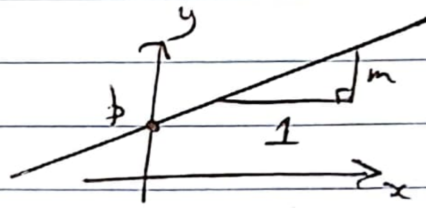
12.5A LINES AND PLANES

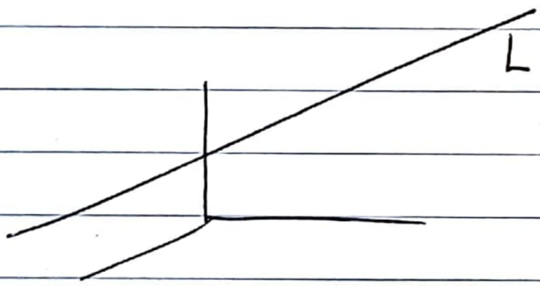
GOAL: Formulae for lines + planes in $\mathbb{R}^3$.

LINES

In $\mathbb{R}^2$ we have $y = mx + b$

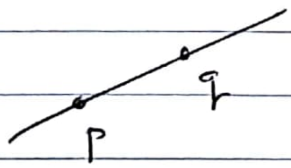


DISCUSSION QN: Can you make sense of concept of SLOPE of a line in space?



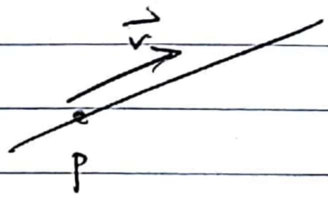
3 WAYS TO DESCRIBE A LINE IN $\mathbb{R}^3$

1. 2 POINTS

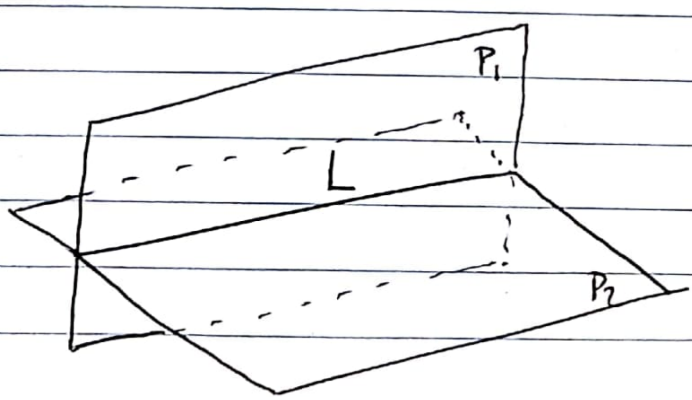


① + ② are related by $\vec{v} = q - p$

* 2. POINT + VECTOR

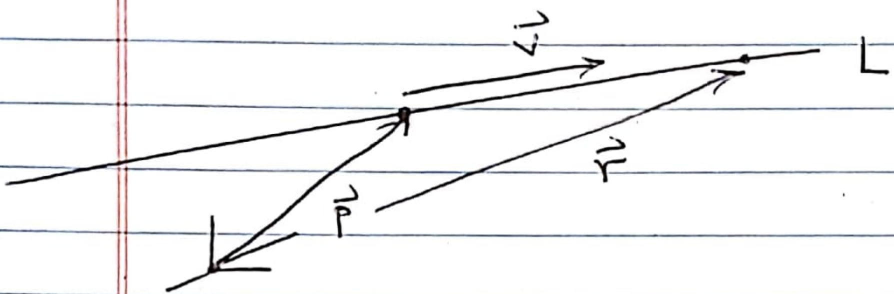


3. INTERSECTION OF 2 PLANES



VECTOR PARAMETRIZATION OF A LINE

LET L be the line thru point p (with position vector $\vec{p}$) that is in direction of a vector $\vec{v}$

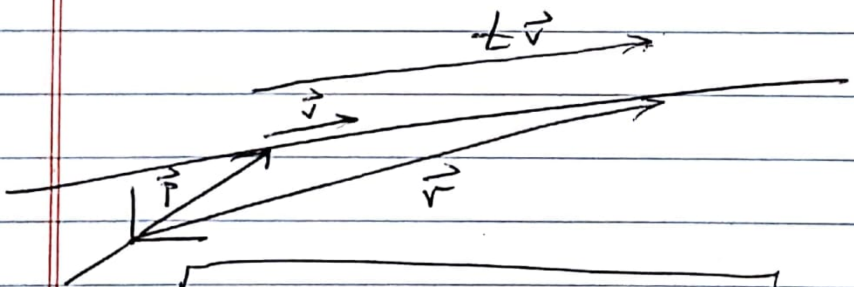


Let $\vec{r}$ be position vector of <u>ANY</u> point on L

<u>GOAL/Q</u> FIND FORMULA for $\vec{r}$ in terms of $\vec{p}$, $\vec{v}$.

