PAR-Constrained Training Signal Designs for MIMO OFDM Channel Estimation in the Presence of Frequency Offsets

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***Abstract*— An important practical issue which has not been incorporated in an optimized way in existing OFDM training signal designs is peak-to-average energy ratio (PAR) of the training signal. Training signals should have low PAR so they do not experience nonlinear distortions at the transmit amplifier and at the same time they should be designed to give a better and more robust estimation performance. In this paper, we study the PAR characteristics of existing OFDM training signals and propose two training signal designs for MIMO OFDM frequency-selective channel estimation in the presence of frequency offsets and PAR constraints. Our proposed training signals achieve more robust channel estimation performance against frequency offsets while satisfying the PAR constraints compared to training signals designed to achieve a fixed low PAR but without any consideration for robustness to frequency offsets.**

I. Introduction

Training signal design for OFDM channel estimation has attracted significant research attention (e.g., for SISO OFDM systems in [1]-[3], for MIMO OFDM systems in [4]-[6]). All training signal designs mentioned above assume no frequency offset. In practice, frequency offset is unavoidable due to local oscillator mismatches. Recently, we presented in [7] training signal designs for MIMO OFDM channel estimation in the presence of frequency offsets.

Another important practical issue which has not been incorporated in an integrated and optimized way in all existing OFDM training signal designs is peak-to-average energy ratio (PAR) of the training signal. Training signals should have low PAR so they do not experience nonlinear distortions at the transmit power amplifier and at the same time they should be designed to give a better and more robust estimation performance. In this paper, we study the PAR characteristics of existing training signals and present new training signal designs for MIMO OFDM channel estimation in the presence of frequency offsets and PAR constraints. Our study reveals that training signals which are most robust to frequency offsets have very large PAR. This observation makes the training design problem under consideration challenging due to conflicting constraints. In this paper, we present two training signal designs which are robust to frequency offsets while satisfying the imposed PAR constraints.

II. Signal Model and MSE-Optimality Conditions

Consider a MIMO OFDM system with K sub-carriers where training signals from N_{Tx} transmit antennas are transmitted over Q OFDM symbols. Since the same channel estimation procedure is performed at each receive antenna, we only need to consider one receive antenna in designing training signals. The channel impulse response (CIR) for each transmit-receive antenna pair (including all transmit/receive filtering effects) is assumed to have L taps and is quasi-static over Q OFDM symbols. Let $[c_{n,q}[0], \ldots, c_{n,q}[K-1]]^T$ be the pilot tones vector of the n-th transmit-antenna at the q-th symbol interval and $\{s_{n,q}[k] : k = -N_g, \ldots, K-1\}$ be the corresponding time-domain complex baseband training samples, including N_g $(\geq L-1)$ cyclic prefix (CP) samples where the superscript T denotes the transpose. Define $\boldsymbol{S}_n[q]$ as the training signal matrix of size $K \times L$ for the n-th transmit antenna at the q-th symbol interval whose elements are given by $[\boldsymbol{S}_n[q]]_{m,l} = s_{n,q}[m-l]$ for $m \in \{0, \ldots, K-1\}$ and $l \in \{0, \ldots, L-1\}$. Let $\boldsymbol{h}_n$ denote the length-L CIR vector corresponding to the n-th transmit antenna.

After cyclic prefix removal at the receiver, denote the received vector of length K at the q-th symbol interval by $\boldsymbol{r}_q$. Then, the received vector over the Q symbol intervals in the presence of a normalized (by the sub-carrier spacing) carrier frequency offset v is

$$\boldsymbol{r} = \boldsymbol{W}(v)\, \boldsymbol{S}\boldsymbol{h} + \boldsymbol{n} \quad (1)$$

$$\text{where} \quad \boldsymbol{r} = [\boldsymbol{r}_0^T\ \boldsymbol{r}_1^T\ \ldots, \boldsymbol{r}_{Q-1}^T]^T \quad (2)$$

$$\boldsymbol{h} = [\boldsymbol{h}_0^T \boldsymbol{h}_1^T \ldots \boldsymbol{h}_{N_{\mathrm{Tx}}-1}^T]^T \quad (3)$$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_0[0] & \boldsymbol{S}_1[0] & \ldots & \boldsymbol{S}_{N_{\mathrm{Tx}}-1}[0] \\ \boldsymbol{S}_0[1] & \boldsymbol{S}_1[1] & \ldots & \boldsymbol{S}_{N_{\mathrm{Tx}}-1}[1] \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{S}_0[Q-1] & \boldsymbol{S}_1[Q-1] & \ldots & \boldsymbol{S}_{N_{\mathrm{Tx}}-1}[Q-1] \end{bmatrix} \quad (4)$$

$\boldsymbol{W}(v) = \mathrm{diag}\{\boldsymbol{W}_0(v),\ e^{\frac{j2\pi v(K+N_g)}{K}}\boldsymbol{W}_0(v),\ e^{\frac{j2\pi v 2(K+N_g)}{K}}\boldsymbol{W}_0(v), \ldots, e^{\frac{j2\pi v(Q-1)(K+N_g)}{K}}\boldsymbol{W}_0(v)\}$, $\boldsymbol{W}_0(v) = \mathrm{diag}\{1, e^{\frac{j2\pi v}{K}}, e^{\frac{j2\pi 2v}{K}}, \ldots, e^{\frac{j2\pi(K-1)v}{K}}\}$, and $\boldsymbol{n}$ is a length-KQ vector of independent and identically-distributed (iid) complex Gaussian noise samples with zero-mean and variance of σ_n^2.

