Economic Evaluation of Wind Farms Based on Cost of Energy Optimization

Jie Zhang* and Souma Chowdhury*
Rensselaer Polytechnic Institute, Troy, New York 12180

Achille Messac†
Syracuse University, Syracuse, NY, 13244

and

Luciano Castillo‡
Rensselaer Polytechnic Institute, Troy, New York 12180

A Response Surface Based Wind Farm Cost (RS-WFC) model is developed to evaluate the economics of wind farms. The RS-WFC model is developed using Extended Radial Basis Functions (E-RBF) for onshore wind farms in the U.S.. This model is then used to explore the influence of different design and economic parameters, including number of turbines, rotor diameter and labor cost, on the cost of a wind farm. The RS-WFC model is composed of three parts that estimate (i) the installation cost, (ii) the annual Operation and Maintenance (O&M) cost, and (iii) the total annual cost of a wind farm. The accuracy of the cost model is favorably established through comparison with pertinent commercial data. Moreover, the RS-WFC model is integrated with an analytical power generation model of a wind farm. A recently developed Unrestricted Wind Farm Layout Optimization (UWFLO) model is used to determine the power generated by a farm. The ratio of the total annual cost and the energy generated by the wind farm in one year (commonly known as the Cost of Energy, COE) is minimized in this paper. The results show that the COE could decrease significantly through layout optimization, to obtain millions of annual cost savings.

Keywords: Cost model, energy, Extended Radial Basis Functions, optimization, response surface, wind farm

Nomenclature

\[
\begin{align*}
N &= \text{Number of wind turbines in a wind farm} \\
P_0 &= \text{Rated power of a turbine (KW)} \\
C_t &= \text{Total annual cost per kilowatt installed ($/KW)} \\
n &= \text{Wind turbine lifetime (year)} \\
C_{in} &= \text{Installation cost per kilowatt installed ($/KW)} \\
C_{ma} &= \text{Cost of material ($)}
\end{align*}
\]

\[
\begin{align*}
C_{O&M} &= \text{Operation and maintenance cost per kilowatt installed ($/KW)} \\
D &= \text{Rotor diameter of a wind turbine (m)} \\
H &= \text{Hub height of a wind turbine (m)}
\end{align*}
\]

*Doctoral Student, Multidisciplinary Design and Optimization Laboratory, Department of Mechanical, Aerospace and Nuclear Engineering, AIAA student member
†Distinguished Professor and Department Chair. Department of Mechanical and Aerospace Engineering, AIAA Lifetime Fellow. Corresponding author. Email: messac@syr.edu
‡Associate Professor, Department of Mechanical Aerospace and Nuclear Engineering, AIAA member
I. Introduction

Wind energy is becoming an increasingly important source of renewable energy, particularly when we seek to reduce the emission of greenhouse gases and to mitigate the effects of global warming. Over the last decade, the global installed wind capacity has grown at an average rate of 28% per year.\(^1\)

In the recent years, the U.S. has emphasized the nation’s need for greater energy efficiency and for a more diversified energy portfolio. This has laid the path for a national effort to explore an energy scenario in which wind would provide 20% of U.S. electricity by 2030.\(^2\)

Efficient planning and resource management is the key to the success of an energy project. Wind is one of the most potent alternative energy resource; however the economics of wind energy is not yet universally favorable to place wind at a competitive platform with coal and natural gas (fossil fuels). Accurate (flexible to local market changes) cost models of wind projects, developed in this paper, would allow investors to better plan their projects, as well as provide valuable insight into the areas that require further development to improve the overall economics of wind energy.

A. Cost Analysis Review

Several models have been developed to estimate the cost for both onshore and offshore wind farms in Europe.\(^3\)–\(^6\) A Geographical Information System (GIS) based cost model for the estimation of the COE is presented in the publication of Structural and Economic Optimization of Bottom-Mounted Offshore Wind Energy Converters (OWECS) in 1998.\(^3\),\(^4\) The model allowed rapid evaluation of the economic viability of certain OWECS concepts over a large geographic area, and the identification of economically suitable sites for locating OWECS. The Energy Research Centre of the Netherlands (ECN) has developed a computer program named Offshore Wind Energy Costs and Potential (OWECOP). This program evaluates the COE for offshore wind energy, using a GIS database. A probabilistic analysis was implemented into the OWECOP cost model to form the OWECOP-Prob.\(^5\) An investigation of the transport and installation cost in offshore wind farm was carried out and implemented in the OWECOP II model of ECN.\(^7\)

