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Explaining bond returns in heterogeneous agent models: The importance of higher-order moments☆

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Abstract

This paper examines the higher-order moments and nonlinear dynamic properties of discount bond returns in an equilibrium heterogeneous agent economy with incomplete markets and borrowing constraints. We find that while it is possible for the economic model to match the mean and variance of the observed bond returns by choosing the parameters such that the borrowing constraint is binding sufficiently often, the implied higher-order moments are at odds with the data. To match the higher-order moments, one needs a model in which the borrowing constraint is binding at times but not too often. In this case, one does not match the first two moments. Using the seminonparametric density estimation and the nonlinear impulse response analysis, we find that our economic model can mimic the asymmetric effect of return shocks on conditional volatility as documented for the real bond returns. However, the economic model has difficulty replicating the dynamic properties observed in the data when the parameter values are chosen to match the first two moments of the observed bond returns. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Recent empirical studies of financial market data have uncovered various interesting non-linear dynamic and higher-order moment properties of asset returns, as summarized below. First, asset returns have been documented as nonlinear time series with strong autoregressive conditional heteroskedasticity (ARCH) (Engle, 1982; Bollerslev, 1986; Engle and Bollerslev, 1986). Second, changes in asset returns have asymmetric (Black, 1976; Christie, 1982, Nelson, 1991; Pagan and Schwert, 1990) and transient (Gallant et al., 1993 and Tauchen et al., 1996) effects on return volatility. Finally, asset returns are more likely to be skewed to the left (a negative skewness) with fat-tails (leptokurtosis) (Clark, 1973; Gallant et al., 1991). However, not much effort has gone into investigating why these properties are observed using structural economic models.[1]

In this paper we examine the nonlinear dynamic properties and higher-order moments of discount bond returns in a heterogeneous agent exchange economy with incomplete markets and borrowing constraints. Our goal is to understand how the higher-order moments of returns are affected by various features of our economic model and explore the nonlinear dynamic properties of returns in the model.

A number of researchers have recently studied asset markets in heterogeneous agent environments.[2] Their results indicate that introducing heterogeneous agents to incomplete markets with market frictions improves the performance of the models in resolving certain asset pricing puzzles. To test if an economic model generates asset returns that match the observed data, most studies adopt the following approach. They first calibrate the model to certain underlying exogenous driving processes such as income and dividends and then solve the model using a numerical alogrithm. Asset returns are then simulated from the economic model, and their mean and standard deviation are compared to the mean and standard deviation of the data. Since asset returns in an economic model are often nonlinear functions of exogenous processes, their distribution may become complicated nonlinear processes even when the model is calibrated

[1] Some recent studies include Den Haan and Spear (1998) which examine the ARCH behavior of interest rates using a dynamic exchange economy with heterogeneous agents, and Kelly and Steigerwald (1998) which study the conditional heteroskedasticity of asset returns using an economic model with asymmetric information.

[2] Mankiw (1986), Kahn (1990), Aiyagari and Gertler (1991), Marcet and Singleton (1991), Huggett (1993), Telmer (1993), Lucas (1994), Heaton and Lucas (1996), and Zhang (1997a,b) examine asset returns in a two agent economy with various market frictions. Huffman (1987) investigates asset pricing and trading volume by employing an overlapping generations model. Ketterer and Marcet (1989) look into trading patterns and hedging dynamics for different market structures. Ingram (1990) studies price volatility and asset returns in an economy consisting of two types of agents — one of which is rational and maximizes her expected life-time utility, while the other type follows rules of thumb. Den Haan (1996,1997) and Krusell and Smith (1997) investigate the asset pricing implications of a model with a continuum of agents and incomplete markets.

to a well-behaved exogenous driving force such as a Gaussian. Thus, the mean and standard deviation are often insufficient to describe the stochastic properties of asset returns. Additional information on the distribution of returns often can be obtained from examining higher-order moments such as skewness and kurtosis. A complete characterization of asset returns, however, would require estimation of their density function. Furthermore, to better understand the dynamic properties of asset returns such as ARCH and the asymmetric effect of changes in returns on return volatility in an economic model, we need additional techniques such as nonlinear impulse response analysis.

In this study we go beyond the existing work by focusing on the higher-order moments and dynamic properties of discount bond returns in our economic model. By calibrating the model to the U.S. quarterly income data, we first explore the relationship between the moments of the simulated returns and features of the model such as agents' risk aversion, income dispersion, and the restrictiveness of borrowing constraints. We then employ the seminonparametric (SNP) method proposed by Gallant and Tauchen (1989) to estimate the density function of the simulated returns. Given the SNP density estimate, we apply the nonlinear impulse response analysis introduced in Gallant et al. (1993) to explore the dynamic properties of our simulated returns. In order to provide the benchmark that our economic model has to confront, the SNP method and the nonlinear impulse response analysis are also applied to the ex post quarterly real returns of discount bonds.

Our main findings are as follows. First, our nonlinear impulse response analysis shows that the conditional volatility of the ex post quarterly real discount bond returns displays quantitatively important asymmetry which resembles the asymmetric effect documented for stock returns. Second, by choosing the parameters of our economic model such that the borrowing constraint is binding sufficiently often, it is possible to match the mean and variance of the observed discount bond returns. However, the implied higher-order moments are then at odds with the data. To match higher-order moments such as skewness and kurtosis, one needs a model in which the borrowing constraint is binding at times but not too often. In this case, one does not match the first two moments. An important implication of the finding is that it can be very misleading to evaluate models by just looking at the first two moments.

Third, the returns simulated from our economic model exhibit the ARCH property for various parameterizations. This can be attributed to the presence of income dispersion and borrowing constraints as explained in Den Haan and Spear (1998). Using the SNP density estimation and the nonlinear impulse response analysis, we find that our economic model can mimic the asymmetric effect of return shocks on conditional volatility found in the ex post quarterly real discount bond returns. Consistent with the results discussed above, we find that if one matches the first two moments one gets a poor match for the nonlinear impulse response functions. However, one gets a good match for the nonlinear impulse response functions if one matches the higher-order moments.

Finally, consistent with the findings on the ex post real returns, the impulse responses of conditional volatility of simulated returns are more persistent than the impulse responses of conditional mean returns. Both the impulse responses of the conditional mean and conditional volatility of quarterly ex post real returns, respectively.

The rest of the paper is organized as follows. Section 2 describes the nonlinear impulse response analysis and the SNP density estimation. The techniques are then applied to quarterly ex post real discount bond returns in Section 3 to provide some benchmark results that our economic model has to confront. Section 4 delineates the economic model. In Section 5 we present the results of our analysis on the simulated returns, and finally Section 6 gives concluding remarks.

2. The nonlinear impulse response analysis and SNP density estimation

In this section, we first discuss the nonlinear impulse response analysis assuming that the conditional density function is known. We then describe the SNP method which allows us to estimate the conditional density for return series.