For the least-squares channel estimate $\hat{\boldsymbol{h}} = (\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H\boldsymbol{r}$, the normalized mean square error (NMSE) is

$$\mathrm{NMSE} = \frac{\mathrm{E}[\|\boldsymbol{h}-\hat{\boldsymbol{h}}\|^2]}{LN_{\mathrm{Tx}}} = \frac{\sigma_n^2\ \mathrm{Tr}\left[(\boldsymbol{S}^H\boldsymbol{S})^{-1}\right]}{LN_{\mathrm{Tx}}}$$
$$+ \frac{\mathrm{Tr}\left[(\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H(\boldsymbol{I}-\boldsymbol{W}(v))\,\boldsymbol{S}\boldsymbol{C}_{\boldsymbol{h}}\boldsymbol{S}^H(\boldsymbol{I}-\boldsymbol{W}(v))^H\boldsymbol{S}(\boldsymbol{S}^H\boldsymbol{S})^{-1}\right]}{LN_{\mathrm{Tx}}} \quad (5)$$
$$\equiv \mathrm{NMSE}_0 + \Delta_{\mathrm{NMSE}} \quad (6)$$

where the first term is the NMSE obtained without any frequency offset and the second term is the extra NMSE caused by the normalized frequency offset v.

In the absence of frequency offsets, the minimum NMSE ($=\sigma_n^2/E_{\rm av}$) is achieved if and only if [6]

$$Condition-A: \sum_{q=0}^{Q-1} \boldsymbol{S}_i^H[q]\boldsymbol{S}_i[q] = E_{\rm av}\boldsymbol{I},\ \forall i \quad (7)$$

$$Condition-B: \sum_{q=0}^{Q-1} \boldsymbol{S}_i^H[q]\boldsymbol{S}_j[q] = \boldsymbol{0},\ \forall i \neq j \quad (8)$$

$$\text{where} \quad E_{\rm av} = \frac{1}{N_{\rm Tx}} \sum_{n=0}^{N_{\rm Tx}-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{K-1} |s_{n,q}[k]|^2. \quad (9)$$

Since using one OFDM training symbol is more robust to frequency offsets than using multiple symbols [7], we will consider one OFDM training symbol and hence, frequency-division multiplexing (FDM) and code-division multiplexing in frequency-domain (CDM-F) pilot structures from [6] which are described in the following, for completeness. Let L_0 be the smallest integer satisfying $L_0 = K/M$ (M is a positive integer) and $L_0 \geq L$. The pilot tone at the k-th sub-carrier for the m-th antenna is given by

$$\text{FDM}: \quad c_m[k] = \sum_{p=0}^{U-1}\sum_{l=0}^{L_0-1} b_m^{(l,p)}\ \delta[k - \frac{lK}{L_0} - i_{m,p}]; \quad (10)$$

$$\sum_{p=0}^{U-1}\sum_{l=0}^{L_0-1} |b_m^{(l,p)}|^2 = KE_{\rm av};\ i_{m,p} \in [0, \frac{K}{L_0}-1];$$

$$i_{m_1,p_1} = i_{m_2,p_2} \text{ only if } (m_1 = m_2\ \&\ p_1 = p_2)$$

$$\text{CDM}: \quad c_m[k] = \sum_{p=0}^{V-1}\sum_{l=0}^{L_0-1} b_m^{(l,p)}\ \delta[k - \frac{lK}{L_0} - i_p]; \quad (11)$$

$$b_m^{(l,p)} = b_0^{(l,p)}\ e^{-j2\pi pm/V}$$

$$i_p \in [0, \frac{K}{L_0}-1];\ i_{p_1} = i_{p_2} \text{ only if } p_1 = p_2$$

where $\{b_0^{(l,p)}\}$ are constant modulus symbols, $1 \leq U \leq K/(N_{\rm Tx}L_0)$, and $N_{\rm Tx} \leq V \leq K/L_0$.