Approximate analytical expression, which represents the cost of a wind farm as a function of various contributing factors, has also been explored in the literature. In Kaldellis and Gavras,\(^8\) a complete cost-benefit analysis model, adapted for the Greek market, was developed to evaluate the pay-back period and the economic efficiency for lifetime operation (10 to 20 years). The result showed that (i) the profitability is particularly sensitive to changes in the capital cost, the capacity factor, the electricity escalation rate, and the initial installation cost; (ii) the profitability is slightly less sensitive to changes in the O&M cost; and (iii) the impact of the turbine rated power and the inflation rate is limited. Kiranoudis et al.\(^9\) evaluated the parameters of the proposed short-cut wind efficiency model, using approximate mathematical expression to represent the installation cost and the annual O&M cost of a wind farm. This cost formulation was later used as the objective function by Sisbot et al.\(^10\) Genetic algorithm was employed to obtain the optimal layout of a wind farm by maximizing the power production capacity.
The Response Surface Based Wind Farm Cost (RS-WFC) model evaluates the cost of a wind farm as a function of various critical design and economic factors. The COE is determined from the RS-WFC model and subsequently minimized to obtain optimal wind farm layouts.

B. Development of RS-WFC Cost Model

The RS-WFC model, presented in this paper, is an extension of the E-RBF cost model introduced in our previous paper by Zhang et al.\textsuperscript{11} The preliminary version of this model\textsuperscript{11} has been accepted to be presented at the ASME 2010 IDETC conference in Montreal, Canada. A power generation model\textsuperscript{12} has been incorporated in this paper. The COE is evaluated by integrating the cost model and the power generation model. The basic steps of the RS-WFC model are summarized as follows:

- **STEP 1:** This step formulates the wind farm cost model using response surface method. Extended Radial Basis Functions (E-RBF) method is adopted to develop the RS-WFC model in this paper. But the RS-WFC model also has the flexibility to use other response surface methods. The RS-WFC model is composed of three parts that estimate (i) the installation cost, (ii) the annual Operation and Maintenance (O&M) cost, and (iii) the total annual cost of a wind farm. The inputs for the RS-WFC model include the following five parameters: (i) the rotor diameter of a wind turbine, (ii) the number of wind turbines in a wind farm, (iii) the cost of construction labor, (iv) the cost of management labor, and (v) the cost of technician labor. The response surface is trained using the data from the Energy Efficiency and Renewable Energy Program at the U.S. Department of Energy.\textsuperscript{13} The accuracy of the model is then illustrated through comparison with pertinent real world data.

- **STEP 2:** This step incorporates a power generation model with the RS-WFC model. The power generation model proposed in the Unrestricted Wind Farm Layout Optimization (UWFLO) technique, recently developed by Chowdhury et al.,\textsuperscript{12} is used in this paper. This technique uses a standard analytical wake model\textsuperscript{14} to determine the growth of wakes; and a standard velocity deficit model\textsuperscript{15} to determine the optimal wind farm layout.

- **STEP 3:** The COE is evaluated and optimized in this step. The COE, used in this model, is measured by Levelized Production Cost (LPC). LPC is defined as the total annual cost divided by the energy generated by a wind farm in one year.

Therefore, the major objective of this paper is to develop a comprehensive cost model of wind farms, and to fill the gap in existing economic models. This paper consists of five sections. In Section II and Section III, the new RS-WFC model, is developed and illustrated. Section IV presents briefly the power generation model implemented by Chowdhury et al.\textsuperscript{12} Section V presents the determination and optimization of the COE. The case of a wind farm with 25 turbines is provided in Section VI. Concluding remarks are given in the last section.