2.1. The nonlinear impulse response analysis

Consider a strictly stationary process $\{y_t\}_{t=-\infty}^{\infty}$, $y_t \in R^M$, with a conditional density function that depends upon at most L lags. Collect the L lags of y_t into $x_{t-1} = (y'_{t-L}, \ldots, y'_{t-1})'$. Denote by $f(y_t \mid x_{t-1})$ the one-step ahead conditional density function. Under the strict stationarity assumption, the functional form of the conditional density is time invariant. Let $g(y_{t-J}, y_{t-J+1}, \ldots, y_t)$ be a time-invariant function of a stretch of the process of length $J+1$, where J is the number of lags in $g(\cdot)$ and $J \leq L-1$. Define $\{\hat{g}_j(x)\}_{j=1}^{\infty}$ as the conditional expectation of the profile $\{g(y_{t-J+j}, \ldots, y_{t+j})\}_{j=1}^{\infty}$ given $x_t = x$:

$$\hat{g}_j(x) = \mathrm{E}[g(y_{t+j-J}, \ldots, y_{t+j}) \mid x_t = x]$$

$$= \int \cdots \int g(y_{j-J}, \ldots, y_j) \left[\prod_{i=0}^{j-1} f(y_{i+1} \mid y_{i-L+1}, \ldots, y_i) \right] \mathrm{d}y_1 \ldots \mathrm{d}y_j \quad (1)$$

where $x = (y'_{-L+1}, \ldots, y'_0)'$ is some fixed initial condition.

There are two special cases in which we are most interested:

$$\hat{g}_j(x) = \mathrm{E}(y_{t+j} \mid x_t = x), \quad j = 1, \ldots, \infty, \quad (2)$$

$$\hat{g}_j(x) = \mathrm{E}[\mathrm{Var}(y_{t+j} \mid x_{t+j-1}) | x_t = x], \quad j = 1, \ldots, \infty. \quad (3)$$

Eq. (2) is the conditional mean profile, denoted $\hat{y}_j(x)$, while Eq. (3) gives the conditional volatility profile, denoted $\mathscr{V}_j(x)$, corresponding to the initial condition of x.

The idea of impulse response analysis is to compare the conditional moment profile starting from a perturbed initial condition to the conditional moment profile starting from a baseline initial condition. The difference across the profiles is the impulse response. Denote by $y_0 \in R^M$ the contemporaneous value and by $y_{-k} \in R^M$, $1 \le k \le L-1$ the lags. Let $\delta y^+, \delta y^- \in R^M$ represent perturbations to the contemporaneous y_0 where δy^+ is a positive shock and δy^- is a negative shock. Denote

$$x^+ = (y'_{-L+1}, y'_{-L+2}, \dots, y'_0)' + (0, 0, \dots, \delta y^{+\prime})', \tag{4}$$

$$x^0 = (y'_{-L+1}, y'_{-L+2}, \dots, y'_0)', \tag{5}$$

$$x^- = (y'_{-L+1}, y'_{-L+2}, \dots, y'_0)' + (0, 0, \dots, \delta y^{-\prime})'. \tag{6}$$

Thus, x^+ is an initial condition corresponding to a positive impulse δy^+ added to contemporaneous y_0. x^- corresponds to a negative impulse, while x^0 is the baseline initial condition with no shock. The profiles $\{\hat{g}_j(x^+)\}_{j=1}^{\infty}$, $\{\hat{g}_j(x^0)\}_{j=1}^{\infty}$, and $\{\hat{g}_j(x^-)\}_{j=1}^{\infty}$ are the forecasts starting from the initial conditions x^+, x^0, and x^-, respectively. Denote $\{\hat{g}_j(x^0)\}_{j=1}^{\infty}$ as the baseline. Profiles deviating from the baseline are then defined as the dynamic impulse responses of $g(y_{t-J+j}, y_{t-J+j+1}, \dots, y_{t+j})$ to shocks δy^+ and δy^-, i.e.,

$$\{\hat{g}_j(x^+) - \hat{g}_j(x^0)\}_{j=1}^{\infty}, \tag{7}$$

$$\{\hat{g}_j(x^-) - \hat{g}_j(x^0)\}_{j=1}^{\infty}. \tag{8}$$

We use Monte Carlo integration to evaluate the integrals of the conditional expectations. Let $\{y_j^r\}_{j=1}^{\infty}$, $r = 1, 2, \dots, N_r$, denote N_r simulated realizations of the process starting from $x = (y'_{-L+1}, \dots, y'_{-1}, y'_0)$. In other words, y_1^r is a random drawing from $f(y_1|x)$ with $x = (y'_{-L+1}, \dots, y'_{-1}, y'_0)'$; y_2^r is a drawing from $f(y_2|x_1)$ with $x_1 = (y'_{-L+2}, \dots, y'_0, y_1^{r\prime})'$, and so forth. Then the forecast of the one-step moment j steps ahead of the function $g(\cdot)$ is approximated as follows:

$$\hat{g}_j(x) = \int \cdots \int g(y_{j-J}, \dots, y_j) \left[\prod_{i=0}^{j-1} f(y_{i+1} \mid y_{i-L+1}, \dots, y_i) \right] \mathrm{d}y_1 \dots \mathrm{d}y_j$$

$$\approx \frac{1}{N_r} \sum_{r=1}^{N_r} g(y_{j-J}^r, \dots, y_j^r). \tag{9}$$

The approximation error tends to zero almost surely as $N_r \to \infty$, under mild regularity conditions imposed on f and g (Gallant et al., 1993).

In our numerical analysis, the one-step conditional density function $f(y_t \mid x_{t-1})$ is approximated by a nonparametric estimate $\hat{f}(y_t \mid x_{t-1}, \theta)$ where θ is the vector of parameters of the density function. Specifically, we choose to implement the SNP density estimate of the form used in Gallant and Tauchen (1989). We next discuss the SNP density estimation.

2.2. The SNP density estimation

The SNP method is based on the notion that a Hermite expansion can be used as a general purpose approximation to a density function. This basic approach can be adapted to the estimation of the conditional density of a multiple time series that has a Markovian structure (the conditional density of the time series given the entire history depends only on a finite number of lags from the past).

The SNP approximation of the density function $f(y_t \mid x_{t-1})$ takes the form of a normal distribution modified by a polynomial:

$$\hat{f}(y_t \mid x_{t-1}, \theta) = \frac{1}{\lambda}[P(z_t, x_{t-1}]^2 \mathrm{n}_M(y_t | \mu_{x_{t-1}}, \Sigma_{x_{t-1}}) \tag{10}$$

where λ is a scalar that makes the density integrate to one, $z_t = R_{x_{t-1}}^{-1}(y_t - \mu_{x_{t-1}})$ is a vector of innovations, $\mathrm{n}_M(y_t \mid \mu_{x_{t-1}}, \Sigma_{x_{t-1}})$ is a normal distribution with mean $\mu_{x_{t-1}}$ (the location function) and variance–covariance matrix $\Sigma_{x_{t-1}} = R_{x_{t-1}} R'_{x_{t-1}}$, and $R_{x_{t-1}}$ (the scale function) is an upper triangular matrix, $P(z_t, x_{t-1})$ denotes a polynomial in z_t of degree K_z whose coefficients are polynomials of degree K_x in x_{t-1}. The constant term of the polynomial is put to one to obtain a unique representation. This normalization means that the leading term of the entire expansion is $\mathrm{n}_M(y_t \mid \mu_{x_{t-1}}, \Sigma_{x_{t-1}})$. The location function $\mu_{x_{t-1}}$ is given by a vector autoregression

$$\mu_{x_{t-1}} = b_0 + b' x_{t-1}. \tag{11}$$

It is assumed to depend on $L_\mu \leq L$ lags. The scale function $R_{x_{t-1}}$ is given by

$$\mathrm{vech}(R_{x_{t-1}}) = \phi_0 + \phi' |e^*_{t-1}| \tag{12}$$

where $\mathrm{vech}(R_{x_{t-1}})$ denotes a vector of length $M(M+1)/2$ containing the elements of the upper triangle of $R_{x_{t-1}}$, $e^*_{t-1} = [(y_{t-L_r} - \mu_{x_{t-L_r-1}}), \ldots, (y_{t-1} - \mu_{x_{t-2}})]$, and $|\cdot|$ denotes elementwise absolute value. The scale function depends on L_r lagged (unnormalized) innovations $(y_t - \mu_{x_{t-1}})$ and $L_\mu + L_r \leq L$ lagged y_t in total. This is an ARCH-type process akin to that proposed by Nelson (1991).