In the presence of frequency offsets, [7] derived the best training signals among those from [6] by minimizing the extra NMSE $\Delta_{\rm NMSE}$. For $K > N_{\rm Tx}L$, the best one (most robust to frequency offsets) is of CDM-F type over all sub-carriers and is given by [7]

$$\{c_k[n]:\ k=0,\ 1,\ \ldots, N_{\rm Tx}-1\} =$$
$$\{\sqrt{E_{\rm av}}\ e^{j\phi_m}\ e^{\frac{-j2\pi mnL}{K}}\ :\ m=0,\ 1,\ \ldots,\ N_{\rm Tx}-1\} \quad (12)$$

where $\{\phi_m\}$ are arbitrary phases. For $K = N_{\rm Tx}L_0$, the best training signals in the presence of frequency offsets require the following additional condition [7]:

Condition-C : For each transmit antenna k, the optimal pilot tone symbols $c_k[n - lN_{\rm Tx} - m]$ for different l are the same.

III. PAR Characteristics of Existing Training Signals

Depending on power amplifier design, allowable PAR of the training signal will vary. Note that the transmit amplifier should be designed to have a large enough linearity range in order to avoid noticeable nonlinear distortion of OFDM data signals [1]. In this section, we will discuss PAR of existing training signals in the literature. The PAR of a continuous-time training signal $s(t)$ is defined as

$$\text{PAR} = \frac{\max_{0\leq t\leq QT}\ |s(t)|^2}{\frac{1}{QT}\int_{t=0}^{QT} |s(t)|^2 dt} \quad (13)$$

where T is the OFDM symbol duration including CP. In practice, PAR is approximated from discrete-time signal $\tilde{s}[n]$ with an appropriate over-sampling factor $N_{\rm up}$ (i.e., at the sampling rate of $N_{\rm up}K$ times the sub-carrier spacing) as

$$\text{PAR} = \frac{\max_n\ |\tilde{s}[n]|^2}{\frac{1}{N_{\rm up}Q(K+N_g)}\sum_{n=0}^{N_{\rm up}Q(K+N_g)-1} |\tilde{s}[n]|^2}. \quad (14)$$

For simplicity and without loss of generality, we consider $Q=1$. The instantaneous training signal energy can be related to the aperiodic autocorrelation $R(m)$ of the corresponding pilot tones $c[n]$ with the sub-carrier spacing of Δ_f as

$$|s(t)|^2 = \sum_{m=-K+1}^{K-1} R(m)e^{j2\pi m\Delta_f t} \quad (15)$$

$$= R(0) + 2\Re\{\sum_{m=1}^{K-1} R(m)e^{j2\pi m\Delta_f t}\} \quad (16)$$

$$\text{where} \quad R(m) = \sum_{n=0}^{K-1-m} c[n]c^*[n+m]. \quad (17)$$

It can be observed from (16) that pilot tones with small aperiodic autocorrelation (i.e., small $R(m)$ for $m \neq 0$) give small PAR.

Consider the training signal designs without frequency offsets from [6]. Optimal training signals of all transmit antennas with CDM(F) allocation have the same PAR since the time-domain signals are just cyclic-shifted versions of one another. For FDM allocation, optimal training signals of all antennas can be easily designed to have the same PAR by using the same pilot symbols on the assigned sub-carriers since shifting in frequency-domain just results in phase rotation of the time-domain signal. Hence, we just need to consider the training signal of the first antenna as far as PAR is concerned.

For CDM(F) allocation, we can design all $\{c_0[n]:\ n = 0,\ 1,\ \ldots,\ K-1\}$. For FDM, we can design $\{c_0[kM]:\ k=0,\ 1,\ \ldots,\ L_0-1; M = K/L_0\}$ which is equivalent to designing all sub-carrier symbols in an OFDM system with L_0 sub-carriers. Hence, in general there is no difference between CDM(F) and FDM regarding PAR of the training signals. We can use very low aperiodic autocorrelation sequences as pilot tones which will give very low PAR (see (16)).

Now, let us consider PAR of the most robust (in the presence of frequency offsets) optimal training signals from [7]. For FDM allocation with $K = LN_{\rm Tx}$, they are given by

$$c_0[n] = \sqrt{N_{\rm Tx}}\ \alpha_1 \sum_{l=0}^{L_0-1} \delta[n - lN_{\rm Tx}]. \quad (18)$$

The corresponding time-domain signal $s_0(t)$ has maximum energy at instants $t = nL_0T_s$, where $1/T_s$ is K times the sub-carrier spacing, since all α_1 are added in phase at these

[1]The PAR constraint could be well above 9 dB for OFDM systems.

instants. Using DFT properties, we obtain the peak energy of $s_0(t)$ as $|\alpha_1|^2/N_{\mathrm{Tx}}$.