II. Cost Model

A. Cost Models Comparison

As we develop a more comprehensive wind farm cost model for the U.S. wind energy market, the benefits and drawbacks of the existing cost models are explored. Five representative cost models are presented in this section. They are (1) the short-cut model;\textsuperscript{9} (2) the cost model for the Greek market;\textsuperscript{8} (3) the OWECOP-Prob cost model;\textsuperscript{5} (4) the JEDI-wind cost model;\textsuperscript{13} and (5) the Opti-OWECS cost model.\textsuperscript{16} A detailed comparison of existing cost models can be found in the previous paper by Zhang et al.\textsuperscript{11}

The input and output structure for each model is shown in Fig. 1. The comparison results between the RS-WFC model and the existing models are given in Table 1. From Table 1, it can be seen that the RS-WFC model: (i) evaluates the Cost of Energy (COE), (ii) evaluates the power generation, (iii) includes life cycle cost, (iv) considers financial parameters, (v) uses appropriate input and output parameter, and (vi) provides analytical expression.
### Table 1. Cost models comparison

<table>
<thead>
<tr>
<th>Location</th>
<th>Short-cut model</th>
<th>Greek model</th>
<th>OWECOP-Prob model</th>
<th>JEDI-Wind model</th>
<th>Opti-OWECS model</th>
<th>RS-WFC model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>onshore</td>
<td>onshore</td>
<td>offshore</td>
<td>onshore</td>
<td>offshore</td>
<td>onshore</td>
</tr>
</tbody>
</table>

#### Input of the cost model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Short-cut model</th>
<th>Greek model</th>
<th>OWECOP-Prob model</th>
<th>JEDI-Wind model</th>
<th>Opti-OWECS model</th>
<th>RS-WFC model</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$P_0$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$D$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$U$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\delta$</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$n$</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Tax</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Financing</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

#### Cost of labor

- $\bar{P}_1$: Analytical model available
- $\bar{P}_2$: Considering life cycle cost
- $\bar{P}_3$: Considering tax
- $\bar{P}_4$: Appropriate input parameters
- $\bar{P}_5$: Appropriate output parameters
- $\bar{P}_6$: Financing parameters available
- $\bar{P}_7$: Considering rotor diameter effect on cost
- $\bar{P}_8$: Including cost of labor
- $\bar{P}_9$: Considering subsidy
- $\bar{P}_{10}$: No parameters difficult to determine
- $\bar{P}_{11}$: Evaluating the power generation
- $\bar{P}_{12}$: Evaluating the COE

#### Output of the cost model

| Total cost       | ✓               | ✓               | ✓                 | ✓               | ✓                | ✓            |
| Installation cost| ✓               |               | ✓                 | ✓               | ✓                | ✓            |
| O&M cost         | ✓               |               | ✓                 | ✓               | ✓                | ✓            |
| $P_{farm}$       | ✓               |               | ✓                 | ✓               | ✓                | ✓            |
| LPC              | ✓               |               |                  |                |                  | ✓            |

#### Pros and Cons

| Pros       | $\bar{P}_1, \bar{P}_{10}$ | $\bar{P}_1, \bar{P}_2, \bar{P}_9, \bar{P}_{10}$ | $\bar{P}_5$ | $\bar{P}_3, \bar{P}_6, \bar{P}_7, \bar{P}_8$ | $\bar{P}_2, \bar{P}_8$ | $\bar{P}_1, \bar{P}_2, \bar{P}_4, \bar{P}_5, \bar{P}_6, \bar{P}_7, \bar{P}_8, \bar{P}_{10}, \bar{P}_{11}, \bar{P}_{12}$ |
| Cons       | $\bar{C}_2, \bar{C}_3, \bar{C}_5$ | $\bar{C}_6, \bar{C}_7, \bar{C}_8$ | $\bar{C}_3, \bar{C}_5, \bar{C}_6$ | $\bar{C}_7, \bar{C}_8$ | $\bar{C}_11, \bar{C}_{12}$ | $\bar{C}_9, \bar{C}_{11}, \bar{C}_{12}$ |

- $\bar{P}_1$: Analytical model available
- $\bar{P}_2$: Considering life cycle cost
- $\bar{P}_3$: Considering tax
- $\bar{P}_4$: Appropriate input parameters
- $\bar{P}_5$: Appropriate output parameters
- $\bar{P}_6$: Financing parameters available
- $\bar{P}_7$: Considering rotor diameter effect on cost
- $\bar{P}_8$: Including cost of labor
- $\bar{P}_9$: Considering subsidy
- $\bar{P}_{10}$: No parameters difficult to determine
- $\bar{P}_{11}$: Evaluating the power generation
- $\bar{P}_{12}$: Evaluating the COE