The Hermite polynomial $P(z_t, x_{t-1})$ is given by

$$P(z_t, x_{t-1}) = \sum_{\alpha=0}^{K_z} \left(\sum_{\beta=0}^{K_x} a_{\alpha\beta} x_{t-1}^{\beta} \right) z_t^{\alpha} \tag{13}$$

Table 1

Parameter setting	Characterization of y_t
$L_\mu = 0, L_r = 0, L_p \geqslant 0, K_z = 0, K_x = 0$	iid Gaussian
$L_\mu > 0, L_r = 0, L_p \geqslant 0, K_z = 0, K_x = 0$	Gaussian VAR
$L_\mu > 0, L_r = 0, L_p \geqslant 0, K_z > 0, K_x = 0$	Non-Gaussian VAR, homogeneous innovations
$L_\mu \geqslant 0, L_r > 0, L_p \geqslant 0, K_z = 0, K_x = 0$	Gaussian ARCH
$L_\mu \geqslant 0, L_r > 0, L_p \geqslant 0, K_z > 0, K_x = 0$	Non-Gaussian ARCH, homogeneous innovations
$L_\mu \geqslant 0, L_r \geqslant 0, L_p > 0, K_z > 0, K_x > 0$	General nonlinear process, heterogeneous innovations

where α and β are non-negative integers, and $z^\alpha = z_1^{\alpha_1} \dots z_M^{\alpha_M}$ with $\sum_{k=1}^{M} \alpha_k \leq K_z$; similarly for x^β. It is assumed that the polynomial depends on $L_p \leq L$ lags of y from x.[3] The hierarchical SNP structure can thus be represented by L_μ, L_r, L_p, K_z and K_x. For notational convenience, hereafter, we use SNP(L_μ, L_r, L_p, K_z, K_x).

To illustrate, consider first the model with $M = 1, L_\mu = 4$, $L_r = 4$, $L_p = 0, K_z = 4$, and $K_x = 0$. The polynomial is of the form

$$P(z_t) = \sum_{\alpha=0}^{4} a_\alpha z_t^\alpha \tag{14}$$

where the $a_0, a_1, \dots, a_4$ are the polynomial coefficients with the constant term $a_0 = 1$ to achieve a unique representation. Both $\mu_{x_{t-1}}$ and $R_{x_{t-1}}$ are linear in $y_{t-1}, \dots, y_{t-4}$. The model has fourteen free parameters: the four free polynomial parameters, the intercept and four slope parameters in $\mu_{x_{t-1}}$, and the intercept and four slope parameters of $R_{x_{t-1}}$.

Now consider $L_p = 1$ and $K_x = 1$ but everything else the same. The polynomial becomes

$$P(z_t, x_{t-1}) = \sum_{\alpha=0}^{4} (a_{0\alpha} + a_{1\alpha} y_{t-1}) z_t^\alpha. \tag{15}$$

The normalization is $a_{00} = 1$. The polynomial has nine free parameters, yielding nineteen free parameters in total.

The relationship between the parameter settings and the characteristics of the density of y_t is summarized in Table 1 (see Gallant et al., 1993).

The parameter vector θ of $\hat{f}(y_t \mid x_{t-1}, \theta)$ consists of the coefficients of the polynomial plus $\mu_{x_{t-1}}$ and $R_{x_{t-1}}$ and is estimated by maximum likelihood. If the number of parameters p_θ grows with the sample size n at the appropriate rate, then the true density, its derivatives, and moments are estimated consistently as shown in Gallant and Nychka (1987).

[3] When $K_x = 0$, $L_p \geq 0$ represents the same model holding all the other parameter settings fixed. In the computer code, L_p is set to 1 when K_x is 0. See Gallant and Tauchen (1995).

To select the optimal SNP model, the following fitting strategy is adopted. We start with a simple Gaussian VAR process and gradually add lags to the location function until the marginal decrease in the objective value (s_n) is very small and some model selection criteria reach a minimum.[4] We then incrementally introduce ARCH by adding lags to the scale function, then non-Gaussian ARCH and finally general nonlinear processes by allowing the polynomial to modify the underlying Gaussian density. We select the models preferred by the selection criteria. The preferred models are then subjected to a battery of diagnostic tests for the goodness-of-fit to determine the final specification.[5]

3. The ex post real returns of discount bonds

In this section, we apply the SNP method and the nonlinear impulse response analysis to quarterly ex post real discount bond returns.[6] The analysis provides the benchmark that our economic model has to confront. We construct quarterly ex post real returns using the Risk Free Rates File compiled by the Center for Research in Security Prices (CRSP) and the consumer price index series extracted from the CITIBASE dataset. The quarterly series starts from the first quarter of 1947 and ends in the fourth quarter of 1993. The returns are unannualized gross returns.

The first row of Table 2 provides the descriptive statistics for the ex post real returns of quarterly discount bonds. The returns are skewed to the left (a negative skewness) and exhibit excessive kurtosis in comparison to the corresponding normal distribution with the same mean and variance. The first-order tocorrelation coefficient is 0.55 and statistically significant at 0.1% level. It is smaller than what has been found for the ex ante real returns (see Litterman and

[4] Specifically, three model selection criteria are calculated for each SNP fit: the Schwarz criterion ($s_n + (p_\theta/2n)\ln(n)$), the Hannan-Quinn criterion ($s_n + (p_\theta/n)\ln[\ln(n)]$), and the Akaike criterion ($s_n + (p_\theta/n)$).

[5] The residuals from each SNP model are examined for both short-term and long-term predictability of the mean (residual levels) and variance (squared residuals). For the long term tests, the residuals and their squares are projected onto annual dummy variables. For our dataset, annual dummy variables $D48, \ldots, D93$ are created such that each dummy variable takes a value of 1 if the observation belongs to that year and zero otherwise. For the short-term tests, the residuals and their squares are projected onto a space formed by the linear, quadratic, and cubic terms of lagged variables (3 lags were used). If an SNP model is the true density then the residuals should be orthogonal to the regressors. The null hypothesis that all regressor coefficients equal zero should not be rejected by an F-test and the R-squared equals zero. Thus, for a given SNP specification, the smaller the R-squared of the regressions, the better the SNP model approximates the true density. We refer to readers to Gallant et al. (1992) for the details on the diagnostic tests.

[6] Similar results are found for the ex post monthly real discount bond returns. The results are not reported here and are available upon request.