For $\frac{N_{\mathrm{Tx}}}{d}$-FDM + d-CDM(F) allocation where $1 < d \leq N_{\mathrm{Tx}}$, the most robust optimal pilot tones are given by

$$c_0[n] = \sum_{m=0}^{d-1} \sqrt{N_{\mathrm{Tx}}/d}\ \alpha_m \sum_{l=0}^{L_0-1} \delta[n - lN_{\mathrm{Tx}} - n_m] \qquad (19)$$

where $\{\alpha_m\}$ are constant modulus symbols, $n_m \in \{0,\ 1,\ \ldots,\ N_{\mathrm{Tx}} - 1\}$, and $n_k \neq n_l$ if $k \neq l$. Then, using DFT properties gives

$$s_0[n] = \sum_{k=0}^{N_{\mathrm{Tx}}-1} \delta[n - kL_0] \sum_{m=0}^{d-1} \frac{\alpha_m}{\sqrt{dN_{\mathrm{Tx}}}}\ e^{-j2\pi n_m kL_0/K}. \qquad (20)$$

Applying Parseval's identity for the DFT, we obtain

$$\sum_{n=0}^{K-1} |s_0[n]|^2 = \sum_{k=0}^{N_{\mathrm{Tx}}-1} |s[kL_0]|^2 = |\alpha_1|^2. \qquad (21)$$

Therefore, we readily obtain the following inequality

$$\max\{|s[kL_0]|^2\} \geq \frac{|\alpha_1|^2}{N_{\mathrm{Tx}}}. \qquad (22)$$

Since the peak energy is greater than or equal to $\max\{|s[kL_0]|^2\}$, we can easily conclude that *the PARs of the most-robust optimal training signals from [7] with FDM+CDM(F) allocation cannot be smaller than those with FDM allocation*. Numerical evaluation of PAR based on the samples at an oversampling factor of 16 shows that in fact the minimum PAR of the most robust optimal training signals from [7] obtained with FDM allocation is smaller than that obtained with CDM(F) or FDM+CDM(F) allocation. Also note that Condition-C introduces some correlation among pilot tones which is not favorable to low PAR.

For a SISO system or a MIMO system with $K > L_0 N_{\mathrm{Tx}}$, the most robust optimal pilot tones in the presence of frequency offsets are given by a CDM-F allocation with $c_0[n] = \alpha_1$. Since the pilot tones are fully correlated, the corresponding PAR is maximum. If we prefer using a training signal with a lower PAR rather than using the most robust training signals from [7], we may choose training signals of FDM, CDM(F) or FDM+CDM(F) allocation with a smaller number ($\geq L_0$) of pilot tones per antenna with an additional property similar to Condition-C (i.e., within each set of equi-spaced L_0 tones, the pilot symbols are the same.). The resulting pilot allocation could also be applied to pilot-data-multiplexed schemes.

For FDM allocation, we have only one choice given by $c_0[n] = \sum_{k=0}^{L_0-1} \alpha_1 \delta[n - kM]$, where $M = K/L_0$ and the corresponding PAR is L_0. For $\frac{N_{\mathrm{Tx}}}{d}$-FDM + d-CDM(F) allocation, we just need to find low correlation pilot tones having a property similar to Condition-C. In other words, each antenna has d sets of equi-spaced L_0 tones of the same symbol and we have d pilot tone symbols to design for low PAR.

IV. PAR-Constrained Training Signal Designs

A. The Frequency-Domain Design

As discussed in the previous section, low PAR of the training signal requires a low correlation property of the pilot tones while training signal's robustness to frequency offsets requires a high correlation property of the pilot tones. Due to these conflicting requirements, it is impossible to obtain a training signal possessing both a very low PAR and the highest robustness against frequency offsets. For scenarios where the training designs discussed in the previous section do not meet the PAR constraint, we propose in the following an algorithm which satisfies the PAR constraint while striving to maintain the robustness against frequency offsets. The algorithm starts with a training signal most robust to frequency offsets presented in [7]. The algorithm gradually replaces some pilot tones with a low correlation sequence in an attempt to lower the PAR. The number of pilot tones replaced is gradually increased until the PAR constraint is satisfied. This approach will be termed frequency-domain (FD) design. As mentioned in the previous section, we only need to design the training signal for the first transmit antenna. The FD design is described below.

1) Calculate the PAR of the training signal most robust to frequency offsets obtained from [7].
2) If PAR $\leq$ PAR$_{\mathrm{desired}}$, the algorithm finishes. Otherwise, set $\lambda = 2$. Replace the last λ pilot tones of the last set of equi-spaced L_0 tones with a length-λ low correlation sequence and calculate the corresponding PAR.
3) If PAR $\leq$ PAR$_{\mathrm{desired}}$, the algorithm finishes. Otherwise, increase λ by one. Replace the last λ pilot tones of the last $\lceil \lambda/L_0 \rceil$ sets of equi-spaced L_0 tones with a length-λ low-correlation sequence and calculate the corresponding PAR. Repeat Step 3.

Note that a pilot tone sequence with low correlation has a very low PAR and hence the above algorithm is guaranteed to satisfy the PAR constraint. The algorithm can be repeated with the training signal of a different pilot allocation type (e.g., FDM over all or some sub-carriers, CDM over some sub-carriers) with a property similar to Condition-C as an initial training signal. Among the obtained training signals satisfying the PAR constraint, the one with smallest NMSE will be chosen. Note that occasionally a training signal with a lower PAR may give a smaller NMSE. Hence, the above procedure can continue for some PARs lower than the PAR constraint and we can choose the training signal with the smallest NMSE among those obtained with PAR less than or equal to the PAR constraint.

