14.7B ABSOLUTE MAX / MIN

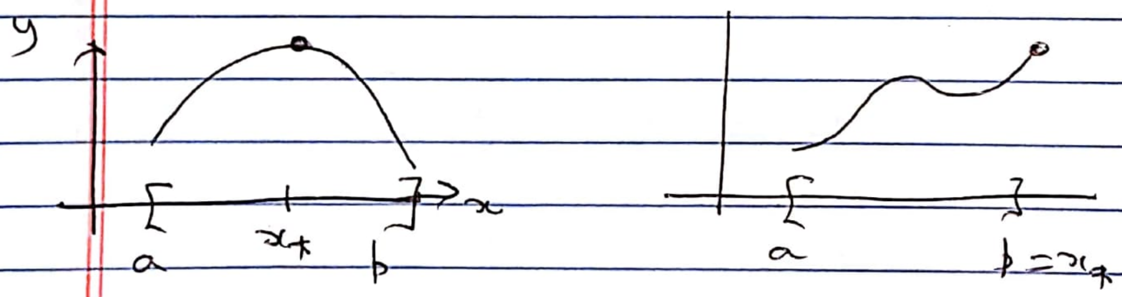
CALC I REVIEW

Suppose $y = f(x)$ is CONTINUOUS on a CLOSED and BOUNDED interval $[a,b]$.

Then f attains an absolute max value at some point $x_* \in [a,b]$.

i.e. $f(x_*) \geq f(x)$ for all $x \in [a,b]$

EX



MAX is either a CPT inside $[a,b]$ or an ENDPT.

STRATEGY TO FIND ABS MAX + MIN

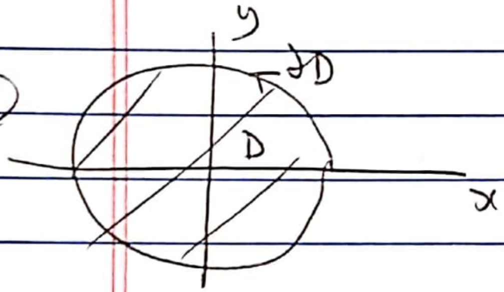
1. Find all CPTS of f in (a,b).
2. Calculate value of f at all pts in ① and at $x=a$ and $x=b$.
3. Take largest + smallest values of f from ②

CALC III $z = f(x,y)$.

Replace interval $[a,b]$ by a DOMAIN D in (x,y)-plane with boundary curve, ∂D.

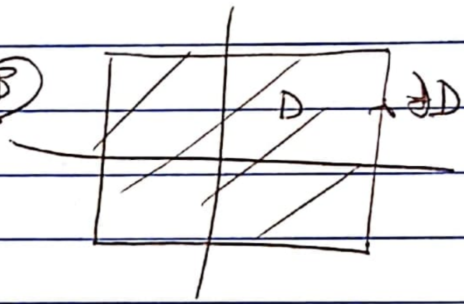
EX

(A)



$x^2+y^2 \leq 1$

(B)



$|x| \leq 1$ and $|y| \leq 1$

We need D to be

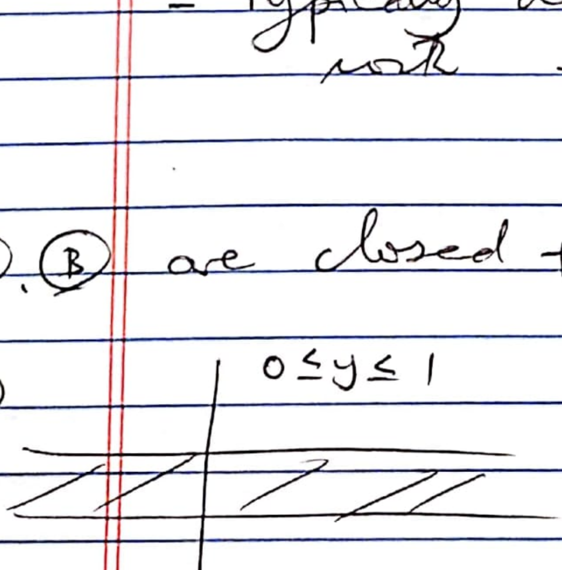
(a) BOUNDED. (subset of a disc of finite radius)

