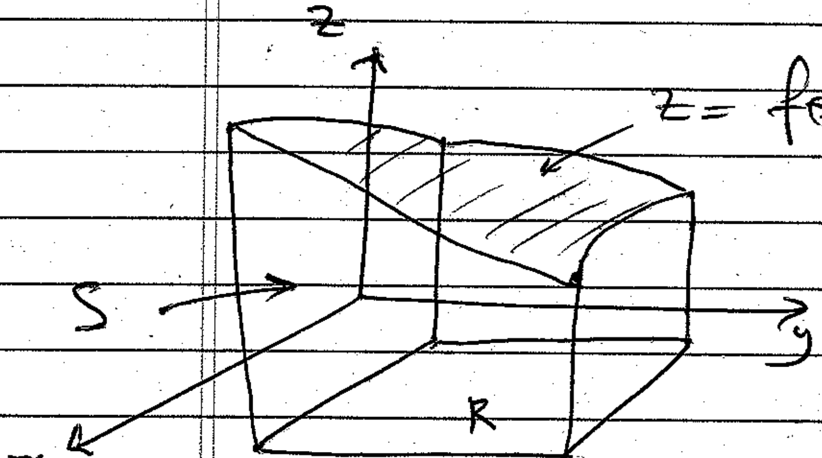
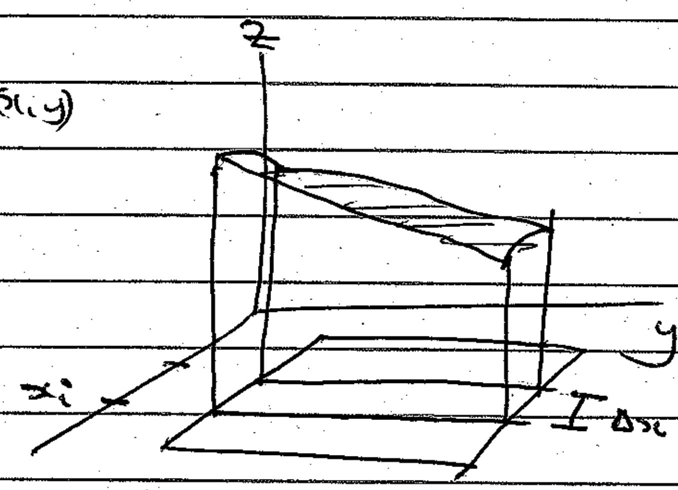
## 15.2 ITERATED INTEGRALS

We compute $\iint_R f\,dA$ by converting it to a pair of nested single variable integrals, each of which we do using FTC.

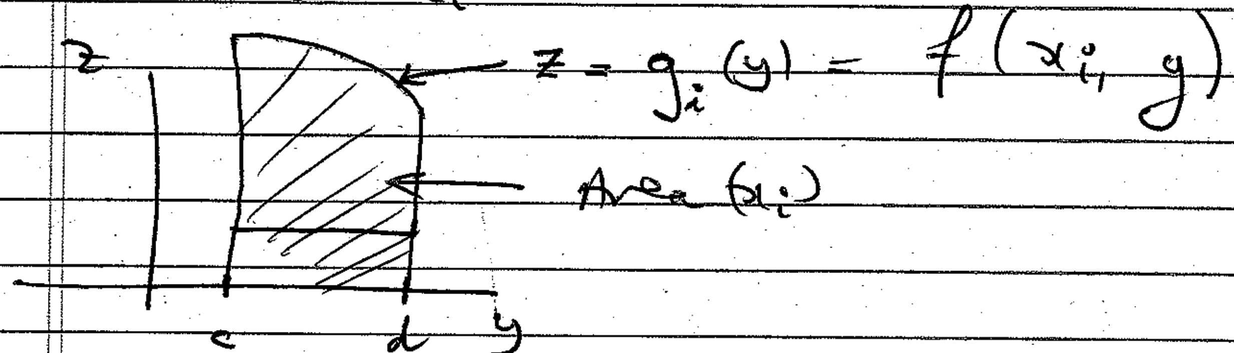
IDEA Vol of Loaf of Bread = Sum of Vols of slices.




ONE SLICE
AT $x = x_i$

FRONT FACE
OF SLICE $x = x_i$



②

So
$$\iint_R f\,dA = \lim_{N_x \to \infty} \sum_{i=1}^{N_x} \begin{pmatrix} \text{Area under } g_i(y) \\ \text{from } y=c \text{ to } y=d \end{pmatrix} \Delta x$$

$$= \lim_{N_x \to \infty} \sum_{i=1}^{N_x} \text{Area}(x_i)\,\Delta x = \int_{x=a}^{x=b} \text{Area}(x)\,dx$$

and
$$\text{Area}(x_i) = \int_{y=c}^{y=d} g_i(y)\,dy = \int_{y=c}^{y=d} f(x_i, y)\,dy$$

FUBINI'S THM

So
$$\boxed{\begin{aligned} \iint_R f\,dA &= \int_{x=a}^{x=b} \left[ \int_{y=c}^{y=d} f(x,y)\,dy \right] dx \\ &\overset{\text{SLICED}}{\underset{\text{OTHER WAY}}{=}} \int_{y=c}^{y=d} \left[ \int_{x=a}^{x=b} f(x,y)\,dx \right] dy \end{aligned}}$$

ITERATED INTEGRAL

THINK! $x$ CONST.

3

EX

1. $$\iint_{[0,1]\times[0,1]} (3x^2+4y^2)\,dA = \int_{x=0}^{x=1}\left[\int_{y=0}^{y=1} (3x^2+4y^2)\,dy\right] dx$$

$$= \int_{x=0}^{x=1} \left[3x^2y + \frac{4}{3}y^3\right]_{y=0}^{y=1} dx$$ PARTIAL ANTIDERIVATIVE WRT $y$

$$= \int_{x=0}^{x=1} \left[\left(3x^2\,1 + \frac{4}{3}\,1^3\right) - 0\right] dx$$

$$= \int_0^1 \underbrace{\boxed{3x^2+\frac{4}{3}}}_{\text{Area }(x)}\,dx = 7/3 = 2\tfrac{1}{3}$$

2. $$\iint_{[0,1]\times[0,2]} (x^2y + 3xy^2)\,dA$$

$$= \int_{y=0}^{y=2}\left[\int_{x=0}^{x=1} x^2y + 3xy^2\,dx\right] dy$$

$$= \int_{y=0}^{y=2} \left[\frac{1}{3}x^3y + \frac{3}{2}x^2y^2\right]_{x=0}^{x=1} dy$$

$$= \int_{y=0}^{y=2} \left(\frac{1}{3}y + \frac{3}{2}y^2\right) dy = 4\tfrac{2}{3}$$

OR DO OTHER WAY...

## (3) REVERSING ORDER OF INTEGRATION

$$\int_{y=0}^{y=1} \int_{x=0}^{x=1} x e^{xy} \, dx \, dy$$

Easier to do y-antider than x-antider. So SWITCH ORDER

$$= \int_{x=0}^{x=1} \int_{y=0}^{y=1} x e^{xy} \, dy \, dx$$

$$= \int_{x=0}^{x=1} \left[ e^{xy} \right]_{y=0}^{y=1} dx$$

$$= \int_{x=0}^{x=1} (e^{x} - 1) \, dx = e - 2$$