Table 2
Moments of returns and impact of changing risk aversion, income dispersion, and borrowing constraints

γ	σ_e^2	$\tilde{B}$	Mean	Std. Dev.	Skewness	Kurtosis	Corr.	Frequency
Data								
—	—	—	1.00228	0.00802	− 1.09434	3.7406	0.55375	—
Complete markets economy								
0.43	0.01115	0.1	1.01309	0.00143	0.00000	3.0000	0.30620	—
Changing the risk aversion coefficient (γ)								
0.43	0.01115	0.1	1.00226	0.00805	0.09906	1.1787	0.67398	0.5540
0.90	0.01115	0.1	0.99223	0.01655	0.12296	1.1834	0.67671	0.5753
1.20	0.01115	0.1	0.98510	0.02180	0.14044	1.1909	0.67849	0.5938
1.50	0.01115	0.1	0.97747	0.02690	0.15972	1.2012	0.67982	0.6039
2.00	0.01115	0.1	0.96383	0.03508	0.19499	1.2227	0.68041	0.6048
Changing income dispersion (σ_e^2)								
0.43	0.0080	0.1	1.00488	0.00672	− 0.02900	1.2657	0.71519	0.5312
0.43	0.0065	0.1	1.00626	0.00608	− 0.14666	1.3100	0.71454	0.5279
0.43	0.0030	0.1	1.00972	0.00401	− 0.48568	1.7943	0.71272	0.4082
0.43	0.0010	0.1	1.01197	0.00228	− 0.79159	2.9456	0.60771	0.2665
Changing the borrowing constraint ($\tilde{B}$)								
0.43	0.01115	0.15	1.00487	0.00730	− 0.24434	1.3045	0.74114	0.4725
0.43	0.01115	0.20	1.00660	0.00673	− 0.53349	1.5810	0.75802	0.4042
0.43	0.01115	0.25	1.00781	0.00623	− 0.77174	1.9577	0.76455	0.3409
0.43	0.01115	0.30	1.00870	0.00578	− 0.98436	2.4149	0.76995	0.2953
0.43	0.01115	0.50	1.01062	0.00454	− 1.65500	4.6657	0.76800	0.1896

Other parameter values are as follows: $\beta = 0.99$, $\eta_1 = 0.004784$, $\rho_1 = 0.3062$, $\sigma_E^2 = 0.0001041$, $\eta_2 = 0.09$, and $\rho_2 = 0.82$. 'Corr' is the first-order autocorrelation and 'Frequency' is the frequency of binding borrowing constraints.

Weiss, 1985; Den Haan, 1995; Den Haan and Spear, 1998). For the return series, the Dickey–Fuller test strongly rejects the null hypothesis of a unit root at the one percent level.[7] Similar results are also reported in Den Haan (1995) and Den Haan and Spear (1998) for ex ante real interest rates.

The Lagrange Multiplier Test for the ARCH proposed by Engle (1982) is conducted for the quarterly real returns. Specifically, we estimate the following model:

$$R_t = \rho_0 + \sum_{i=1}^{p} \rho_i R_{t-i} + u_t, \quad u_t \sim \mathrm{N}(0, h(u_{t-1}, \dots, u_{t-q})) \tag{16}$$

[7] The results are not reported and are available upon request.

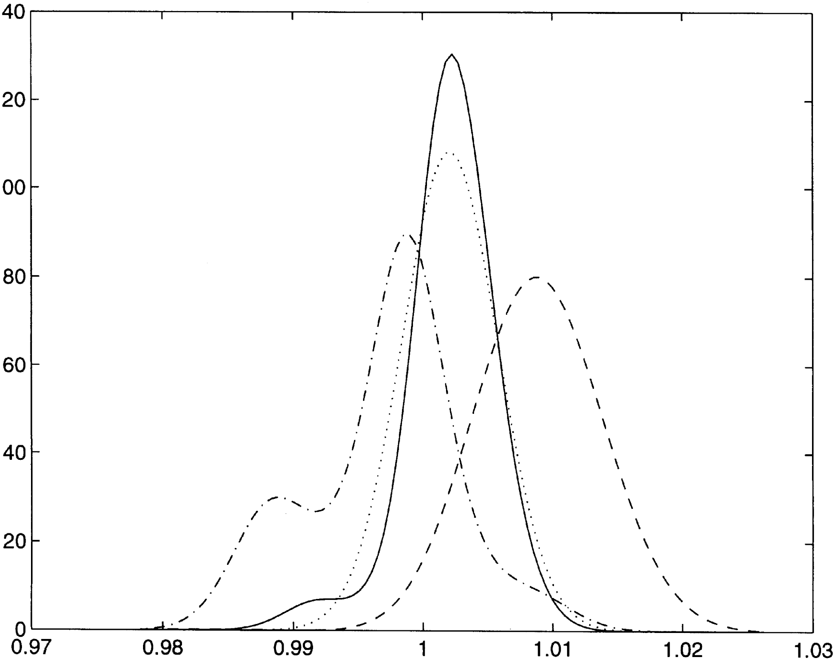

Fig. 1. Conditional probability density function for quarterly ex post real gross returns at three conditioning sets: (1) Solid line: the PDF at the conditioning set that all lagged returns are set at the unconditional mean; (2) broken line: the PDF at the conditioning set that all lagged returns are set at one standard deviation (unconditional) below the unconditional mean; and (3) dashed line: the PDF at the conditioning set that all lagged returns are set at one standard deviation (unconditional) above the unconditional mean. Dotted line: the normal PDF with the same conditional mean and conditional variance when all lagged returns are set at the unconditional mean.

where $h(u_{t-1},\dots,u_{t-q}) = \theta_0 + \sum_{j=1}^{q}\theta_j u_{t-j}^2$. Eq. (16) is estimated with a two-stage procedure. The number of lags in the first equation (p) is chosen to be three according to the Schwarz model selection criterion. Engle (1982) shows that the product of the number of observations and the R-square statistic from the second regression asymptotically has a chi-square distribution with q degrees of freedom under the null hypothesis that no ARCH is present. We test the null hypothesis of no ARCH at various lags ranging from one to three. The p-values for the tests are consistently less than 0.00001. We thus overwhelmingly reject the null hypothesis of no ARCH. This is consistent with the finding of Den Haan and Spear (1998) in which ex ante real interest rates are found to exhibit ARCH.

To perform the nonlinear impulse response analysis, we first estimate the conditional density function for the ex post real returns. Using the model selection procedure discussed above, we choose SNP(3,3,1,4,1) for our quarterly

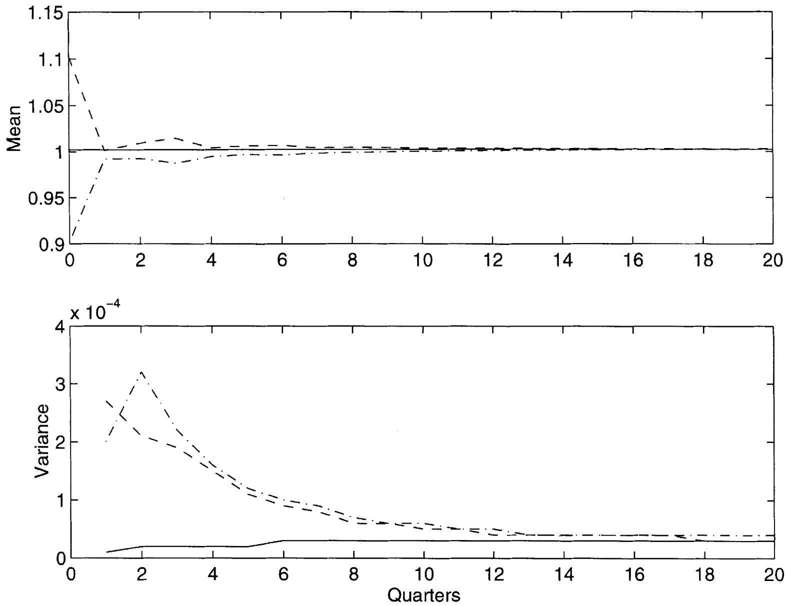

Fig. 2. Impulse responses of quarterly ex post real gross returns to a 10% return shock. Solid line: baseline response, dashed line: response to the positive shock, broken line: response to the negative shock.

