Math 2415 Spring 2016
Friday Problem Session on 14.7

A guided example

Consider the function \( z = f(x, y) = x^2 + 3y^2 - 2xy - 3x \). The critical points of this function are the points in the \( xy \)-plane that satisfy the equations

\[
0 = \frac{\partial f}{\partial x} = 2x - 2y - 3 \tag{1}
\]

\[
0 = \frac{\partial f}{\partial y} = 6y - 2x. \tag{2}
\]

Equations (1) and (2) are the equations of a pair of lines in the \( xy \)-plane. The critical points are the points that lie on both of these lines (since both equations need to hold at a critical point). By sketching the two lines you can see that in this example there can only be one critical point of \( f \).

In general the critical point equations

\[
0 = \frac{\partial f}{\partial x}(x, y) \tag{3}
\]

\[
0 = \frac{\partial f}{\partial y}(x, y) \tag{4}
\]

are the equations of a pair of curves in the \( xy \)-plane. The critical points are the points that lie on both curves. Often you can sketch these two curves and use your sketch to determine how many critical points there are and roughly where they are located. Then you can use this geometric information to guide an algebraic calculation to solve Equations (3) and (4) and thereby find the precise location of the critical points. **Use this combined geometric-algebraic technique in Problems 2-6 below.**

Problems

1. If you didn’t do Active Learning Models Project #4: Limits last week, do it today.
2. 14.7.7
3. 14.7.9
4. 14.7.11
5. 14.7.13
6. 14.7.17
7. 14.7.19
8. 14.7.21
9. 14.7.35
10. 14.7.37

**Challenge Problem**

Active Learning Models Project #7: Max/Min/Saddle