There are several works on sequences with low periodic or aperiodic correlation properties. In our algorithm, a sequence with low aperiodic autocorrelation property is required. Examples of such sequences are Newman's sequence [8] and Schroeder's sequence [9]. The length-P Newman's sequence is given by

$$c[m] = e^{j\pi m^2/P},\ m = 0,\ 1,\ \ldots, P-1. \qquad (23)$$

The length-P Schroeder's sequence is defined by

$$c[m] = c[0]\ e^{-j\pi(m+1)m/P},\ m = 1,\ 2,\ \ldots, P-1 \qquad (24)$$

where $c[0] = e^{j\phi}$ is arbitrary. It can be easily checked that the Schroeder's sequence is similar to the Newman's sequence except that it has a minus sign in the phase and an additional phase term of $(\pi m/P)$. These two differences do not affect the PARs when these sequences are used as pilot tones. In this paper, we simply adopt the Newman sequence.

B. The Time-Domain Design

In the following, we will discuss an alternative approach termed as time-domain (TD) training design which uses single-carrier-type training signals with CP. Based on the training signal design from [7], we just need to consider the following training signals (in time-domain):

$$s_{k,q}[n] = \sum_{i=0}^{d_{k,q}-1} A_{k,q,i}\delta[n - l_{k,q,i}],\ k = 0, \ldots, N_{\mathrm{Tx}} - 1 \quad (25)$$

$$d_q \equiv \sum_{k=0}^{N_{\mathrm{Tx}}-1} d_{k,q} \leq \frac{N}{L} \quad (26)$$

$$\sum_{q=0}^{Q-1}\sum_{i=0}^{d_{k,q}-1} |A_{k,q,i}|^2 = E_{\mathrm{av}},\ \forall k \quad (27)$$

where $d_{k,q}$ is the number of non-zero samples of the q-th signaling interval (excluding CP) for the k-th transmit antenna, and for each q, $\{l_{k,q,i} :\ \forall k, i\}$ are any permutation of $\{m_p\}$ with $m_{p+1} - m_p \geq L$, $K + m_0 - m_{d_q-1} \geq L$, and $0 \leq m_p \leq K - 1$. Let V_l denote the l-th diagonal element of $\boldsymbol{V} \equiv (\boldsymbol{I} - \boldsymbol{W}(v))$ from (5). Then the training signal design from [7] that minimizes the extra NMSE becomes finding training signals that minimize the following function:

$$\mathrm{Tr}[\boldsymbol{X_d}] = \sum_{k=0}^{N_{\mathrm{Tx}}-1}\sum_{m=0}^{L-1}\sigma_m^2 \left|\sum_{q=0}^{Q-1}\sum_{i=0}^{d_{k,q}-1}|A_{k,q,i}|^2 V_{m+l_{k,q,i}+Kq}\right|^2. \quad (28)$$

For small values of v, we have $V_l = 1 - e^{j2\pi k_l v/K} \simeq -j2\pi k_l v/K$ where $k_l = \lfloor l/K \rfloor N_g + l$. Utilizing the fact that V_l increases approximately linearly with l, we obtained in [7] the most robust pilot tones (against frequency offsets) as given in (12) whose time-domain signal is given by

$$\begin{aligned}&\{|s_k[n]| :\ k = 0,\ 1,\ \ldots, N_{\mathrm{Tx}} - 1\}\\ &= \{\sqrt{E_{av}}\delta[n - mL] : m = 0,\ 1,\ \ldots, N_{\mathrm{Tx}} - 1\}. \quad (29)\end{aligned}$$

Intuitively, when satisfying Conditions A and B, (12) (and hence (29)) simply allocates training signal energies to the leading samples (in time-domain) so that they are weighted by the smallest V_l values. In our TD design, we apply the same approach while satisfying PAR constraints. We start with the most robust training signal from (12) whose corresponding PAR is K. If the PAR constraint is less than K, we modify the training signal from (12) in the time-domain as follows:

$$\begin{aligned}&\{s_k[n] :\ k = 0,\ 1,\ \ldots, N_{\mathrm{Tx}} - 1\}\\ &= \{\sum_{i=0}^{d-1} A_{m,i}\ \delta[n - mL - i] : m = 0,\ 1,\ \ldots, N_{\mathrm{Tx}} - 1\} \quad (30)\end{aligned}$$