real returns.[8] The SNP model is a general nonlinear process with three lags in both the location and scale functions and a polynomial of degree 4 in z_t and 1 in x_{t-1} with one lag. Fig. 1 shows the conditional density plots for quarterly ex post real returns at three conditioning sets: (1) all lagged returns are set at the unconditional mean; (2) all lagged returns are set at one standard deviation (unconditional) below the unconditional mean; and (3) all lagged returns are set at one standard deviation (unconditional) above the unconditional mean. For comparison, we also plot the normal density with the same conditional mean and conditional variance when all lagged returns are set at the unconditional mean. For the first conditioning set, the density is skewed to the left with excessive kurtosis in comparison to the normal. It also exhibits a fat tail on the left side. The conditional density for the second conditioning set shows qualitatively similar features to the first density plot except that the fat tail on the left side is more pronounced and becomes a hump. The density plot for the third conditioning set resembles a normal distribution with a symmetric bell shape.

Given the estimated density, we can perform nonlinear impulse response analysis to characterize the dynamic properties of the ex post real returns. The

[8] The estimation and diagnostic test results are not reported and are available upon request.

baseline initial condition is set at the unconditional mean of the data, $x^0 = \bar{x} = [1/(T - L + 1)]\sum_{t=L+1}^{T} x_t$, where $L = \max(L_\mu + L_r, L_p)$. The perturbed initial conditions are 10% of average gross returns:

$$x^{+} = \bar{x} + (0, \ldots, 10\%\bar{R})' \tag{17}$$

$$x^{-} = \bar{x} + (0, \ldots, -10\%\bar{R})' \tag{18}$$

where $\bar{R}$ is the unconditional mean of ex post real returns. The results are presented in Fig. 2.[9]

The plot shows that the impulse responses of the conditional mean are symmetric around the baseline conditional mean profile with no shocks and die out quickly. A number of empirical studies have reported that risk free rates are persistent. For instance, Den Haan (1995) finds that the nominal yields of three month treasury bill have an AR(1) coefficient around 0.9. Litterman and Weiss (1985) and Den Haan (1995) find that the ex ante real returns of three month treasury bill have an AR(1) coefficient of 0.77. Our results thus indicate that the ex post real returns can be less persistent than nominal returns and ex ante real returns.

In contrast, the impulse responses of the conditional volatility of the ex post real returns are very persistent. They increase sharply and die out very slowly regardless of the sign of shocks. This implies the existence of ARCH, consistent with our previous tests and the findings of Den Haan and Spear (1998). The impulse responses also show an asymmetric effect: a negative shock increases return volatility more than a positive shock of the same magnitude. In the next section, we formulate a heterogeneous agent general equilibrium model with incomplete markets and borrowing constraints, and explore to what extent the empirical results documented here and in previous research can be explained by our economic model.

4. The economic model

Assume that there are two types of infinitely lived agents and one perishable consumption good in the economy. The preferences of agent i are represented by the following utility function:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U^i(c_t^i)\right], \quad i = 1, 2 \tag{19}$$

where β is the discount factor, and c_t^i is the consumption of agent i at time t. $U^i(\cdot)$ is assumed to be continuous, concave, strictly increasing, and continuously differentiable.

[9] The number of simulations, N_r, is set at 20,000.

There is only one asset present in the financial market — a one period discount bond. Holding one unit of the bond at time t entitles the holder to a risk free claim to one unit of the consumption good at time $t+1$. Agents in the economy receive their income from two sources, endowment of labor income and payoffs to their asset holdings. The first source is exogenous and follows a stochastic process while the second source is endogenous due to the endogeneity of asset holdings. Agents are not permitted to trade contracts written on their uncertain future labor income. Since the market for contingent claims on future labor income is not available, the payoffs for the existing assets cannot span the whole state space. Therefore, the asset market is incomplete. Finally at time t, agent i's budget constraint is given by

$$c_t^i + p_t b_t^i \leq e_t^i + b_{t-1}^i \tag{20}$$

where b_t^i and e_t^i are agent i's bond holdings and endowment of labor income at time t, respectively, and p_t is the price of bond at time t.

Market incompleteness implies that agents will adjust their asset holdings to accommodate different realizations of exogenous states. Restrictions on asset holdings are therefore necessary to rule out default and/or a Ponzi scheme in equilibrium. Following convention, we impose a priori bound on an agent's asset holdings:[10]

$$b_t^i \geq B,\ B \leq 0 \text{ for } i = 1, 2. \tag{21}$$

Since closed-form solutions for the model are infeasible, we must resort to numerical methods. From the several numerical algorithms that are available to solve dynamic equilibrium models (see Judd, 1998), we choose the policy function iteration algorithm proposed in Coleman (1990).

To conduct the numerical simulation, we use the following constant relative risk aversion (CRRA) one-period utility function:

$$U^i(c_t^i) = \frac{c_t^{i\,1-\gamma} - 1}{1-\gamma} \quad \text{for } i = 1, 2. \tag{22}$$

The exogenous state variables in this case include the growth rate of aggregate endowment income and agent one's share of the endowment.[11] Denote by $E_t = e_t^1 + e_t^2$ the aggregate endowment income, and by $\tilde{e}_t^1$, agent one's share of the total endowment income. We use the following specification for the law of

[10] A sensible alternative to the ad hoc borrowing constraint used here is proposed in Zhang (1997a,b) which is based on the condition under which agents have no incentive to default.

[11] Since there are two types of agents, knowing the share of agent one's endowment income allows us to calculate the share of agent two's endowment income.

motion of the exogenous state variables:[12]

$$\ln\left(\frac{E_t}{E_{t-1}}\right) = \eta_1 + \rho_1 \ln\left(\frac{E_{t-1}}{E_{t-2}}\right) + \varepsilon_{1t}, \tag{23}$$

$$\tilde{e}_t^1 = \eta_2 + \rho_2 \tilde{e}_{t-1}^1 + \varepsilon_{2t}, \tag{24}$$

where ε_{1t} and ε_{2t} are independent normal random variables with mean zero and constant variance σ_E^2 and σ_e^2, respectively. The above specification combined with the CRRA utility function allows us to normalize variables such as consumption, individual endowment, and bond holdings to create a stationary model suitable for numerical analysis.