- $\bar{C}_1$: Analytical model not available
- $\bar{C}_2$: No life cycle cost considering
- $\bar{C}_3$: No tax
- $\bar{C}_4$: Too many input parameters
- $\bar{C}_5$: Too less input parameters
- $\bar{C}_6$: Neglecting financing parameters
- $\bar{C}_7$: Cannot include the rotor diameter effect on cost
- $\bar{C}_8$: Neglecting cost of labor
- $\bar{C}_9$: No subsidy
- $\bar{C}_{10}$: Including input parameters difficult to determine
- $\bar{C}_{11}$: Cannot evaluate the power generation
- $\bar{C}_{12}$: Cannot evaluate the COE
B. Extended Radial Basis Function (E-RBF) Approach

Typical response surface methods include: (i) Quadratic Response Surface Methodology (QRSM), (ii) Radial Basis Functions (RBFs), (iii) Extended Radial Basis Functions (E-RBF), and (iv) Kriging. The E-RBF approach, which has been illustrated to be a robust response surface technique by Mullur and Messac,\textsuperscript{17,18} is adopted in this paper. The E-RBF approach uses a combination of radial and non-radial basis functions, which possesses the appealing properties of both types of basis functions: (i) the effectiveness of the multiquadric RBFs, and (ii) the flexibility of the N-RBFs.\textsuperscript{17}

C. RS-WFC Cost Model

The RS-WFC cost model is divided into three parts to estimate (i) the installation cost, $C_{in}$, (ii) the annual O&M cost, $C_{O&M}$, and (iii) the total annual cost of a wind farm, $C_t$. As discussed above, the number of coefficients of the RS-WFC cost model, $n_u = (3m + 1)n_p$, is based on the number of data points. Table 2 shows the number of coefficients for each part of the RS-WFC cost model. $C_{LC}$, $C_{LT}$, and $C_{LM}$ represent the wage per hour for construction labor, technician labor and management labor, respectively; $N$ is the number of turbines in a farm and $D$ is the rotor diameter. Thus, although the RS-WFC cost model presents a mathematical expression, the coefficients ($n_u$) have not been listed in this paper.

A brief description of the RS-WFC cost model of wind farms is given as follows. The detailed formulation of the RS-WFC cost model can be found in our previous paper presented at the ASME 2010 IDETC conference in Montreal, Canada.\textsuperscript{11}
1. Installation Cost

Installation costs consist of the following five parts: (i) the cost of wind turbines, (ii) the support structure cost, (iii) the equipment cost, (iv) the material cost, and (v) the construction labor cost. The installation cost is based on the national average cost adjusted for geographic differences in the construction labor cost. The installation cost model is developed using data from 40 different states of the U.S..

2. Annual O&M Cost

The input parameters for the model include three parts: (i) the number of wind turbines in a wind farm, (ii) the management labor cost, and (iii) the technician labor cost of all states in the U.S. (except the state of New York). The output of the model is the corresponding annual O&M cost.

3. Total Annual Cost

This subsection develops a model that estimates the total annual cost of a wind farm based on the number and the rotor diameter of wind turbines. The model can be expressed as (Table 2)

\[ C_t = f(N, D) \] (1)

The total annual cost data, \( C_t \), is obtained from the sum of annual installation cost and O&M cost.

\[ C_t = \frac{1}{n} C_{in} + C_{O&M} \] (2)

where \( n \) is the lifetime of a wind turbine. It is assumed that the installation cost is divided equally for each year during the lifetime of a wind turbine.

The rotor diameter, which is directly related to the rated power and the power generated by a wind farm, is a key parameter of a wind turbine. In order to investigate the effect of the rotor diameter on the cost of a wind farm, a survey of leading wind turbines has been done. The survey data is provided in our previous paper,\(^{11}\) which is used in the current research. We explore that wind turbines with different rotor diameters may have the same rated power. Here, for every specific rated power, an average rotor diameter value is selected, which is given in Table 3.

| \( D(m) \) | 49 | 55 | 59.2 | 65 | 80.5 | 82 |
| \( P_0(MW) \) | 0.60 | 0.85 | 1.00 | 1.25 | 1.50 | 1.65 |
| \( D(m) \) | 84.25 | 88 | 92.13 | 100 | 101 |
| \( P_0(MW) \) | 2.00 | 2.10 | 2.30 | 2.50 | 3.00 |

The input parameters to the total annual cost model are the number and the rotor diameter of wind turbines installed in a wind farm, and the output is the total annual cost of a wind farm. All the 101 sets of data points are obtained from the state of New York,\(^{13}\) and the selected sample data is shown in Table 4.

