MATH 6390 (Fall 2013) Project 1
Due Wed Oct 2nd

(A) Do Problem 7.19 (page 41) from Chapter 7 of Moler. Use the `events` feature of MATLAB's ODE solver to determine the time t_f so that $y(t_f) = 0$. (See Section 7.10 of Moler. Make a MATLAB movie of the sprinter's jump in parts (c) and (d).

(B) This problem is motivated by material in **[OL, #20]**.

Implement and verify the correctness of the order 3 predictor-corrector (PECE3) algorithm with a constant step size based on the order 3 Adams-Bashforth and Adams-Moulton formulae given on page 14 of the lecture notes on ODEs. [The predictor-corrector algorithm is described on page 16 of the lecture notes for the case of the order 1 Euler and backward Euler methods.]

For the first two time steps use the fourth-order Runga-Kutta method, RK4. (see `http://en.wikipedia.org/wiki/RungeKutta_methods`)

Apply your code to the scalar ODE-IVP

$$\frac{dy}{dt} = y^2 - 5t, \tag{1}$$

$$y(0) = 1, \tag{2}$$

with a final time of $t = 1$. Implement your PECE method in a function with inputs and outputs given by

```
[TOUT,YOUT] = PECE3(ODEFUN,TSPAN,Y0,DELTATIME),
```
(3)

much as in `ode45`.

Next write a function that assesses the accuracy and efficiency of `PECE3` as follows.

1. Graphically compare the results you obtain to those obtained using `ode113` with relative and absolute tolerances of 10^{-13}.

2. Next verify that your implementation is globally third-order accurate by calculating the error

$$E = |y_{\texttt{PECE3}}(1) - y_{\texttt{ode113}}(1)| \tag{4}$$

as a function of the step-size, h, for $h = 10^{-1}$, 10^{-2}, $10^{-3}, \cdots, 10^{-K}$, for an appropriate choice of integer K.

   (a) Plot $\log_{10}(E)$ as a function of $\log_{10}(h)$ and estimate the slope of the line on your log-log plot to verify that the global error is $O(h^3)$.

(b) Using MATLAB's `tic` and `toc` functions, plot the computational time for `PECE3` as a function of h. (A log-log plot might be more useful.) and compare to the time for `ode113` with the tolerances given above.

(C) Implement an ODE solver that uses the midpoint and trapezoid methods discussed on pages 10 and 11 of the lecture notes to control the local error of the midpoint method, using a user-specified error tolerance, `tol`. We estimate the local error, d_n, of the n-th step of the midpoint method by

$$d_n \approx |y_n^{\texttt{mid}} - u_n^{\texttt{trap}}|, \tag{5}$$

where $y_n^{\texttt{mid}}$ is obtained from $y_{n-1}^{\texttt{mid}}$ using one step of the midpoint method and $u_n^{\texttt{trap}}$ is obtained from $y_{n-1}^{\texttt{mid}}$ using one step of the trapezoid method. Use the following step-size selection rules:

1. If $d_n < 0.5\texttt{tol}$ then accept the step and double the step size for the next step: $h_{n+1} = 2h_n$.
2. If $0.5\texttt{tol} \leq d_n < \texttt{tol}$ then accept the step and keep the current step size: $h_{n+1} = h_n$.
3. If $d_n \geq \texttt{tol}$ then reject the step. Try the step again with half the current step size: $h_n = 0.5h_n$.

Test your method by comparison to the flame ODE discussed in the Lecture ODE Solvers III.

$$\frac{dy}{dt} = y^2 - y^3, \tag{6}$$

$$y(0) = \delta \tag{7}$$

for $\delta = 10^{-1}$ and with a final time of $t_{\text{final}} = 2/\delta$, and for and $\delta = 10^{-3}$ with a final time of $t_{\text{final}} = 1.25/\delta$. Define the global error of the midpoint method to be

$$E = |y^{\texttt{mid}}(t_{\text{final}}) - y^{\texttt{exact}}(t_{\text{final}})| \tag{8}$$

where $y^{\texttt{exact}}$ is given in terms of the Lambert W function. Plot the global error versus `tol` over a suitably chosen range of tolerance values. (Once again use a log-log plot.) For the largest and smallest values of the tolerance, plot the step size, h_n, versus n. Comment.

Report

Use MATLAB's publish command to generate pdf files containing your code, results, and a discussion. **Submit your MATLAB .m files and .pdf files in a single tar file.**

Grading Scheme

40 points for each of (A), (B), (C): 15 points for your code, 15 points for results, 5 points for code comments and 5 points for discussion of results.

Total Points = 120.