(b) CLOSED (contains its boundary)
- Typically defined by inequalities with $\leq$ rather than $<$

EX

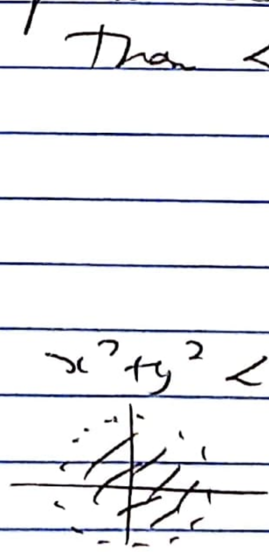
(A), (B) are closed + bounded

(C) $0 \leq y \leq 1$



NOT BOUNDED

(D) $x^2+y^2 < 1$



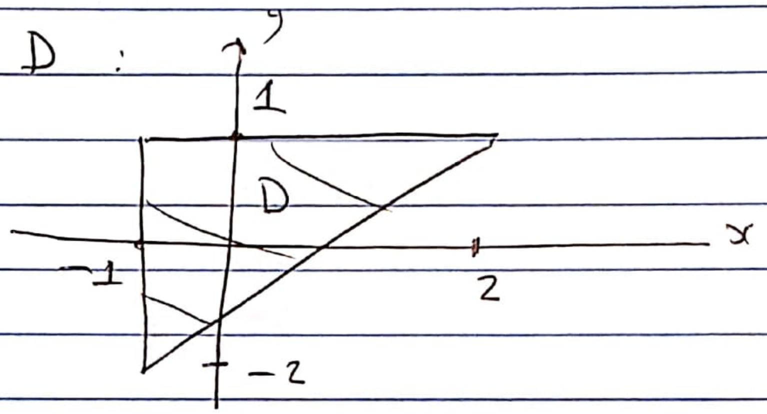
NOT CLOSED

EXS

1 [EASTER EGG IN TOBLERONE BOX 3D MODEL]

Find ABS MAX + MIN of $z = f(x,y) = x^2 + 2xy + 3y^2$

on domain D:



0 Since f is CTS on closed + bounded domain f has ABS MAX + MIN.

1 Find Values of f at all CPTS inside D.

$0 = \nabla f = (2x + 2y, 2x + 6y) = (0, 0)$

at $2x + 2y = 0$
$2x + 6y = 0$

SUBTRACT $4y = 0$

So $(x,y) = (0,0)$ is only CPT.

$(0,0)$ is in D ✓

$\boxed{f(0,0) = 0.}$ (A)

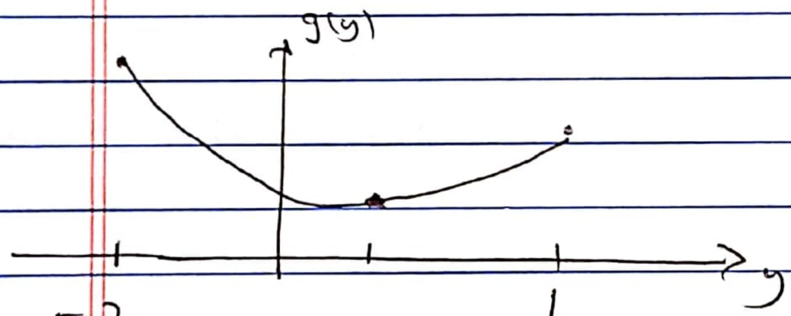
2. Find ABS MAX + MIN of f on ∂D.

∂D consists of 3 lines.
This gives us 3 CALC I ABS max/min problems.