In the RS-WFC cost model, the wind turbine lifetime (\( n \)), the number of years financed (\( n_{fi} \)), the percentage financed (\( \theta \)), and the interest rate (\( \eta \)) are specified as 20 years, 10 years, 80%, and 10%, respectively.
Table 4. Sample data for developing total annual cost model

<table>
<thead>
<tr>
<th>$D(m)$</th>
<th>$P_0(MW)$</th>
<th>$N$</th>
<th>$C_t($$/KW$$)$</th>
<th>$D(m)$</th>
<th>$P_0(MW)$</th>
<th>$N$</th>
<th>$C_t($$/KW$$)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.85</td>
<td>10</td>
<td>132.01</td>
<td>80.5</td>
<td>1.5</td>
<td>10</td>
<td>131.48</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>20</td>
<td>132.01</td>
<td>80.5</td>
<td>1.5</td>
<td>20</td>
<td>131.26</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>30</td>
<td>131.63</td>
<td>80.5</td>
<td>1.5</td>
<td>30</td>
<td>129.65</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>40</td>
<td>131.05</td>
<td>80.5</td>
<td>1.5</td>
<td>40</td>
<td>128.85</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>50</td>
<td>130.49</td>
<td>80.5</td>
<td>1.5</td>
<td>50</td>
<td>127.93</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>70</td>
<td>129.41</td>
<td>80.5</td>
<td>1.5</td>
<td>70</td>
<td>126.46</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>80</td>
<td>129.14</td>
<td>80.5</td>
<td>1.5</td>
<td>80</td>
<td>126.41</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>90</td>
<td>128.36</td>
<td>80.5</td>
<td>1.5</td>
<td>90</td>
<td>126.36</td>
</tr>
<tr>
<td>55</td>
<td>0.85</td>
<td>100</td>
<td>127.82</td>
<td>80.5</td>
<td>1.5</td>
<td>100</td>
<td>126.39</td>
</tr>
</tbody>
</table>

In this section, the Extended Radial Basis Function (E-RBF) approach was presented, and a new RS-WFC cost model in the context of wind farm was developed using this approach. The validation of the model and the estimated results are discussed in the next section.

III. Cost Model Validation

In this section, we illustrate briefly the RS-WFC cost model and present the estimated installation cost, annual O&M cost and total annual cost of a wind farm.

A. Estimated Installation Cost

The installation cost model is developed using 40 data points, which is sufficient to ensure the accuracy of the model. Figure 2 shows the relation between the installation cost and the cost of construction labor. It can be seen that the installation cost increases with the cost of construction labor. The installation cost per kilowatt installed increases from $1,982/$KW to $2,040/$KW while the cost of construction labor changes from $10/h to $28/h, which is increased by 2.93%. The result suggests that the installation cost is not particularly sensitive to the cost of construction labor, since the labor cost is a relatively small percentage of the installation cost.

![Figure 2. Relation between the installation cost and the labor cost](image-url)
B. Estimated Annual O&M Cost

The RS-WFC annual O&M cost model is illustrated by estimating the O&M cost of a wind farm in the state of New York. The input parameters include three parts: (i) the number of wind turbines, (ii) the cost of technician labor, and (iii) the cost of management labor.

Figure 3(a) shows the relation between the annual O&M cost and the number of wind turbines. It can be observed that, when the number of wind turbines increases from 10 to 100, the annual O&M cost decreases sharply from $26.67/KW to $21.60/KW, approximately 19.01%. This manifests that the number of wind turbines exerts a great influence on the annual O&M cost of a wind farm. Roughly, the annual O&M cost increases one dollar (per kilowatt installed) for each 20 wind turbines. However, it can also be seen that when the number of wind turbines is small (less than 20), the change in the annual O&M cost is not clearly evident.