The law of motion for the growth rate of aggregate endowment income is calibrated to quarterly nonfinancial income series for the United States reported in the Survey of Current Business. The income data covers the period from the first quarter of 1959 to the fourth quarter of 1993 and includes wage and salary disbursements, other labor income, and nonfarm proprietors' income. The estimates for the constant (η_1) and the autoregressive coefficient (ρ_1) are 0.004784 and 0.3062, respectively. Both estimates are statistically significant at one percent level. The variance for the growth rate of aggregate endowment (σ_E^2) is 0.0001041. For the process of the individual share of endowment, we choose the constant (η_2) to be 0.09 and the quarterly first-order autoregressive coefficient (ρ_2) to be 0.82. This implies a unconditional mean of 0.5 and a monthly first-order autoregressive coefficient of 0.9 which are consistent with Den Haan and Spear (1998). We also choose the discount factor (β) to be 0.99 which is consistent with our quarterly model. We then experiment with various values for the risk aversion coefficient (γ ranges from 0.43 to 2.0), the variance for the error term of the individual share of endowment (σ_e^2 ranges from 0.001 to 0.01115), and the borrowing constraint as a fraction of total endowment, $\tilde{B} = B/E$ (ranging from 0.1 to 0.5). In our baseline model the parameter values are chosen such that simulated returns match quarterly ex post real returns of discount bonds in the mean and standard deviation. Alternative values for these three parameters (γ, σ_e^2, and $\tilde{B}$) are then used to explore the relationship between the properties of our economic model and the moments of returns simulated from our model.

We apply the Hermite–Gauss quadrature rule to discretize the exogenous processes. With each of the two state variables (growth rate of aggregate endowment and share of agent one's endowment) taking two possible outcomes, there are four possible states for the discretized exogenous stochastic process. The high endowment growth rate is 1.7% and the low is -0.33%. The mean endowment growth rate is 0.69% with standard deviation of 1.02% per quarter.

[12] The same specification is also used in Den Haan and Spear (1998).

For the baseline parameterization, the high individual share of endowment is 60.56% and the low is 39.44%. The mean individual share of endowment is 50% with standard deviation of 10.56%.

5. Simulated returns

In this section we first discuss the relationship between features of our model and the moments of the simulated returns.[13] For comparison, we also report the results for the complete markets economy for the baseline parameter values. We then examine the density functions and nonlinear dynamic properties of the simulated returns for various parameter settings in our economic model.

5.1. Moments of simulated returns and properties of the economic model

In Table 2, we present the moments of returns implied in a complete markets economy when the parameters are set at the baseline values. For the baseline parameterization, the complete markets economy implies a counterfactually high mean discount bond returns and low standard deviation in comparison to the data. Further, under the log normality assumption for the growth rate of aggregate endowment the net return is normally distributed. This leads to a skewness of zero and a kurtosis of three. The first-order autocorrelation is 0.3062 the same as that for the growth rate of aggregate endowment. It is thus lower than what is observed in the data.

We next report the moments of returns simulated from our economic model for various values of the risk aversion coefficient ranging for 0.43 to 2.0 (a risk aversion coefficient of zero means risk neutral). We have the following observations. First, the economic model can match the first two moments for certain parameterization (the baseline model).[14] However, the same parameterization fails to match higher-order moments such as skewness, kurtosis, and the first-order autocorrelation. While the observed quarterly ex post real bond returns have a distribution skewed to the left with excess kurtosis, the simulated quarterly bond returns for the baseline model show a slight skewness to the right with less kurtosis compared to a normal distribution with the same mean and variance. The simulated returns also have a first-order autocorrelation of 0.67.

Second, as agents become more risk averse, the mean gross return decreases while the volatility of returns (measured by standard deviation) increases.

[13] We calculate the moments using 10,000 simulated observations after discarding the first 5,000 simulated values to remove the possible transient effects.

[14] Because we attempt to match two moments using three free parameters, there are many parameter combinations that allow us to match the first two moments. The results reported in the paper are for one such parameterization. But the results are robust to alternative parameter choices.

Intuitively, as agents become more risk averse, they are more likely to resort to the financial market to buffer shocks to their endowment income. Given the fixed borrowing limit, agents thus face binding constraints more frequently (the frequency of binding constraints increases from 55.4% to 60.48%) when their risk aversion coefficient increases from 0.43 to 2.0. Because equilibrium bond prices go up when agents face binding constraints, the returns are lower when agents hit the borrowing limit. As a result, the mean returns are lower when agents are more risk averse. Further, because the change in returns is much larger as agent's status changes from nonbinding (binding) to binding (nonbinding) that remains unchanged, the return volatility will exhibit a hump shape as the frequency of binding constraints goes up: the volatility is higher when the frequency is in the middle than when it is either very low (the constraint rarely binds) or very high (the constraint almost always binds).[15] The increase in return volatility observed here thus implies that while the frequency of binding constraints gets higher as the risk aversion coefficient becomes larger, the level of the frequency is still too low to drive down the standard deviation.

Third, the higher-order moments, skewness, kurtosis, and the first-order autocorrelation, all increase as agents become more risk averse. The results can again be explained by agents facing binding constraints. As stated above, a high risk aversion is associated with more frequently binding borrowing constraints. Therefore, as agents become more risk averse, the frequency that the borrowing constraint is nonbinding decreases. Consequently, the outcome of high returns, which occurs when the borrowing constraint is nonbinding, behaves more like outliers in the right tail. This leads to a distribution skewed to the right (a positive skewness) with a higher kurtosis. Furthermore, when agents frequently face binding borrowing constraints, the returns are more persistent. This is because agents are more likely to face binding borrowing constraints next period if they face binding constraints this period, holding everything else fixed. Therefore, a low realized current return due to agents facing binding constraints is more likely to be followed by a low return as agents are more likely to face binding constraints next period. Consequently, as agents become more risk averse, the first-order autocorrelation increases. The change in the coefficient, however, is very small.

To investigate how the moments of simulated returns vary with income dispersion between agents, in Table 2 we report the moments of simulated returns of various levels of the variability of agent one's share of endowment income, ranging form 0.001 to 0.008. We make the following observations. First, as income dispersion shrinks (σ_e^2 decreases), the mean return increases and the standard deviation of returns decreases. Intuitively, as income dispersion becomes narrower, agents become more homogeneous. This reduces the benefit of

[15] This may not be true in an economy with more than two types of agents.

using the bond market for consumption smoothing. As a result, agents face binding borrowing constraints less frequently (the frequency of binding constraints decreases from 53.12% to 26.65%), which leads to lower bond prices, higher returns and lower volatility of returns. Second, as income dispersion shrinks, the distribution of returns becomes more skewed to the left with a higher kurtosis. The intuition is as follows. As income dispersion shrinks, the borrowing constraint becomes binding less frequently. The realization of low returns, which occurs when the borrowing constraint is binding, behaves more like outliers in the left tail. This leads to a distribution that is skewed to the left (a negative skewness) with a higher kurtosis. Third, the first-order autocorrelation also decreases as the income dispersion between agents decreases. This is attributed to the less frequently binding borrowing constraints and is consistent with the explanation provided above for varying risk aversion coefficients.

In Table 2 we also examine the impact of the restrictiveness of the borrowing constraint on the moments of simulated returns for various borrowing limits ranging from 15 to 50 percent of total endowment income. We have the following observations. Consistent with our previous results of changing income dispersion, the mean return increases and the standard deviation of returns decreases as agents are allowed to borrow more. The intuition is the same as above. As the borrowing limit increases, the constraint becomes less restrictive and agents thus face binding constraints less frequently (the frequency of binding constraints decreases from 47.25% to 18.96%). Further, as the borrowing constraint becomes less restrictive, the distribution of the simulated returns becomes more skewed to the left with a higher kurtosis. The intuition is the same as in the case discussed above when income dispersion shrinks. An interesting observation in this case is that the autocorrelation shows a slight hump shape as the borrowing limit increases. When the borrowing limit is set at a very high level, agents hit the bound very infrequently and the persistence in returns decreases. When the borrowing limit becomes tighter, agents hit binding constraints more frequently, and this generates more comovement in the simulated returns. However, the standard deviation of the returns also increases. As a result, the autocorrelation coefficient could become smaller.