<u>ANS</u> There is a real # t so that

$$\vec{r} = \vec{p} + t\vec{v}$$



WRITE $\boxed{\vec{r}(t) = \vec{p} + t\vec{v}}$ $t \in \mathbb{R}$

- This is the <u>Vector Parametrization</u> of L.

<u>THINK</u> t = TIME
$\vec{r}(t)$ = POSITION (on line) at time t

We call t a <u>parameter</u> along L.

EX $\vec{p} = (1,2,3)$, $\vec{v} = (4,6,9)$

$$\vec{r}(t) = \vec{p} + t\vec{v} = (1,2,3) + t(4,6,9)$$

$$= (1+4t,\ 2+6t,\ 3+9t)$$
$$= (x(t),\ y(t),\ z(t))$$

A SCALAR PARAMETRIZATION (or COORDINATE FORM) of L is the

$$x = 1+4t$$
$$y = 2+6t$$
$$z = 3+9t$$

NOTES

① PLUG in $t=1$ to get another pt on L :

$$\vec{q} = \vec{r}(1) = (5, 8, 12)$$

② Since L is also the line thru $\vec{q}$ in dirn $\vec{v}$
we get a second parametrization of same line :

$$\vec{r}_2(t) = \vec{q} + t\vec{v} = (5+4t,\ 8+6t,\ 12+9t)$$

③ Since L is also the line thru $\vec{p}$ in dirn $2\vec{v}$
we get a 3rd parn of same line :

$$\vec{r}_3(t) = \vec{p} + t(2\vec{v}) = (1+8t,\ 2+12t,\ 3+18t)$$

(4) **Q** How many parametrizations does L have?

A ∞ #: One for each choice of point $\vec{p}$ on L and ~~direction~~ vector $\vec{v}$ parallel to L.

PHYSICS INTERPRETATION

IF $\vec{r}(t) = \vec{p} + t\vec{v}$ = POSITION at time t along L

THEN

$$\vec{p} = \vec{r}(0) = \text{INITIAL POSITION}$$

$$\vec{v} = \text{VELOCITY (Constant)}$$

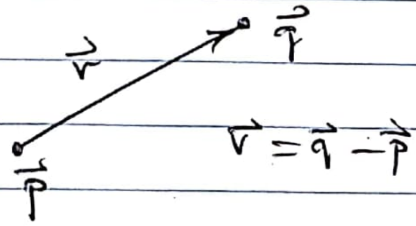
as

$$\text{VELOCITY} = \frac{\text{CHANGE IN POSITION}}{\text{CHANGE IN TIME}}$$

$$= \frac{\vec{r}(t) - \vec{r}(0)}{t - 0} = \frac{(\vec{p} + t\vec{v}) - \vec{p}}{t} = \vec{v}.$$

SPECIAL EXS

(A) LINE SEGMENT FROM $\vec{p}$ TO $\vec{q}$.



$$\vec{r}(t) = \vec{p} + t\vec{v}$$

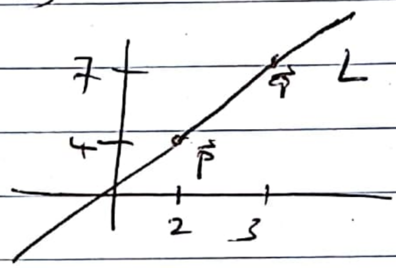
$$= \vec{p} + t(\vec{q} - \vec{p}) = (1-t)\vec{p} + t\vec{q} \quad \text{FOR } 0 \le t \le 1$$

as $\vec{r}(0) = \vec{p}$ and $\vec{r}(1) = \vec{q}$.

(B) CASE OF A LINE IN $\mathbb{R}^2$

EX $\vec{p} = (2,4)$, $\vec{q} = (3,7)$

$\vec{r}(t) = \vec{p} + t(\vec{q} - \vec{p}) = (2,4) + t((3,7) - (2,4))$
$= (2+t, 4+3t)$



$x = 2+t$
$y = 4+3t$

Q
CONVERT TO $y = mx + b$

HINT ELIMINATE t

A $t = x - 2$
So $y = 4+3t = 4+3(x-2) = 3x - 2$