Figure 3(b) presents the relation between the annual O&M cost and the cost of management labor. As shown in Fig. 3(b), the annual O&M cost rises following the cost of management labor increases.

C. Estimated Total Annual Cost

The variation of the total annual cost, with the number and the rotor diameter of wind turbines, is discussed in this subsection. The error is evaluated by

\[ \text{Error} = \frac{|f_p - f_{ref}|}{f_{ref}} \]  

(3)

where \( f_p \) represents the estimated value and \( f_{ref} \) is the reference value. The reference installation costs are obtained from the Energy Efficiency and Renewable Energy Program at the U.S. Department of Energy.

The estimated result is illustrated through comparison with the commercial wind farm data in the state of New York region, which is shown in Table 5. It can be seen that the largest error is 0.60% for case 9 (the rotor diameter and the number of wind turbines are 100 and 30, respectively), and the estimated total annual costs are nearly equal to the reference values for the other eight cases.

Figure 4(a) presents the relation among the total annual cost, the number of wind turbines, and the rotor diameter of a wind turbine in a 3D surface. Figure 4(b) shows the relation between the total annual cost and the rotor diameter of a wind turbine. When the number of wind turbines is selected as 50, it can be seen from Fig. 4(b) that the total annual cost decreases from $131.3/KW to $126.4/KW (approximately 3.73%) when the rotor diameter of a wind turbine increases from 50m to 100m. It can also be observed that the total annual cost decreases slowly when the rotor diameter is less than 70m; then the total annual cost begins to decrease sharply when the rotor diameter changes from 70m to 85m. If the rotor diameter continues to increase beyond 85m, the change in the total annual cost is particularly limited. Figure 4(b) and Table 3 indicate that the use of small wind turbine is not generally cost effective.
Table 5. Total annual cost estimated by the RS-WFC cost model for New York state

<table>
<thead>
<tr>
<th>Case number</th>
<th>( D(m) )</th>
<th>( P_0(MW) )</th>
<th>( N )</th>
<th>( C_t/$KW ) Reference</th>
<th>( C_t/$KW ) RS-WFC</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
<td>0.60</td>
<td>50</td>
<td>131.32</td>
<td>131.60</td>
<td>0.21%</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>0.85</td>
<td>60</td>
<td>129.94</td>
<td>129.93</td>
<td>0.01%</td>
</tr>
<tr>
<td>3</td>
<td>59.2</td>
<td>1.00</td>
<td>40</td>
<td>130.66</td>
<td>130.53</td>
<td>0.10%</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>1.25</td>
<td>50</td>
<td>129.22</td>
<td>129.55</td>
<td>0.26%</td>
</tr>
<tr>
<td>5</td>
<td>80.5</td>
<td>1.50</td>
<td>60</td>
<td>126.98</td>
<td>126.67</td>
<td>0.24%</td>
</tr>
<tr>
<td>6</td>
<td>82</td>
<td>1.65</td>
<td>50</td>
<td>127.45</td>
<td>127.32</td>
<td>0.10%</td>
</tr>
<tr>
<td>7</td>
<td>84.25</td>
<td>2.00</td>
<td>40</td>
<td>127.61</td>
<td>127.90</td>
<td>0.23%</td>
</tr>
<tr>
<td>8</td>
<td>88</td>
<td>2.10</td>
<td>60</td>
<td>127.93</td>
<td>126.40</td>
<td>0.01%</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>2.50</td>
<td>30</td>
<td>127.93</td>
<td>127.16</td>
<td>0.60%</td>
</tr>
</tbody>
</table>

Figure 4(c) shows the relation between the total annual cost and the number of wind turbines. When the rotor diameter is 82m, the total annual cost decreases from $131.48/KW to $126.38/KW (approximately 3.88%) while the number of wind turbines increases from 10 to 100. It can also be observed that the total annual cost does not change significantly when the number of wind turbines increases beyond 60.

In this section, the RS-WFC cost model was illustrated and a number of parameters related to the cost of a wind farm were analyzed.