where $d = \lceil E_{\mathrm{av}}/E_{\mathrm{peak}} \rceil$ and E_{peak} is the allowable peak sample energy defined by the PAR constraint [2]. Now, we consider the design of $\{A_{m,i}\}$. In the frequency-offset-robust design from [7], $\boldsymbol{S}^H\boldsymbol{S} = E_{\mathrm{av}}\boldsymbol{I}$ and $\boldsymbol{Y} = \boldsymbol{S}^H\boldsymbol{V}\boldsymbol{S}$ is a diagonal matrix. The TD design attempts to closely follow these diagonal conditions by suppressing off-diagonal elements of $\boldsymbol{S}^H\boldsymbol{S}$ and $\boldsymbol{Y}$ as follows. With the structure in (30), the off-diagonal elements of $\boldsymbol{S}^H\boldsymbol{S}$ are just the aperiodic autocorrelation and cross-correlation of $\{A_{m,i}\}$. Similarly, the off-diagonal elements of $\boldsymbol{Y}$ are the weighted (by V_l) aperiodic autocorrelation and crosscorrelation of $\{A_{m,i}\}$. By simply neglecting the weighting, the TD design finds $\{A_{m,i}\}$ with low aperiodic autocorrelation and crosscorrelation. If we set $s_{m+l}[n] = s_m[n - lL]$, the aperiodic crosscorrelation becomes the same as aperiodic autocorrelation and we just need to find low aperiodic autocorrelation sequence of length d for which we adopted the Newman's sequence.

[2]In the design, the maximum PAR allowed may be set smaller than the PAR constraint to allow PAR re-growth due to filtering.

V. Performance Results and Discussions

We evaluated the FD algorithm starting from CDM and FDM pilot structures. Each starting pilot tone vector possesses the robustness property against frequency offsets. For the CDM structure, the number of sets of L_0 equi-spaced tones used for each antenna is V. For the FDM structure, it is U. When $V = UN_{\mathrm{Tx}}$, the total number of pilot tones for all antennas is the same for both CDM and FDM structures. Note that for $N_{\mathrm{Tx}} = 1$, CDM and FDM structures are the same. We considered an OFDM system with $K = 64$ sub-carriers in an 8-tap multipath Rayleigh fading channel. In this case, $L_0 = L = 8$ and the maximum number of transmit antennas that can be supported while possessing channel identifiability is 8. In calculating PAR, $N_{\mathrm{up}} = 16$ is used. Figures 1-3 present the channel estimation NMSE of the PAR-constrained training signals with different CDM and FDM structures in the presence of frequency offsets for $N_{\mathrm{Tx}} = 1$, 4, and 8, respectively. The following observations are in order:

1) At $v \leq 0.01$, the NMSE differences among different training structures are insignificant. As v increases, the NMSE differences increase, more noticeably with mild PAR constraints.
2) For $N_{\mathrm{Tx}} < K/L_0$, training structures with larger V or U give better NMSE with mild PAR constraints while those with smaller V or U are better with stringent PAR constraints. This can be attributed to the following two facts: (i) a smaller V or U has PAR advantage as discussed in the previous section, and (ii) the training structure with a larger V or U resembles more closely to the training structure most robust to frequency offsets (i.e. a CDM structure using all sub-carriers).
3) For $N_{\mathrm{Tx}} < K/L_0$, when the same number of total pilot tones is used in the CDM and FDM structures, the CDM structure yields a slightly better NMSE. The FDM structure has a slight PAR advantage while the CDM structure (being more similar to the most robust structure) has a slight advantage against frequency offsets. The latter appears to outperform the former, hence giving a slight advantage for the CDM structure.
4) As N_{Tx} increases, the NMSE differences among different PAR constraints become much smaller; and when $N_{\mathrm{Tx}} = K/L_0$, the NMSE differences are insignificant.
5) For $N_{\mathrm{Tx}} = K/L_0$, the FDM structure gives a marginal NMSE advantage at mild PAR constraints. The reason can be explained as follows. There is no difference

between CDM and FDM in term of robustness against frequency offsets since the most robust structure can be either CDM or FDM when $N_{\mathrm{Tx}} = K/L_0$. On the other hand, the FDM structure has a slight PAR advantage which results in a marginal NMSE advantage for FDM when the PAR constraint is imposed.

6) Overall, in the presence of frequency offsets and PAR constraints, the CDM training structure is recommended for the following reason. The FDM structure has a marginal advantage only when $N_{\mathrm{Tx}} = K/L_0$. For $N_{\mathrm{Tx}} < K/L_0$, the CDM structure has a slight advantage. Most practical systems would have $N_{\mathrm{Tx}} < K/L_0$ and hence the CDM structure would be a better choice.

Next, we compare the channel estimation NMSE among the training signals obtained from our FD and TD designs and those with a fixed low PAR. The CDM pilot structure with $V = K/L_0$ is used for all training signals. The low PAR training signals are of CDM type where one of the antennas uses a Newman's sequence over all sub-carriers. Note that PARs for different antennas are the same as discussed in the previous section. Figures 4-6 show the NMSE performance at a SNR of 10 dB for $N_{\mathrm{Tx}} = 1, 4$, and 8, respectively. Figures 7-8 present NMSE for $N_{\mathrm{Tx}} = 4$ at SNR values of 0 dB and 20 dB, respectively. Our proposed training signals achieve better performance than the training signals with a fixed low PAR. The following remarks are in order.