5.2. Nonlinear dynamics of simulated returns

Our nonlinear dynamic analysis focuses on the simulated returns from two different parameter settings. The first set of parameter values are chosen to match the mean and standard deviation of the observed quarterly ex post real returns of discount bonds (the baseline model). We choose the second set of parameter values such that the simulated returns match the observed returns in skewness (all parameter values are the same as in the baseline model except for the following parameters: $\gamma = 0.33$, $\sigma_e^2 = 0.00351$, and $\tilde{B} = 0.19$). For each parameter setting, we simulate 50,000 observations from our economic model.[16] For the second parameterization, the mean gross return is 1.0108, the standard deviation is 0.0027, the kurtosis is 3.0945, the first-order autocorrelation is 0.7125, and the frequency of binding borrowing constraints is 25.84%.

We first apply the ARCH lagrange multiplier test to the simulated returns from both the baseline and the alternative models to see if our economic model can generate returns with the ARCH effect documented for the data. The number of lags for the first stage regression is chosen to be 7 for the baseline model and 2 for the alternative model, according to the Schwarz criterion. We then test the null hypothesis of no ARCH for various lags ranging from one to seven for both models in the second stage. The p-values of the tests are consistently below 0.00001. The null hypothesis of no ARCH effects is therefore overwhelmingly rejected for all the lags used in the second stage regression for both the baseline and the alternative models. The results are consistent with the findings on the ex post quarterly real returns of discount bonds reported in Section 3. Similar findings on the short-term interest rates are reported in Den Haan and Spear (1998) and are attributed to the combined effect of income dispersion and borrowing constraints.

Applying the estimation and model selection techniques discussed in Section 2 to the simulated returns for both parameterizations, we find that the preferred SNP model for the simulated returns of the baseline parameterization is SNP(7, 3, 1, 4, 3) and the preferred SNP model for the simulated returns of the alternative parameterization is SNP(2, 4, 1, 2, 2).[17] Both SNP models are general nonlinear processes. The first model has 7 lags in the location function, 3 lags in the scale function, and a polynomial of degree 4 in z_t and 3 in x_{t-1} with one lag. The second model has 2 lags in the location function, 4 lags in the scale function, and a polynomial of degree 2 in both z_t and x_{t-1} with one lag.

In Fig. 3 we plot the conditional density functions of the two SNP models along with a normal distribution with the same conditional mean and conditional variance as the simulated returns. We again plot each estimated density at three conditioning sets as we did for the data. We have the following observations. The estimated density for the baseline model (the top panel) at the first conditioning set shows a hump in the right tail. This is due to the fact that the simulated returns are relatively high when agents do not face binding constraints. Comparing with the density of quarterly ex post real returns, we see that more mass of the distribution lie below 1.0, implying counterfactually high percentile of negative net rate of returns. Given that the economic model matches the mean and standard deviation of the real returns, this finding shows that it can be misleading to evaluate an economic model by only examining the

[16] The first 5000 observations are discarded to remove possible transient effects.

[17] For the simulated returns, only the short-term tests are required.

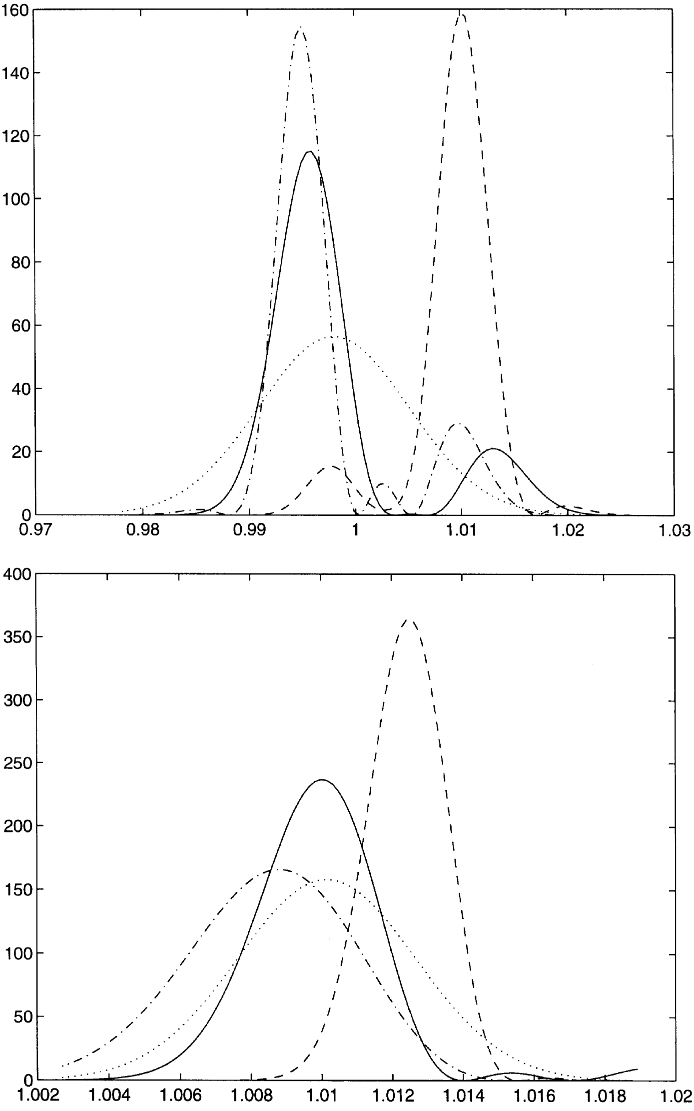

Fig. 3. Top panel: probability density function for the gross returns simulated from the baseline model. Bottom panel: probability density function for returns simulated from the alternative model. Solid line: the PDF at the conditioning set that all lagged returns are set at the unconditional mean; Broken line: the PDF at the conditioning set that all lagged returns are set at one standard deviation (unconditional) below the unconditional mean; dashed line: the PDF at the conditioning set that all lagged returns are set at one standard deviation (unconditional) above the unconditional mean; dotted line: the normal PDF with the same conditional mean and conditional variance when all the lagged returns are set at the unconditional mean.

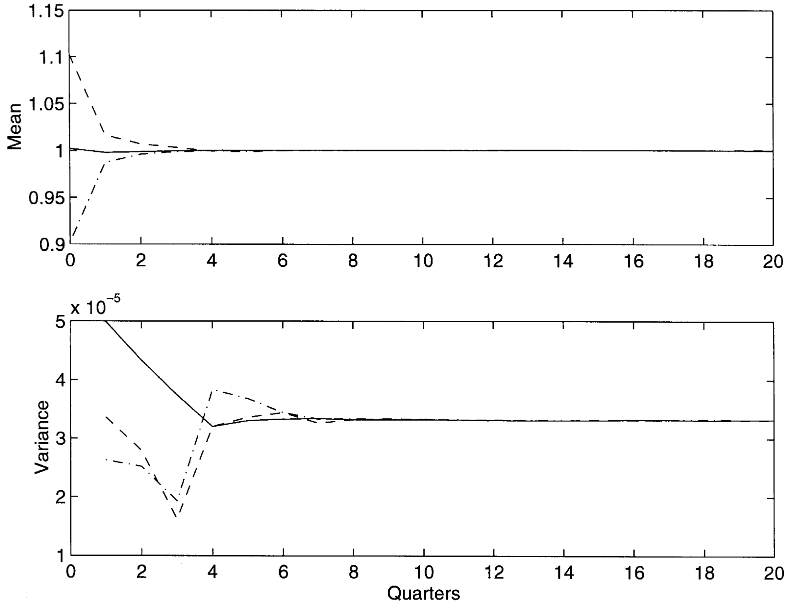

Fig. 4. Impulse responses of simulated gross returns in the baseline model to a 10% return shock. Solid: baseline, dashed: response to the positive shock, broken: response to the negative shock.

first two moments of returns. The density plot for the second conditioning set is very similar to the first except that it is more skewed to the left. The density plot at the third conditioning set, however, is qualitatively different. The hump on the right side is now much more pronounced than the one on the left indicating that borrowing constraint becomes binding very infrequently.