IV. Power Generation Model

Sorensen and Nielsen\textsuperscript{19} showed that the total power extracted by a wind farm is significantly less than the simple product of the power extracted by a standalone turbine and the number of wind turbines in the farm. This difference is attributed to the mutual shading effect of wind turbines.\textsuperscript{15} This calls for a power generation model that effectively accounts for the velocity deficits created by the wake effect. The power generated by a wind farm \((P_{farm})\), with \( N \) wind turbines, is evaluated as a sum of the power generated by each individual turbine, which is given by

\[
P_{farm} = \sum_{j=1}^{N} P_j
\]

A. The Unrestricted Wind Farm Layout Optimization (UWFLO) Power Generation Model

The UWFLO power generation model used in this paper has been developed by Chowdhury et al.\textsuperscript{12} A rectangular wind farm of given dimensions, consisting of \( N \) turbines, is considered in the model. Figure 5 shows the input and output structure of the UWFLO power generation model. \( C_p \) and \( a \) represent the power coefficient and the induction factor of a wind turbine, respectively.

B. Wake Model

The wake model, used in this paper, is adopted from Frandsen et al.\textsuperscript{14} This model employs the control volume concept that relates the thrust and the power coefficients to the velocity deficit. The growth of the wake behind any Turbine-\( j \) is given by

\[
D_{wake,j} = (1 + 2a\bar{s})D_j
\]

\[
\bar{s} = \frac{s}{D_j}
\]
(a) The total annual cost based on the number and the rotor diameter of wind turbines

(b) The total annual cost based on the rotor diameter of a wind turbine

(c) The total annual cost based on the number of wind turbines

Figure 4. Relation between the total annual cost and the input factors

Figure 5. Input and output structure of the UWFLO power generation model
where $D_{wake,j}$ is the diameter of the expanding wake at a distance $s$ behind Turbine-$j$. The parameter $\alpha$ is the wake spreading constant which is evaluated by

$$\alpha = \frac{0.5}{\ln \left( \frac{z_H}{z_0} \right)}$$  \hspace{1cm} (7)$$

where $z_H$ and $z_0$ are the average hub height of the turbines and the average surface roughness of the wind farm region, respectively. The wind velocity deficit in the wake is given by

$$U = \left[ 1 - \frac{2a}{(1 + 2\alpha s)^2} \right] U_j$$  \hspace{1cm} (8)$$

where $a$ is the induction factor, which can be derived from the coefficient of thrust. Induction factor is one of the design characteristics of a wind turbine rotor.

V. Optimizing Cost of Energy

A. Cost of Energy (COE)

COE is measured by Levelized Production Cost (LPC), which is given by

$$COE = \frac{C_t \times P_0 \times N \times 1000}{AEP_{net}}$$  \hspace{1cm} (9)$$

where $AEP_{net}$ is the net annual energy production ($KWh$). $AEP_{net}$ is evaluated as

$$AEP_{net} = 365 \times 24 \times P_{farm}$$  \hspace{1cm} (10)$$

which neglects the variation of the wind power plant.

B. Wind Farm Optimization Problem Formulation

The objective of optimization is to minimize the COE for a wind farm. The optimization problem is formulated as follows.

$$\min_{V} : \quad f = \frac{C_t \times P_0 \times N \times 1000}{AEP_{net}(V)}$$  \hspace{1cm} (11)$$

subject to

$$g_1(V) \leq 0$$  \hspace{1cm} (14)$$

$$g_2(V) \leq 0$$  \hspace{1cm} (15)$$

$$0 \leq x_i \leq x_{farm}$$  \hspace{1cm} (16)$$

$$0 \leq y_i \leq y_{farm}$$  \hspace{1cm} (17)$$

where $x_i$ and $y_i$ are the coordinates of the wind turbines in the farm. The inequality constraint $g_1$ is the minimum clearance required between any two adjacent turbines, which is given as

$$g_1(V) = \sum_{i=1}^{N} \sum_{\substack{j=1 \atop j \neq i}}^{N} \max \left( (D_i + D_j + \Delta_{min} - d_{ij}) , 0 \right)$$  \hspace{1cm} (18)$$

$$d_{ij} = \sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2}$$  \hspace{1cm} (19)$$

where $\Delta_{min}$ is the minimum clearance required between the outer edge of the rotors of two adjacent turbines. The inequality constraint $g_2$ ensures the location of wind turbines within the fixed size rectangular wind farm.
which is expressed as

\[ g_2(V) = \frac{1}{2N} \left\{ \frac{1}{x_{farm}} \sum_{i=1}^{N} \max(-x_i, x_i - x_{farm}, 0) + \frac{1}{y_{farm}} \sum_{i=1}^{N} \max(-y_i, y_i - y_{farm}, 0) \right\} \]  

(20)

where \( x_{farm} \) and \( y_{farm} \) represent the extent of the rectangular wind farm in the \( x \) and \( y \) directions, respectively. The number of design variables, in this optimization problem, is equal to \( 2N \).

