4) the first term dominates for small ω and the second term for large ω . Therefore, the final solutions are found after integration:

$$S_x(\omega) = \frac{\sqrt{2}\varepsilon \bar{n} \ell}{\sqrt{D\omega}} , \quad \text{for small } \omega ; \quad (5)$$

$$S_x(\omega) = \frac{2\sqrt{2D}\varepsilon \bar{n}}{\ell \sqrt{\omega^3}} , \quad \text{for large } \omega . \quad (6)$$

Notice that for low frequencies the spectrum varies as $f^{-1/2}$, while for high frequencies it goes as $f^{-3/2}$. There will be a transition region between these two asymptotes with the cross-over point at

$$\omega_t = \frac{2D}{\ell^2} = \frac{1}{\tau_d} , \quad (7)$$

where τ_d is the average diffusion time over the length ℓ . It is characteristic behavior of diffusion spectra for all dimensional cases that the high-frequency asymptote is unchanged, but the low-frequency variation is functionally very different. A figure showing the asymptotic behavior of 1-D Ion Diffusion noise as described by equations (5) and (6) is below.

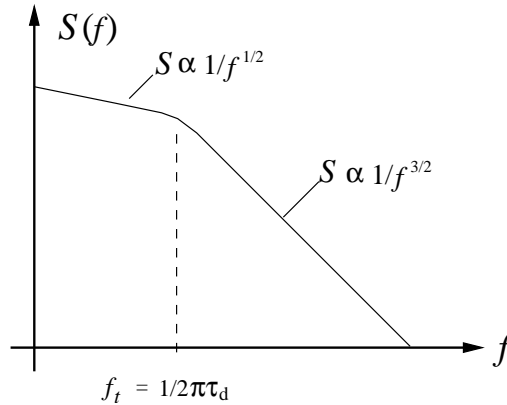


Figure 1: The frequency spectrum of Ion Diffusion noise