(a) $\boxed{x = -1}$

$$g(y) = f(-1, y) = 1 - 2y + 3y^2 \quad \text{on } [-2, 1]$$

$$0 = g'(y) = -2 + 6y \Rightarrow y = 1/3$$

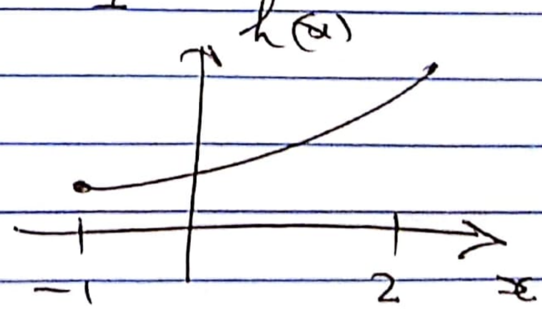


$$\boxed{\begin{array}{lll} f(-1, 1/3) = 2/3 & & \text{(B)} \\ f(-1, -2) = 17 & & \text{(C)} \\ f(-1, 1) = 2 & & \text{(D)} \end{array}}$$

(b) $\boxed{y = 1}$ $\quad h(x) = f(x, 1) = x^2 + 2x + 3$ on $[-1, 2]$

$$0 = h'(x) = 2x + 2 \Rightarrow x = -1$$

$$\boxed{\begin{array}{lll} f(-1, 1) = 2 & \text{(D)} & \text{again} \\ f(2, 1) = 11 & \text{(E)} & \end{array}}$$



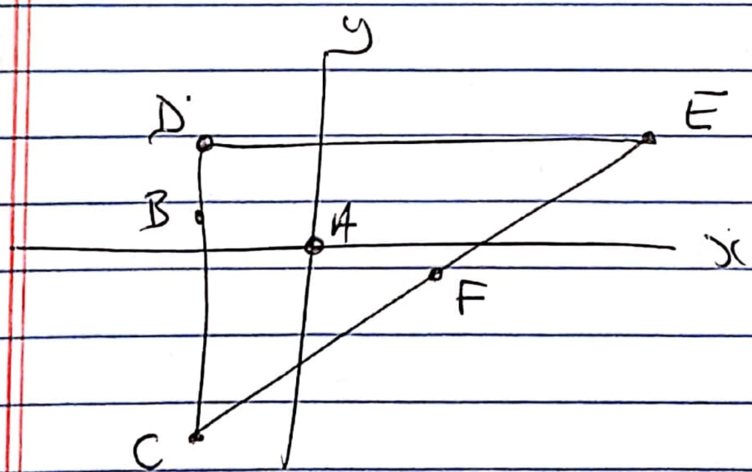
c $\boxed{y = x - 1}$

$$k(x) = h(x, x-1)$$
$$= x^2 + 2x(x-1) + 3(x-1)^2$$
$$= 6x^2 - 8x + 3 \qquad \text{on } [-1, 2]$$

$$0 = k'(x) = 12x - 8 \quad @ \quad x = 2/3.$$

$$\boxed{f(2/3, -1/3) = 1/3 \quad F}$$

Endpts taken care of above : C and E



ABS MIN is 0 @ (0, 0) A

ABS MAX is 17 at (−1, −2) C

3. $z = f(x,y) = y^2 - x^2$ on $x^2+y^2 \le 1$

(a) CPTS IN D

$$0 = \nabla f = (2x, -2y) = (0,0) \text{ at}$$

$(x,y) = (0,0)$ in D ✓

$$\boxed{f(0,0) = 0.}$$

(b) ABS MAX/MIN on ∂D

This time Parametrize Circle

$$\vec{r}(t) = (\cos t, \sin t) \qquad 0 \le t \le 2\pi$$

Restriction of f to ∂D is

$$g(t) = f(\vec{r}(t)) = \sin^2 t - \cos^2 t$$
$$= -\cos 2t$$

$$0 = g'(t) = 2\sin 2t \quad @\ t = 0, \pi/2, \pi, 3\pi/2$$

t	x	y	$f(x,y)$	
*	0	0	0	
0	1	0	-1	MIN
$\pi/2$	0	1	1	MAX
π	-1	0	-1	MIN
$3\pi/2$	0	-1	1	MAX