1) For any N_{Tx}, the NMSE differences between our proposed training signals and those with a fixed low PAR become larger at larger SNR and frequency offset.
2) As N_{Tx} increases, these NMSE differences become smaller. When $N_{\mathrm{Tx}} = K/L_0$, these NMSE differences become insignificant.
3) For $N_{\mathrm{Tx}} < K/L_0$, the TD design gives better performance than the FD design most of the time. When $N_{\mathrm{Tx}} = K/L_0$, the FD design results in a better performance than the TD design.

VI. Conclusions

In this paper, we studied the PAR characteristics of the existing training signals for MIMO OFDM channel estimation in the absence/presence of frequency offsets. The requirements on the training signals to possess low PAR and be robust against frequency offsets are conflicting. We presented two training signal designs (frequency-domain (FD) and time-domain (TD)) which are robust to frequency offsets while satisfying the PAR constraints. Both designs give better channel estimation performance than using training signals with a fixed low PAR. For $N_{\mathrm{Tx}} < K/L_0$, the TD design has a channel estimation performance advantage over the FD design but when $N_{\mathrm{Tx}} = K/L_0$, the FD design is better.

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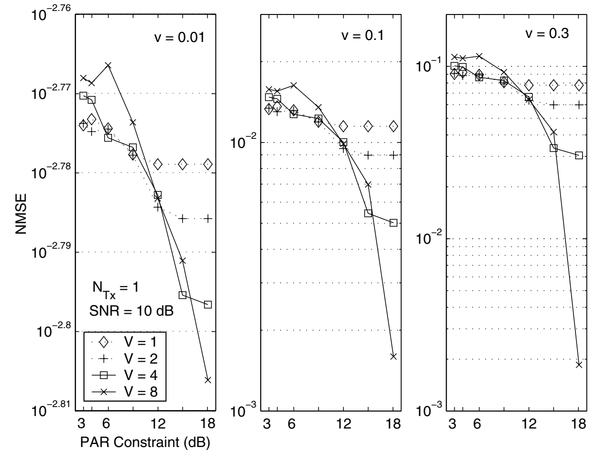

Fig. 1. The NMSEs for different training structures ($N_{\mathrm{Tx}} = 1$)

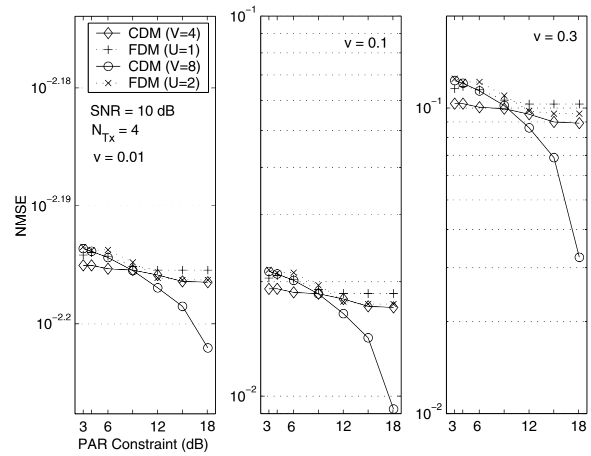

Fig. 2. The NMSEs for different training structures ($N_{\mathrm{Tx}} = 4$)

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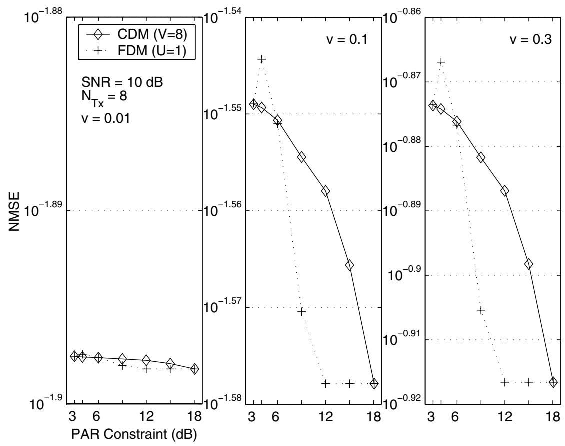

Fig. 3. The NMSEs for different training structures ($N_{\mathrm{Tx}} = 8$)