VI. Case Study

A wind farm, consisting of a \( 5 \times 5 \) array of ENERCON E-82 wind turbines, is selected in this study. The parameters of the wind farm and the wind conditions are given in Table 6 and Table 7, respectively. The power coefficient \( (C_p) \) curve and the induction factor \( (\alpha) \) curve are illustrated in Fig. 6(a) and Fig. 6(b), respectively. In Fig. 6(a), the square dot represents the real power coefficient data provided by the wind turbine manufacturer,\(^{21}\) and the curved line represents the corresponding quartic curve fit (of the data). In Fig. 6(b), the square dot represents the real induction factor data provided by the wind turbine manufacturer;\(^{21}\) and the curved line also represents the corresponding quartic curve fit (of the data). It is assumed that the wind farm is developed in the State of New York.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of wind turbines, ( N )</td>
<td>25</td>
</tr>
<tr>
<td>Type of wind turbines</td>
<td>ENERCON E-82</td>
</tr>
<tr>
<td>Length of the wind farm, ( x_{farm} )</td>
<td>1200m</td>
</tr>
<tr>
<td>Breadth of the wind farm, ( y_{farm} )</td>
<td>800m</td>
</tr>
<tr>
<td>Rated power of each wind turbine, ( P_0 )</td>
<td>2MW</td>
</tr>
<tr>
<td>Rotor diameter of a wind turbine, ( D )</td>
<td>82m</td>
</tr>
<tr>
<td>Hub height, ( H )</td>
<td>85m</td>
</tr>
<tr>
<td>Cut-in wind speed</td>
<td>2m/s</td>
</tr>
<tr>
<td>Cut-out wind speed</td>
<td>28m/s</td>
</tr>
<tr>
<td>Rated wind speed</td>
<td>13m/s</td>
</tr>
<tr>
<td>Average surface roughness, ( z_0 )</td>
<td>0.7m</td>
</tr>
</tbody>
</table>

Table 6. Wind farm parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average wind speed</td>
<td>8.35m/s</td>
</tr>
<tr>
<td>Wind direction</td>
<td>0° with positive ( x )-axis</td>
</tr>
<tr>
<td>Air density</td>
<td>1.2kg/m³</td>
</tr>
</tbody>
</table>

Table 7. Wind conditions

A. Optimization Result

The total annual cost per kilowatt installed \( (C_t) \), evaluated using the RS-WFC cost model, is \( 129,997\$/KW \). The convergence history is shown in Fig. 7. It can be seen from Fig. 7(b) that the optimization converges after approximately 45000 function evaluations, during which the COE decreases by 5.405\%, from \( 0.0999\$/KWh \) to \( 0.0949\$/KWh \). Figure 7(a) shows that the power generated by the farm increases from \( 7.4245\text{MW} \) to
7.8117 MW, approximately 5.22%. Thus, the resulting annual cost savings for the wind farm is approximately 5.3% of the total annual cost, which is approximately $342,151.

VII. Conclusion

This paper developed a model to explore the economics of wind farms by integrating the cost and power generation. A Response Surface Based Wind Farm Cost (RS-WFC) model was developed, based on the commercial data. The RS-WFC model can estimate (i) the installation cost, (ii) the annual O&M cost, and (iii) the total annual cost of a wind farm. In addition, the RS-WFC model uses mathematical expression to estimate the cost of a wind farm, which can be used to investigate the influence of various cost factors. The resulting cost model can be a useful tool for wind farm planning. The UWFLO power generation model was combined with the RS-WFC model to evaluate the COE of a wind farm. This comprehensive cost model typically caters to the interests of investors, wind farm planning engineers, and policy makers.

VIII. Acknowledgements

Support from the National Found from Awards CMMI-0533330, and CMII-0946765 is gratefully acknowledged.
References


---

14 of 14

American Institute of Aeronautics and Astronautics