The estimated density for the alternative model (the bottom panel) shows that the net rate of returns are almost always positive, which is in contrast to the distribution of quarterly ex post real returns. The conditional density estimate at the first conditioning set also shows a hump in the right tail while the estimated density for the quarterly real returns exhibits a hump on the left. For either the second or the third conditioning set, the density resembles a normal with a symmetric bell shape.

Given the estimated conditional density functions for the simulated returns, we can perform the nonlinear impulse response analysis. We set the baseline initial condition at the unconditional mean of simulated returns. As we did for the ex post real returns, we also choose the perturbed initial conditions to be 10% of average simulated returns. We then compute the conditional mean and conditional variance profiles for the simulated returns for up to 20 quarters for all three initial conditions. The results are plotted in Figs. 4 and 5.[18] We have the

[18] The number of simulations, N_r, is set at 20,000.

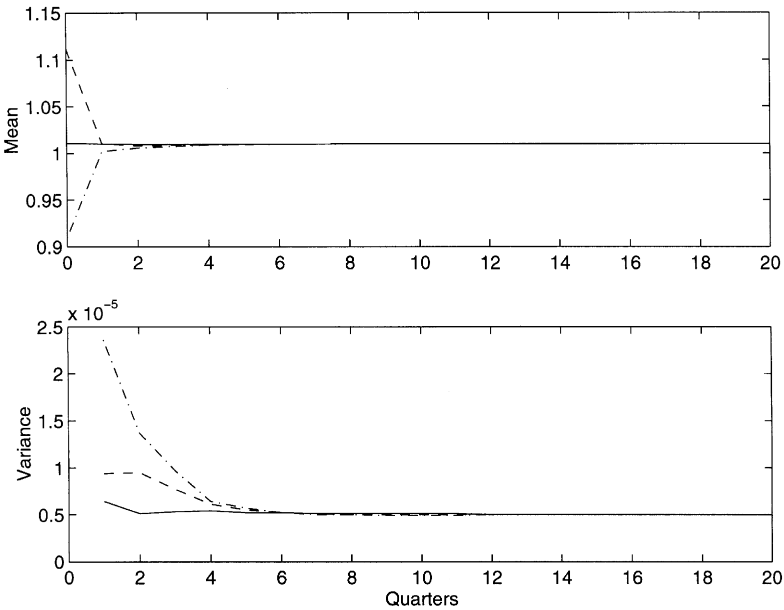

Fig. 5. Impulse responses of simulated gross returns in the alternative model to a 10% return shock. Solid line: baseline response, dashed line: response to the positive shock, broken line: response to the negative shock.

following observations. The conditional mean responses to return shocks are symmetric around the baseline conditional mean profile with no shocks and die out quickly for both returns simulated from the baseline and the alternative models. The conditional volatility responses, however, are more persistent for the simulated returns. The volatility responses also exhibit an asymmetric effect to return shocks. A negative shock increases the volatility more than a positive shock of the same magnitude. A possible explanation is the following. In our model a negative return shock is associated with agents moving from facing nonbinding constraints to binding constraints. Since agents are more likely to face binding constraints next quarter if they face binding constraints this quarter, the frequency of facing binding constraints is likely to be higher when the return shock is negative than when it is positive. As we have explained before, variance of returns increases when the frequency of binding constraints goes up as long as the level of the binding frequency is not too high. Thus, a negative return shock will more likely lead to higher return volatility than a positive return shock of the same magnitude.

While the volatility responses for the returns simulated from the alternative model resemble that of quarterly ex post real discount bond returns, the volatility responses for the returns simulated from the baseline model exhibit different features in comparison to the volatility responses of the data.

Specifically, the conditional volatility responses for the baseline model decrease initially and then start to increase and finally converge to the longterm conditional variance. The intuition is as follows. For the conditional variance profile with no shock, initially, the conditional mean profile lies below the long-term conditional mean return and is increasing (from quarter one to quarter four). Recall that in the baseline model a low return realization is associated with a binding borrowing constraint. The frequency of agents facing binding constraints therefore decreases as the conditional mean return increases. This leads to a decreasing conditional variance. For the conditional variance profiles with shocks, because initially the conditional mean returns are either far above (for a positive return shock) or far below (for a negative return shock) the long-term conditional mean return, the borrowing constraint is either rarely binding or almost always binding. The conditional variance are therefore very low. However, as the conditional mean profiles fall below the conditional mean with no shocks (which happens between the third and fourth quarters), the conditional variance start to increase because the frequency of agents facing binding borrowing constraints is higher than that in the baseline case with no shocks. The results thus imply that if one matches the first two moments one gets a poor match for the nonlinear impulse response functions for this class of economic models.

We also find that the impulse responses of the simulated returns in general are less persistent than that of their observed counterpart. The mean responses for the simulated returns die out in about 4 to 6 quarters while the mean responses for quarterly ex post real returns die out in about 6 to 8 quarters. The volatility responses for the simulated returns die out in about 10 to 12 quarters while the volatility responses for quarterly ex post real returns die out in about 18 quarters. One possible explanation for the finding is that the exogenous driving forces in the model are not persistent enough. In our analysis we use the AR(1) specification for both the aggregate endowment growth rate and the individual endowment share. Allowing more lags in the exogenous driving forces may increase the persistence in the mean and volatility responses of returns in the economic model.[19]

6. Concluding remarks

In this study we examine the higher-order moments and nonlinear dynamic properties of real discount bond returns in a heterogeneous agent exchange economy with incomplete markets and borrowing constraints. We find that the economic model can match the mean and standard deviation of quarterly ex post real discount bond returns for certain parameterization. However, the

[19] This also increases the dimension of state space which may complicate the numerical solution.

model fails to match the higher-order moments such as skewness, kurtosis, and the first-order autocorrelation for the same parameter values. To match the higher-order moments, one needs a model in which the borrowing constraint is binding at times but not too often. This illustrates the tension between matching the first two moments and matching the higher-order moments using this class of models.

Utilizing the SNP density estimation and the nonlinear impulse response analysis, we find that the economic model can mimic the asymmetric effect of return shocks on conditional volatility of returns as documented for quarterly ex post real returns. We also find that while the impulse responses of conditional volatility for the model matching the skewness resemble that of the data the impulse responses of conditional volatility for the model matching the first two moments exhibit different features. This shows that the class of models examined here may have difficulty replicating the dynamic properties observed in the data when the parameter values are chosen to match the first two moments of observed discount bond returns.

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