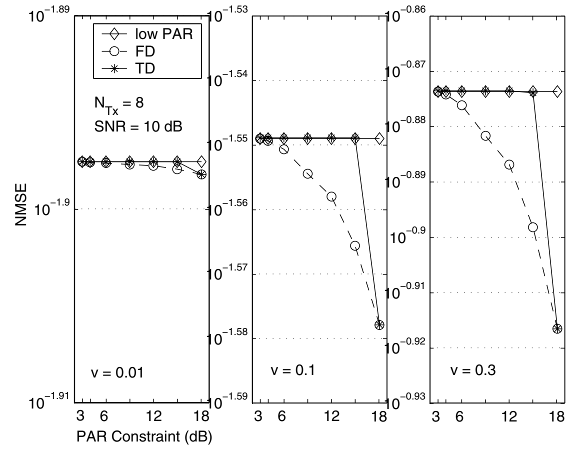

Fig. 6. The NMSEs for training signals with a fixed low PAR and those obtained from FD and TD designs ($N_{\mathrm{Tx}} = 8$, SNR = 10 dB)

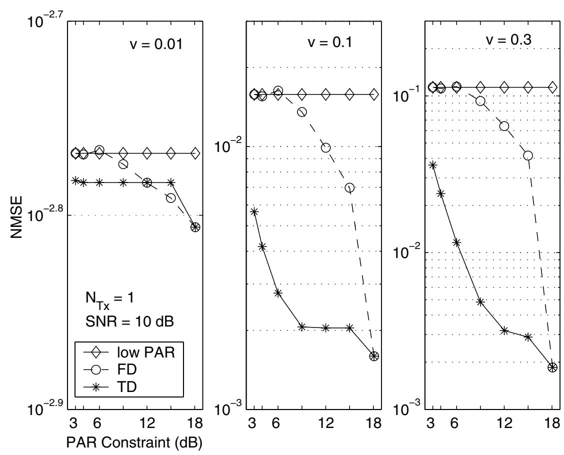

Fig. 4. The NMSEs for training signals with a fixed low PAR and those obtained from FD and TD designs ($N_{\mathrm{Tx}} = 1$, SNR = 10 dB)

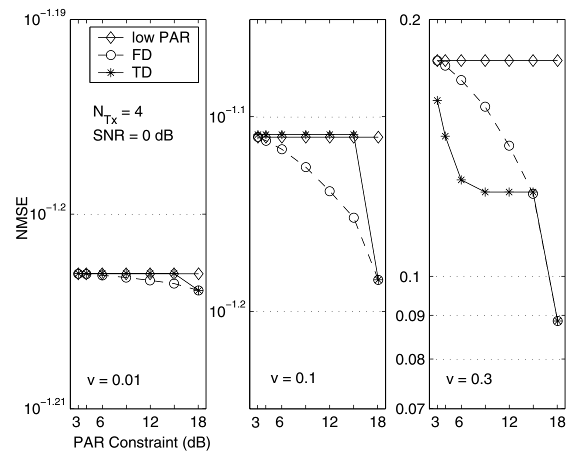

Fig. 7. The NMSEs for training signals with a fixed low PAR and those obtained from FD and TD designs ($N_{\mathrm{Tx}} = 4$, SNR = 0 dB)

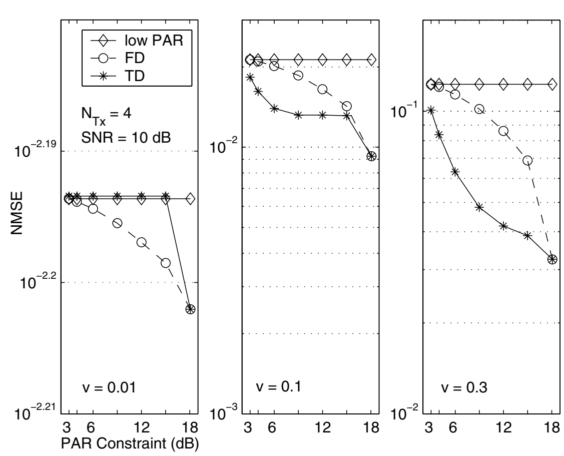

Fig. 5. The NMSEs for training signals with a fixed low PAR and those obtained from FD and TD designs ($N_{\mathrm{Tx}} = 4$, SNR = 10 dB)

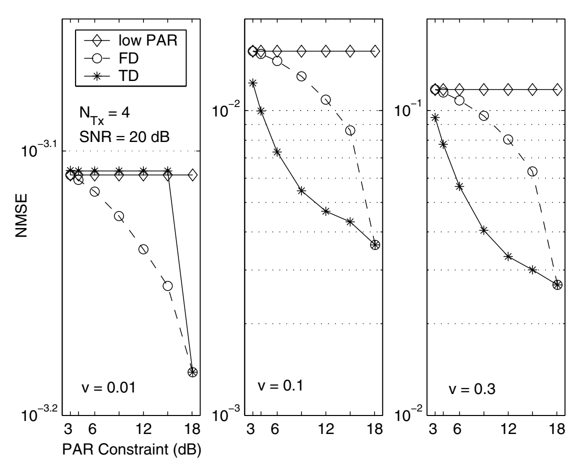

Fig. 8. The NMSEs for training signals with a fixed low PAR and those obtained from FD and TD designs ($N_{\mathrm{Tx}} = 4$, SNR = 20 dB)